The Area Function

By Steve Ouellette

Time required
45 minutes

ACTIVITIES eXCHANGE **ID: 8298**

Activity Overview

In this activity, students will look at the graph of $g(x)=\int_a^x f(t)\,dt$ *with the goal of understanding Part 1 of the Fundamental Theorem of Calculus. Specifically, students will see that g(x) is related to the area under the graph of f(t) from a to x. They will also notice that the graph of g(x) is the antiderivative of f(t), thereby providing graphical support for Part 1 of the Fundamental Theorem of Calculus.*

Concepts

- *The Fundamental Theorem of Calculus, Part 1*
- *Area under a curve as a function of x*

Teacher Preparation

Students should already be familiar with Part 2 of the Fundamental Theorem of Calculus, often referred to as the Evaluation Theorem. This activity should accompany the introduction of Part 1 of the Fundamental Theorem of Calculus, which students often have difficulty understanding. This activity will help provide strong visual support for this theorem by relating g(x) to an "area function" as well as the antiderivative of f(t).

- *The screenshots on pages 74 and 75 demonstrate expected student results. Refer to the screenshots on page 76 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "8298" in the quick search box.***

Classroom Management

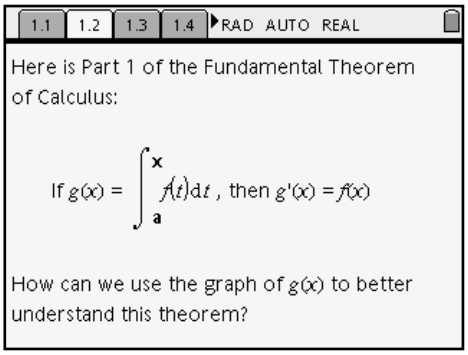
- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. The student worksheet is intended to guide students through the main ideas of the activity and provide a place to record their observations.*
- *Students will manipulate premade sketches, rather than create their own constructions. Therefore, a basic working knowledge of the TI-Nspire handheld is sufficient.*
- *After each trace is completed, students should advance to the Lists & Spreadsheet page, position the cursor in the formula cell (gray) for Column A, and press enter twice to clear the data. Students should then repeat this process for Column B. If these steps are not followed, there will be too much data, causing the device to run slowly.*
- *When students are tracing out g(x), it is recommended that they move the cursor slowly from the left side of the screen to the right side of the screen only once. This will also limit the amount of data collected in the Lists & Spreadsheet application.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the activity is completed successfully.*

TI-Nspire™ Applications

Graphs & Geometry, Lists & Spreadsheet, Notes

Two focus questions define this activity:

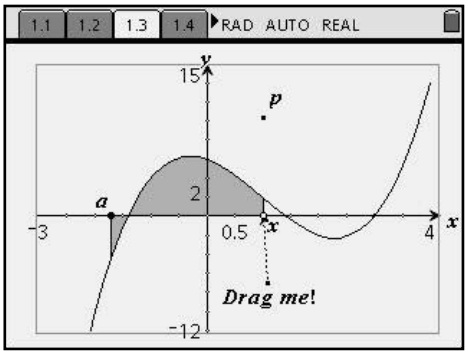
- What does Part 1 of the Fundamental Theorem of Calculus *mean*?
- How is the graph of $g(x) = \int_a^x f(t)\,dt$ related to the graph of $f(t)$?



Some discussion should follow the posing of these questions. You may also choose to evaluate $\int_a^x f(t)\,dt$ using the Evaluation Theorem for a given function $f(t)$ (such as $f(t) = t^2$) and then take the derivative of this result. This will demonstrate that $g'(x) = f(x)$. You may also want to tell students to think of the derivative of g as $f(t)$ evaluated at the upper limit of integration, x.

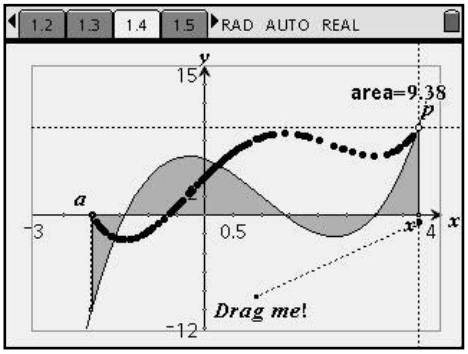
Problem 1 – Investigating the graph of *g*(*x*)

Step 1: Tell students that they will work cooperatively by starting with a study of the graph shown on page 1.3. Students will be asked to explain the relationship among the points p, x, and a. They should notice that p has x-coordinates equal to x and y-coordinates equal to the area under $f(t)$ from a to x. Students should also recognize that p increases when $f(t)$ is positive and decreases when $f(t)$ is negative.



