MATH AND SCIENCE @ WORK

AP* STATISTICS Educator Edition

SPACEWALK TRAINING

Instructional Objectives

Students will

- explain what is meant by the direction, form and strength of the overall pattern of a scatter plot;
- define the correlation coefficient, *r*, and describe what it measures;
- write a regression equation and interpret the meaning of the slope and *y*-intercept in context of the problem;
- explain what is meant by extrapolation and interpret a situation where extrapolation occurs;
- make predictions based on the correct mathematical model; and
- plot residuals and interpret their graphical form.

Degree of Difficulty

For the average student in Advanced Placement Statistics, this problem is at a moderate to advanced difficulty level.

Class Time Required

This problem requires 40 minutes.

- Introduction: 5 minutes
- Student Work Time: 30 minutes
- Post Discussion: 5 minutes

Background

This problem is part of a series of problems that apply Math and Science @ Work in NASA's scientific laboratories.

The Neutral Buoyancy Laboratory (NBL) is a 202 ft (62 m) long, 102 ft (31 m) wide and 40 ft (12 m) deep pool located inside the NASA Sonny Carter Training Facility in Houston, Texas. The NBL allows astronauts to train for spacewalks in an environment resembling microgravity (weightlessness). Due to the size of the NBL, two different training activities can be performed at either end of the pool simultaneously. The NBL is large enough to hold full-sized mockups of the space shuttle cargo bay, flight payloads (like the Hubble telescope) and the International Space Station (ISS). Astronauts wear pressurized Extra-vehicular Mobility Unit (EMU) suits, which weigh approximately 280 lbs (127 kg) while training in the NBL. They are assisted by at least four professional scuba divers wearing

Grade Level
11-12

Key Topic
Linear Regression

Degree of Difficulty
Moderate to Advanced

Teacher Prep Time
5 to 10 minutes

Class Time Required
40 minutes

Technology
- TI-Nspire™ Learning Handhelds
- TI-Nspire document: *Spacewalk_Training.tns*

AP Course Topics
Exploring Data:
- Exploring Bivariate Data

NCTM Standards
Data Analysis and Probability

regulation scuba gear. For every hour the astronaut plans to spend on a spacewalk, the team will spend seven hours training in the water. On a training day at the NBL, astronauts normally spend up to six consecutive hours in the pool. The scuba divers, however, are limited to five hours of dive time per day and this time is broken into at least two different dives.

Figure 1: Neutral Buoyancy Lab (NBL) located in the Sonny Carter Training Facility in Houston, TX

Figure 2: An astronaut trains in the NBL and is assisted by professional scuba divers.

Neutral buoyancy is the term used to describe an object that has an equal tendency to float as it does to sink. In water, items can be made neutrally buoyant using a combination of weights and flotation devices. In such a state, even a heavy object can be easily manipulated, as is the case in the microgravity of space. However, there are two important differences between neutral buoyancy as achieved in the NBL and the weightlessness of space. First, suited astronauts training in the NBL are not truly weightless. While the suit/astronaut combination is neutrally buoyant, the astronauts feel their weight while in the suit. The second is that water drag hinders motion, making some tasks easier to perform in the NBL than in microgravity and other tasks more difficult. While these differences are recognized by spacewalk trainers and astronauts, neutral buoyancy is currently the best method available for astronauts to train for spacewalks.

AP Course Topics

Exploring Data: Describing patterns and departures from patterns

- Exploring bivariate data
  - Analyzing patterns in scatterplots
  - Correlation and linearity
  - Least-squares regression line
  - Residual plots

NCTM Standards

Data Analysis and Probablility

- Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them
  - Understand the meaning of measurement data and categorical data, of univariate and bivariate data and of the term variable
  - Understand histograms, parallel box plots and scatterplots and use them to display data
- Select and use appropriate statistical methods to analyze data
  - For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools

- Recognize how linear transformations of univariate data affect shape, center, and spread
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled

Problem and Solution Key (One Approach)

Students are given the following problem information within the TI-Nspire document, Spacewalk_Training.tns, which should be distributed to their TI-Nspire handhelds. The data in Table 1 is provided to the students in three different tables on TI-Nspire pages 1.5, 2.2 and 3.2.

Even though we may not realize it, pressure is always acting on our bodies due to the weight of the air in the atmosphere we live in. At sea level, 14.7 pounds per square inch (psi) of atmospheric pressure is acting on our bodies. Our body compensates for this pressure by pushing back with the same force. When we surround ourselves with water instead of air, the weight of the water produces additional pressure on our bodies. The deeper we go, the more pressure we feel. During training in the NBL, astronauts and the scuba divers that assist them are in water as deep as 40 feet while experiencing varying pressure on their bodies. Astronauts' suits are pressurized at the same pressure as they are when they are working in space on a spacewalk. This means that astronauts have more pressure acting on them than scuba divers at the same depth. Table 1 shows the pressure gage values experienced by scuba divers and astronauts at different depths of water. It is important to note that pressure gage readings do not take into account the 14.7 psi of atmospheric pressure at sea level.

