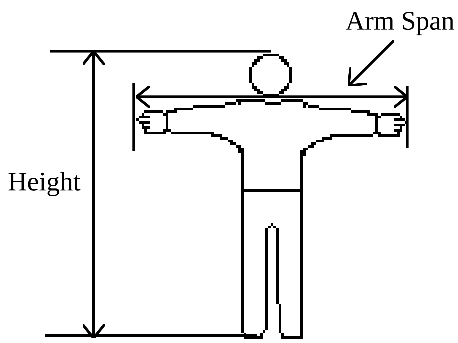
EXPLORATIONS

Name ____________________

Date ____________________

Activity 3

How Do You Measure Up?



Does increasing the amount of time practicing a sport increase performance levels in that sport? Does decreasing the speed at which a car is driven increase the gasoline mileage for that car? As a person gets older, does the person's hat size increase?

In situations like these, questions are being asked about how one quantity is related to another. Answering such questions usually requires one to collect and analyze several sets of data pairing the two quantities.

The Problem

In this activity, you will investigate the relationship between a person's height and arm span (the distance from fingertip to fingertip of a person's outstretched arms).

To begin, you will need to collect data on both quantities from several members of your class.

Using a tape measure or meter stick marked in centimeters, determine the heights and the arm spans, to the nearest whole centimeter, of two or three members of your class.

- Record the measurements in Table 3.1 in the **Questions** section and then complete the table with measurements collected by other members of your class.
- Examine the measurements you have recorded in the tables and answer #1 through #4 in the **Questions** section of this activity.

In this activity, students are introduced to the use of scatterplots for investigating possible relationships between two quantitative variables. It may take two to three class periods to complete all of the problems.

The variables under examination are two body measurements: height and arm span. Students collect data, produce a scatterplot of the data, and interpret any patterns exhibited by the scatterplot.

The language of coordinate systems is used in this activity. You may want to acquaint students with some of the associated terminology either before or during this activity.

In addition to examining lists of numbers, it is often helpful to display data in a graph or plot and look for visual clues that might suggest possible relationships between two variables. One way to graph paired data, like the height and arm span measurements, is to construct a scatterplot. A scatterplot is simply a graph of all of the ordered pairs of data on a single coordinate system. Before you plot the data, it is important to examine the data for maximum and minimum values so that appropriate measurement scales can be determined for the axes on the coordinate grid.

✐ With the help of others in your group, determine an appropriate scale for each of the horizontal and vertical axes on the **Scatterplot of Height vs. Arm Span** in #5 of the **Questions** section. Plot the information you have recorded using height along the horizontal axis and length of arm span along the vertical axis. Then answer #6 through #10.

Using the Calculator

Since students have had the opportunity to produce a scatterplot on paper, they will better understand some of the decisions that need to be made when instructing a calculator to produce a scatterplot of the same data. Once they can quickly plot data points on the calculator, they can use their time to focus on interpretation of the graphical displays.

Calculator technology also can be useful in searching for relationships between two quantities like height and arm span. In fact, your graphing calculator can draw scatterplots like the one you created. To do so, you must first place your data into the calculator's **LIST** storage.

Calculating the Results

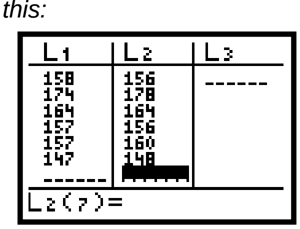
1. Press [STAT] **4:ClrList** [2nd] [L1] [,] [2nd] [L2] [,] [2nd] [L3] [ENTER] to clear the needed lists. (The third list **L3** will be used to store additional values computed later in this activity.)

2. Press [STAT] **1:Edit** to gain access to the calculator's list storage.

3. Type the numbers representing student heights into the first list (**L1**) on your calculator. Type a number, press [ENTER], and repeat until all numbers have been entered.

4. Press [▶] to move to the second list. Type the numbers representing student arm spans into the second list (**L2**). Be sure to enter the arm span for each student on the same row as the student's height.

When students enter the data, it will look similar to this:

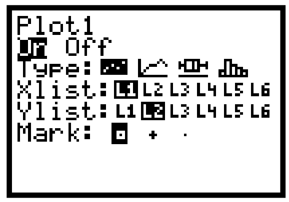


Once the paired data have been entered, you need to enter information on two windows before the calculator can produce a scatterplot graph. First, you must tell the calculator you want it to draw a scatterplot, and then you must define the intervals and the scales you want it to use for the coordinate system, just like you did when you drew a scatterplot on paper.

