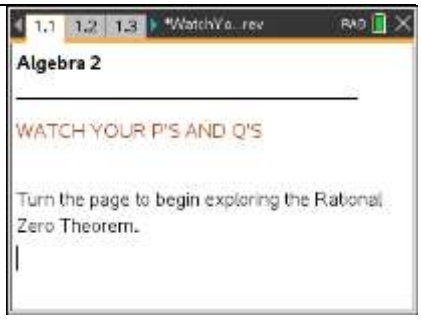
Watch Your P's and Q's

ALGEBRA 2

TEACHER NOTES

Objectives

- Students will apply the Rational Zero Theorem to final all rational zeros of a polynomial.
- Students will divide on polynomial by another to obtain a quotient and a remainder.
- Students will approximate the real roots of a polynomial equation by graphing and identifying the number of real roots.



Vocabulary

- Rational Zero Theorem
- Rational zeros
- Synthetic Division
- Polynomial
- Irrational zeros
- Velocity

About the Lesson

- Students should have already begun to observe graphs of quadratic functions. Students should also be able to use the quadratic formula and synthetic division to solve polynomial equations.
- Remind students that the zeros of a polynomial are those values for which the polynomial is equal to zero (f(x) = 0).
- This activity is intended to be **teacher-led** with students in **small groups**. You should seat your students in pairs so they can work cooperatively on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- Teaching time: Approximately 30-minutes.

TI-Nspire™ Navigator™

- Send out the WatchYourPsandQs.tns file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Lesson Materials

- Compatible TI Technologies:

TI- Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This lesson includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products, including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the lesson for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials.

Lesson Files:

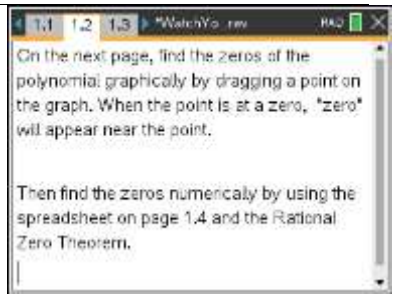
Student Activity

- WatchYourPsandQs_student.pdf

TI-Nspire document

- WatchYourPsandQs.tns

Problem 1: Pages 1.2 to 1.4 on the handheld.



1. Page 1.2 introduces the activity.

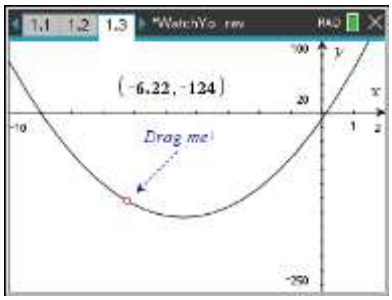
   Write is the Rational Zero Theorem?

Answer: The **Rational Zero Theorem** states that all potential rational zeros of a polynomial are of the form $\frac{P}{Q}$, where P represents all positive and negative factors of the last term of the polynomial and Q represents all positive and negative factors of the first term of the polynomial.

For this polynomial, $7x^2 + 62x - 9$, the possible rational zeros are:

$$\frac{P}{Q} = \frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 7} = \pm 1, \pm 3, \pm 9, \pm \frac{1}{7}, \pm \frac{3}{7}, \pm \frac{9}{7}$$

2. Move to page 1.3 and locate the zeros of the function by dragging the point on the graph.

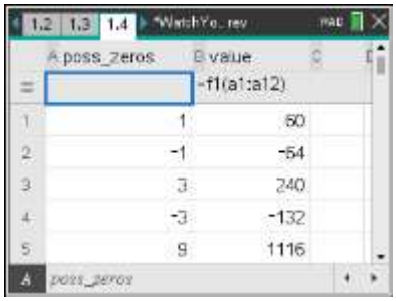


   What are the zeros?

   One page 1.3, have students move the point on the graph to find all zeros of the polynomial, recording their results. When the point has reached a zero, the word "zero" appears. Remind students that zeros of a polynomial are the x-values when the y-values are zero. The location (x,0) is referred to as the x-intercept.

   Answer: The zeros occur at x = –9 and 0.1

3. Move to page 1.4 and use the spreadsheet to find the exact zeros of the function.



   What are the exact zeros?

   On page 1.4, possible zeros have been entered into the first column of the spreadsheet.