Step 2: Next, students will advance to page 1.4 and drag point x, which will trace out $g(x)$ as shown. Students will see more clearly that $g(x)$ increases when $f(t)$ is positive and decreases when $f(t)$ is negative. Students will also see that the local minimum and maximum values of $g(x)$ correspond to the x-intercepts of $f(t)$.

Make sure that students see $g(x)$ as an "area function" as well as the antiderivative of $f(t)$. This second observation ties in directly with Part 1 of the Fundamental Theorem of Calculus.



Problem 2 – Investigate another area function

Students will use their knowledge from Problem 1 to answer the questions posed in this problem before dragging point *x*. They should obtain the following answers:

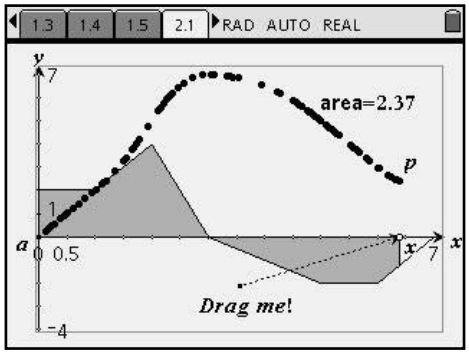
a) $g(0) = 0$, $g(1) = 2$, $g(2) = 5$, $g(3) = 7$, $g(6) = 3$

b) g is increasing on the interval (0, 3)

c) g is decreasing on the interval (3, 7)

d) g has a local maximum value at $x = 3$

e) the graph of g is shown to the right



Problem 3 – Another look at the function from Problem 1

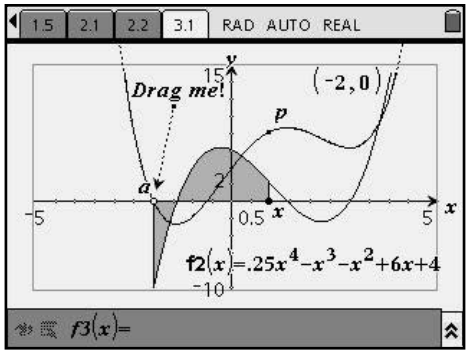
Step 1: Students will use the Evaluation Theorem as shown, with a lower limit of $a = -2$ to find a formula for g defined by the following:

$$g(x) = \int_{-2}^{x} \left(t^3 - 3t^2 - 2t + 6\right) dt$$

$$\int_{-2}^{x} \left(t^3 - 3t^2 - 2t + 6\right) dt = \left[\frac{1}{4}t^4 - t^3 - t^2 + 6t\right]_{-2}^{x}$$

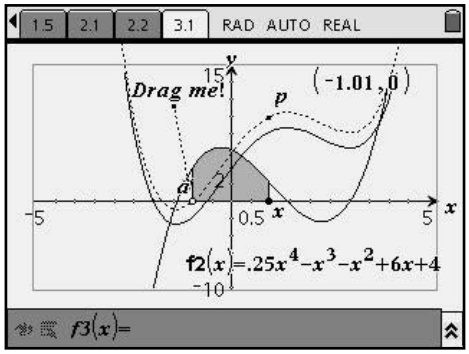
$$= \frac{1}{4}x^4 - x^3 - x^2 + 6x - \left(-4\right)$$

$$= \frac{1}{4}x^4 - x^3 - x^2 + 6x + 4$$



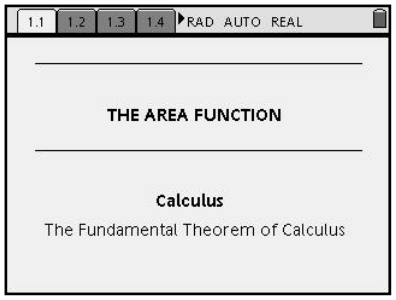
By graphing $g(x) = \frac{1}{4}x^4 - x^3 - x^2 + 6x + 4$ as **f2**(*x*), students will verify they have found $g(x)$. This will also support the notion that $g(x)$ is the antiderivative of $f(t)$.