*It is important that all other plots and functions be turned off or cleared before attempting to produce a graph of **Plot1**. To turn off all statistical plots, students can press* [2nd] [STAT PLOT] 4 [ENTER]. *To turn off all functions, they can press* [2nd] [Y-VARS] ***5:On/Off*** *then **2:FnOff**.*

1. Press [2nd] [STAT PLOT] **1:Plot1** to select **Plot1**.

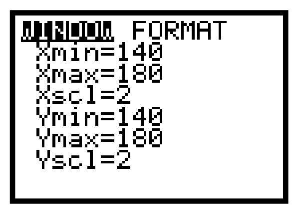
2. Edit the window so that yours looks like the one at the right. To highlight a selection, use the blue arrow keys to move the blinking cursor to the desired location and press [ENTER].



By selecting the first option available in **Type**, you are selecting a scatterplot.

3. When you have finished making the changes in **Plot1**, press [WINDOW] and edit the numbers to match those you used when you constructed the scatterplot on paper.

The window screen settings for the sample data are shown below:



Note that **Xmin** and **Xmax** refer to the minimum and maximum values to be used along the horizontal axis height. **Xscl** defines the distance between reference or *tick* marks used along that axis. **Ymin**, **Ymax**, and **Yscl** define the same parameters for the vertical axis.

✎ Record your selected values in the window shown in #11 in the **Questions** section.

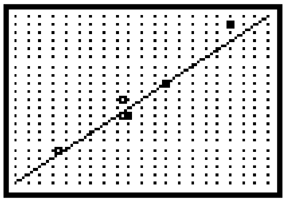
4. Press [GRAPH] to view the scatterplot.

5. You may prefer to view the plot with grid points visible. On a TI-82, you can add grid points to the plot by pressing [WINDOW], selecting **FORMAT**, pressing [▼] and [▶] to select **GridOn**, and pressing [ENTER].

On the TI-80 calculator, grid points can be added to a graph by pressing [2nd] [DRAW] 9 [ENTER]. *On the TI-83, press* [2nd] [FORMAT], *then press* [▼] *and* [▶] *to move to **GridOn**, and press* [ENTER].

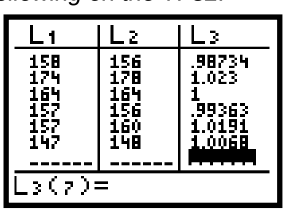
6. Press [GRAPH] again, and grid points should appear on the scatterplot. The distance between grid points is determined by the settings you have defined for **Xscl** and **Yscl**.

A scatterplot of the sample data, along with a graph of ***y = x****, as shown on a TI-82 is given below. Students will add the line in step 8.*



*An examination of ratios is reasonable anytime one naturally expects that a measure of zero on one variable would be associated with a measure of zero on the other, that is, whenever one variable is expected to vary directly (****y = k*x****) with the other.*

Computing the ratio of ***L2*** *to* ***L1*** *using the sample data results in the following on the TI-82:*



Although it is expected that most of the values will lie near 1, it is possible that some will not. This is fine. Rather than accepting the typical or "average" ratio as 1, you could instead use the arithmetic average of all the computed ratios. This average, or mean, can be found by computing the mean of ***L3****. From the home screen, press* [2nd] [LIST]*, highlight* ***Math*** *and then press* 3 [2nd] [L3] [ENTER] *to compute and display this mean value. For the sample data this mean is approximately 1.005.*

7. You can view the coordinates of points on this plot by pressing [TRACE] and using [▶] and [◀] to highlight different points. Check that the traced values agree with those you have collected.

When you examined your paper version of this scatterplot, you were asked to draw a line that connected points where the first and second coordinates were equal. To do this on your calculator, you will need to add the graph of the line defined by **y = x**.

8. To add this line to your scatterplot, simply press [Y=] [X,T,Θ] [GRAPH].

The fact that the height and arm span data seem to lie very close to the line **y = x** (where **x** represents height and **y** represents arm span) is an interesting finding. In fact, it is just this type of algebraic relationship that statisticians are often looking for when they examine sets of paired data. Sometimes in the search for such relationships, it is helpful to examine the ratios of two quantities being studied.

In this study, you are comparing height and arm span so let's use the calculator to compute the ratio of arm span to height for each class member for whom you have collected data.

9. To obtain this list of ratios:

a. Press [STAT] 1 to gain access to the calculator's list storage.

b. Press [▶] twice to move to **L3**.

10. Press [▲] to move to the top so that **L3** is highlighted.

11. Press [2nd] [L2] [÷] [2nd] [L1] [ENTER] to define **L3** as a list of ratios of height to arm span.

✎ Go to the **Questions** section and answer #12 through #15.

Questions

Table 3.1. Height and Arm Span

Name	Height (cm)	Arm Span (cm)

Although student names are not used in the analysis of any collected data, it is important that the body measurements be linked to the same person. In the Problems for Additional Exploration, students are asked to investigate possible relationships between overall height and shoe length. Identifying the data with the person from whom it was collected will save making height measurements again if that problem is assigned. Sample measurements from six real students are provided in the table below.