   To evaluate the polynomial from the previous page for each value,

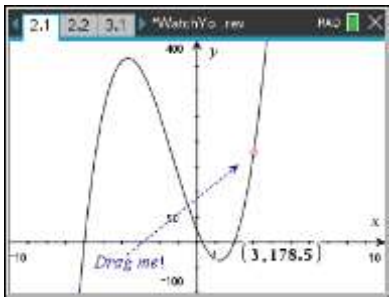
have students enter the formula, **f1**(a1:a12), into the formula cell (gray) for Column B.

Answer: The zeros are x = -9 and x = 1/7

TI-Nspire Navigator Opportunity: *Screen Capture*

This would be a good place to do a screen capture to verify students are able to grab and drag the point and find all the zeros.

Problem 2: Pages 2.1 to 2.2 on the handheld.

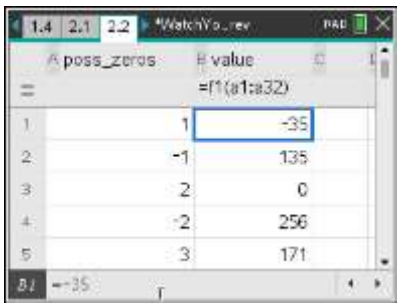


4. Drag the point on page 2.1 to locate the zeros of the function.

   What are the zeros?

   On page 2.1, the students will once again find the zeros by dragging the point on the graph.

   Answer: Zeros at x = -6, 0.3, and 2



5. Move to page 2.2 and identify all of the possible rational zeros using the Rational Zero Theorem and enter the results in the spreadsheet. Use the column labeled **value** to find the zeros of the function.

   Answer: The possible zeros are as follows:

   {±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24, ±1/7, ±2/7, ±3/7, ±4/7, ±6/7, ±8/7, ±12/7, ±24/7}

   The possible zeros should be entered into Column A.

   What are the zeros?

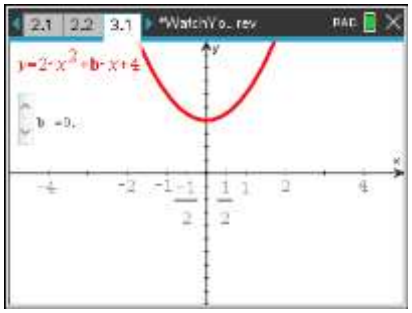
   The formula to be entered into Column B is **f1**(a1:a32).

   Answer: Zeros at x = -6, 2/7, and 2

TI-Nspire Navigator Opportunity: *Quick Poll*

You may choose to use Quick Poll throughout the activity to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask. In Problem 1, you may ask students to list the zeros of the graph on page 1.3, or to list several of the rational zeros possible using the Rational Zero Theorem.

Problem 3: A graphical look at the possible zeros of a parabola.



6. Click the slider to change the value of b (which changes the parabola).

   How is it possible that the graph sometimes has zeros that do not cross at any of the possible rational zeros that are labeled on the graph?

   Answer: Sometimes the graph crosses at **irrational** zeros. The students may not make the connection that the rational zero theorem is **ONLY** good for finding possible **rational** zeros.

Exercises: You may use pages 4.1 and 4.2 to verify your answers to the exercises below.

The worksheet exercises are designed to test the understanding of the Rational Zero Theorem. Have students complete the remaining exercises individually or in small groups. A graph and spreadsheet for student use are provided on pages 4.1 and 4.2.

1. Use the method described in the activity to find the rational zeros for $-10x^3 + 15x^2 + 16x - 12$.

   Answer: 2

2. How could synthetic division be used to help find the other zeros for the polynomial in Exercise 1?

Answer: Students can use synthetic division with the known zero of 2. This would reduce the polynomial to a quadratic which can be solved using the quadratic formula.

Use synthetic division to find the zeros of the polynomial.

Use synthetic division with the known zero of 2. This would reduce the polynomial to a quadratic which can be solved using the quadratic formula.

Answer: The zeros are 2, and $\frac{-5 \pm \sqrt{265}}{20}$

3. Is it possible for a polynomial to not have any rational zeros? Explain.

   Answer: Yes. For example the polynomial $f(x) = x^2 - 5$ has only irrational roots.

   Is it possible for a polynomial to no have any zeros at all? Explain.

   Answer: Yes. For example the polynomial $f(x) = x^2 + 5$ has no REAL roots.

4. An object is launched vertically from a point s_0 above the ground at an initial speed of v_o feet per second. Its vertical distance above the ground is given by the equation $s = -16t^2 + v_0 t + s_0$. Determine how long an object with velocity of 300 ft/sec will stay in the air if thrown upwards from a height of 5 feet.

   Answer: 18.8 seconds