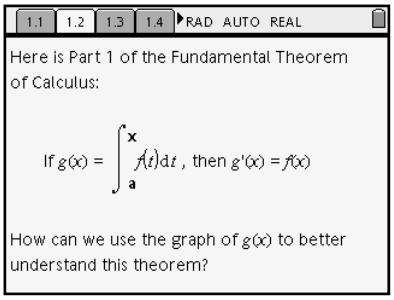
Step 2: Next, students will move point *a*, which will result in a vertical translation of $g(x)$. However, students should realize that $g'(x)$ is still equal to $f(x)$ since the slope of $g(x)$ will be the same for any value of *a*. Students should think of this vertical translation as a family of curves that make up the general antiderivative of *f*. This should also help students recognize that Part 1 of the Fundamental Theorem of Calculus holds for any value of *a*.

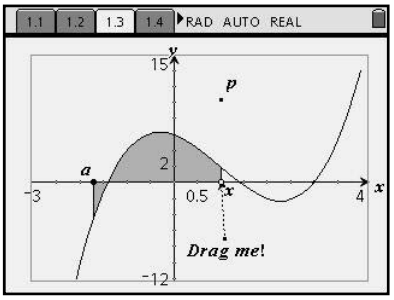


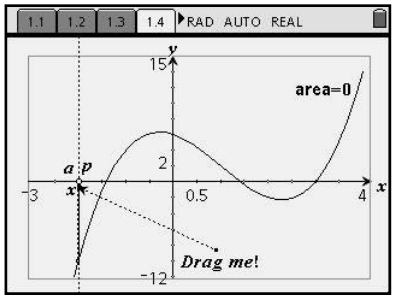
ACTIVITIES eXCHANGE

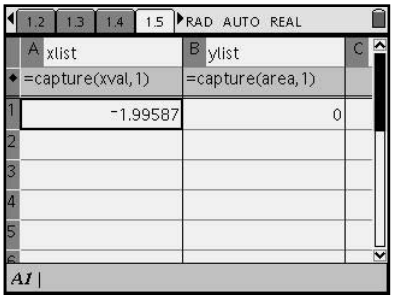
Visit education.ti.com/exchange to download activity files, including the student .tns file CalcAct03_AreaFunction_EN.tns. Enter "8298" in the quick search box.

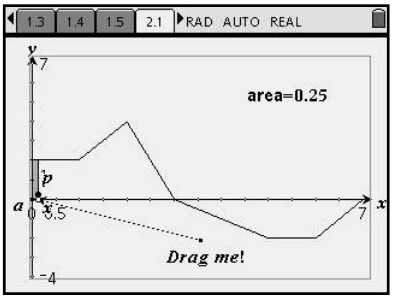


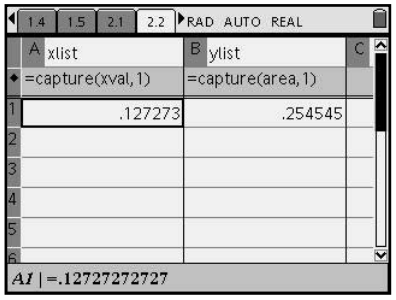


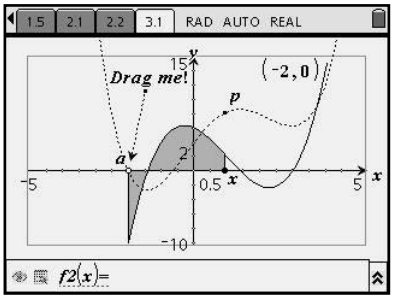












What's My Model?

By Pat Flynn

Time required
45 minutes

ACTIVITIES eXCHANGE **ID: 8518**

Activity Overview

Students will investigate several different regression models and determine which of the models makes the most sense, based upon a real-world situation (cooling a cup of hot chocolate).

Concepts

- *Mathematical modeling using regression curves*
- *Forecasting*

Teacher Preparation

This investigation illustrates to students how to strategically choose an appropriate regression model instead of blindly choosing.

- *The activity pages that follow provide a brief overview of the steps students will complete as the activity progresses. The student worksheet provides more detailed instructions as to how to use TI-Nspire learning technology to complete each step.*
- *The screenshots on pages 78–80 demonstrate expected student results. Refer to the screenshots on page 81 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "8518" in the quick search box.***

Classroom Management

- *This activity is intended to be **teacher-led**, with breaks for independent student practice. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds. The majority of the ideas and concepts are only presented in **this** document; so be sure to cover all material necessary for students' total comprehension.*
- *The student worksheet StatAct01_WhatsMyModel_worksheet_EN is intended to guide students through the main ideas of the activity while providing more detailed instruction on how to perform specific actions using the tools of the TI-Nspire handhelds. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*

TI-Nspire™ Applications

Calculator, Graphs & Geometry, Lists & Spreadsheet, Notes