Name	Height (cm)	Arm Span (cm)
Jill	158	156
Brett	174	178
Jo	164	164
Julie	157	156
Asha	157	160
Cherise	147	148

1. How often is a person's height greater than their arm span?

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2. How often is height less than arm span?

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In the sample data provided, height was greater than arm span one-third of the time, height was less than arm span one-half the time, and the two measurements were equal one-sixth of the time.

3. How often are height and arm span equal?

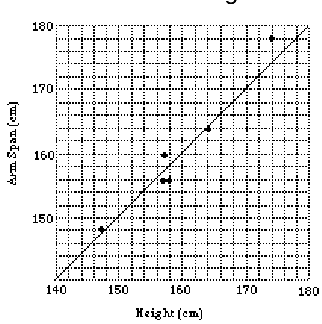
In general, arm span measurements are approximately equal to height measurements on the same individual. Variations from this general relationship are natural and to be expected within any set of collected data.

4. Do you see any other relationships between height and arm span? Describe these below.

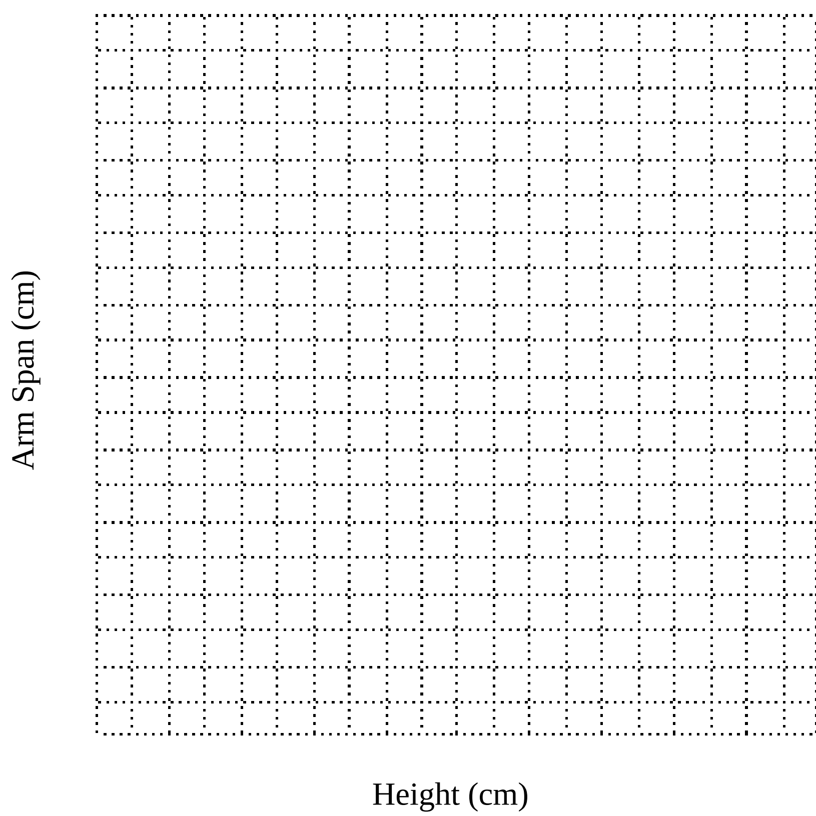
Return to page 22.

In the sample data, heights range from 147 cm to 174 cm and arm spans from 148 cm to 178 cm. Each dimension of the grid is bordered by 20 grid squares so a distance which is a multiple of 20 would be appropriate on each scale. Using 140 cm to 180 cm on each axis would be an appropriate range for the set of sample data.

A scatterplot of the sample data is given below along with the graph of the line ***y = x*** *on the same coordinate grid.*



5. **Scatterplot of Height vs. Arm Span**



6. In general, as height increases, what happens to arm span? How is this illustrated in the scatterplot?

In general, as height increases (as we move from left to right across the graph) the associated arm span measurements also increase (get higher on the graph).

7. In what ways does the scatterplot, when compared to the table of data, make it easier to see relationships between height and arm span?

Since the scatterplot orders the data on the basis of increasing heights, and tables of data do not necessarily do so, patterns such as this are often much easier to see from a graphical presentation than from an unordered numerical presentation of the data.

8. Find three or four points on your coordinate system (they do not have to be points of the scatterplot) where the horizontal and vertical coordinates are equal, for example (150, 150). Draw a straight line through these points; this line will serve as a reference line for comparing heights to arm spans. Some of the points of the scatterplot lie above this line, some lie below this line, and others may lie on this line. Find several points that lie above this line. What is true about the relationship between height and arm span for each of these points?

Be certain that students understand that the points they are to locate are not necessarily points that are part of the scatterplot. They can be any points located on the coordinate grid.

Points that lie above the line represent students whose arm spans are larger than their heights.

9. How would you describe the relationship between height and arm span for points that lie below the line?

Points that lie below the line represent students whose arm spans are smaller than their heights.

If a point in the scatterplot lies on the line, then the arm span and height of the student are equal.

10. If a point lies on this line, what relationship exists between height and arm span?

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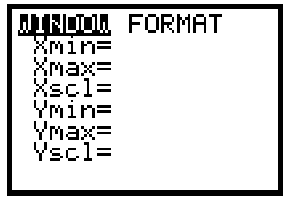
*Return to page 22, **Using the Calculator**.*

11. Record your settings for the Plot Window:



Return to page 22, step 4.

*Since height is represented on the x-axis and arm span on the y-axis, **y = x** indicates that arm span and height are equal.*

12. Explain why the line **y = x** contains points where the two coordinates are equal.

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If the ratio of arm span to height is less than one, then the measure of the arm span is less than the measure of the corresponding height.

13. Find a value in **L3** that is less than 1. What is the relationship between the height and arm span used to calculate this value?

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If **L2** ÷ **L1** ≈ **1**, *then a person's arm span and height are approximately equal in measure.*

14. If we assume that **L2** ÷ **L1** ≈ **1**, what relationship exists between arm span and height?

Note: *The symbol ≈ means* approximately equal to.

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15. Based upon this relationship, predict the arm span measurement for a person who is known to be 170 cm tall? How often do you think your estimate would be too high? Too low? Explain your responses.

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A person found to be 170 cm tall would be expected to have an arm span of approximately 170 cm. The best estimate of how often this would be too high (or too low) could be based upon the percent of the points in the scatterplot that lie above (or below) the line $y = x$*.*

Problems for Additional Exploration

The instructions for this first problem assume that students still have the data on heights and arm spans in the first three lists on their calculators. It is also assumed that data for shoe length will be entered for individuals in the same order used for heights and arm spans.

Some variation in the ratios of shoe length to overall height are to be expected but, as with the arm span and height comparisons, most ratios should center around some value. The average of the ratios should serve as a good approximation to use in describing a relationship between the two quantities. This ratio (call it R) would serve as the best guess for the slope of a line that "fits" the points on the scatterplot. Students could enter the expression **Y1 = RX** *(where R is replaced with the computed mean of the shoe lengths to overall height ratios) into the* $\boxed{\text{Y=}}$ *menu. They could then* $\boxed{\text{GRAPH}}$ *both the scatterplot and the line on the same window to observe the goodness of the "fit."*

1. Do you think there is a relationship between a person's height and the length of their shoe (shoe length, not shoe size)? Investigate this question by doing the following:

a. Collect data on the shoe lengths of class members. Enter this data in Table 2 below. Make certain to record the new information with the correct individual.

Table 3.2. Height and Shoe Length

Name	Height (cm)	Shoe Length (cm)

b. Clear lists **L4** and **L5** on your calculator.

c. Enter the shoe length data into **L4**.

d. Produce a scatterplot of height (**L1**) versus shoe length (**L4**) where height is represented along the horizontal axis and shoe length is represented along the vertical axis.

Do the plotted points appear to form a pattern? Describe any patterns you see.

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e. Calculate **L5** as the ratio of **L4** to **L1**; that is, calculate the ratio of shoe length to height.

Find the mean of the ratios. From the home screen, press [2nd] [LIST], press [▸] to highlight **MATH**, press 3 [2nd] [L5] [ENTER]. State an approximate relationship that would permit you to estimate a person's shoe length if you knew the person's height.

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2. Describe a process you could use to determine and describe a relationship between hand span (the distance beween the tip of the thumb and the tip of the little finger when the fingers of the hand are stretched open) and length of the forearm (the distance from the inside of the elbow to the base of the hand at the wrist joint).

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We would be interested in seeing the results of student investigations into problems 2 and 3. Encourage students to write to us providing their data, graphical displays, and analyses. Send your class results to:

Dr. Christine Browning
Department of Mathematics and Statistics
Western Michigan Univ.
Kalamazoo, MI 49008

We will return correspondence.

The variables height, arm span, and shoe length are examples of quantitative variables. The variables favorite color of M&M, make of automobile, and gender are examples of categorical variables.

Another interesting ratio that students might enjoy investigating is that of height to bellybutton height. Interestingly enough, this ratio will be very close to the golden ratio

$$\frac{1+\sqrt{5}}{2} \approx 1.618.$$

The golden ratio, also called the divine proportion, finds its way into many other settings including art, architecture, and plant growth.

3. Make up a question that concerns two *quantitative* variables (variables whose values are numbers) of interest to you. Collect data on these two variables and use lists and scatterplots to look for and describe possible relationships.