Table 1: Pressure gage values of scuba divers and astronauts

Water Depth (feet)	Scuba Diver Pressure Gage Reading (psi)	Astronaut Pressure Gage Reading (psi)
29	12.6	16.8
30	13.0	17.2
31	13.4	17.6
32	13.9	18.1
33	14.3	18.5
34	14.7	18.9
35	15.2	19.4
36	15.6	19.8
37	16.0	20.2
38	16.5	20.7

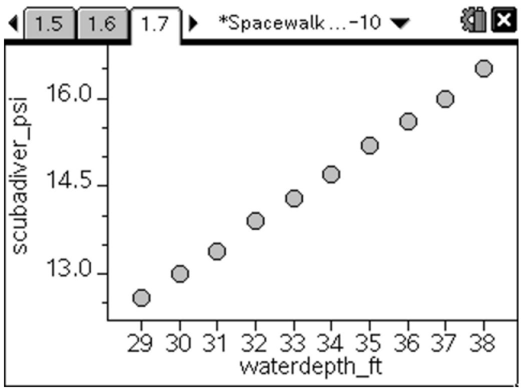
A. Follow the instructions on TI-Nspire pages 1.4-1.6 to plot the gauged pressure experienced by scuba divers (psi) vs. hte water depth (ft). Determine the correct mathematical model to fit the data and give statistical evidence to support your decision by answering the related embedded questions found in the TI-Nspire document.

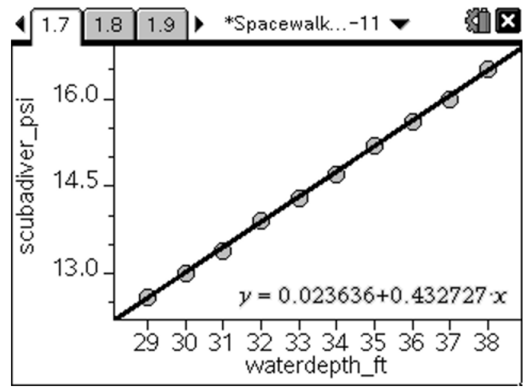
   Students are given the data about the water depth and scuba diver pressure from Table 1 and are given the following instructions:

   - *On the next page, press **menu** and add a **Data & Statistics** page.*

- *Click on the bottom and left side of the screen to add the data to plot. Use* ***waterdepth_ft*** *as the explanatory variable (on x-axis) and use* ***scubadiver_psi*** *as the response variable (on y-axis).*
- *Select the appropriate regression model by pressing* ***menu > Analyze > Regression****.*

The following screenshots show the student results.





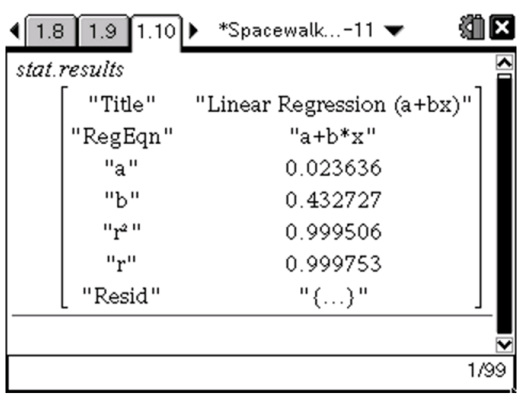
Embedded Questions:

1.8 Interpret the slope and *y*-intercept in the context of the problem.

The slope is the change in pressure of 0.433 psi for every foot of water depth. The y-intercept is 0.024, which is the amount of pressure as the diver enters the water.

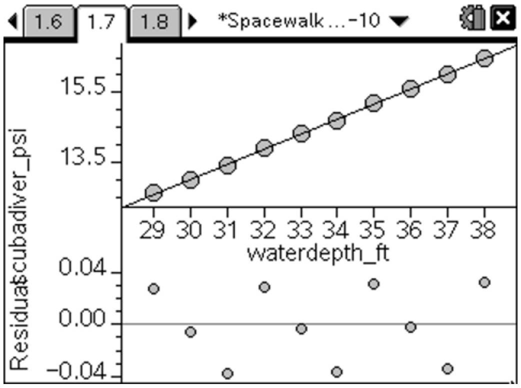
1.9 Determine the correlation coefficient and interpret its value.

To calculate, go to the next page, press **menu**, add a **Calculator** page, press **menu** again, select **Statistics>Stat Results** and press **Enter**.



The r value is very close to 1 indicating a very strong, positive, linear relationship between water depth and the pressure felt by the scuba diver.

Add a residual plot to the graph on page 1.7 by going to **menu > Analyze > Residual > Show Residual Plot**.



1.12 The plotted data shows what type of association between depth and pressure?

A strong, positive association

1.13 Describe the association based on the residuals of the graph.

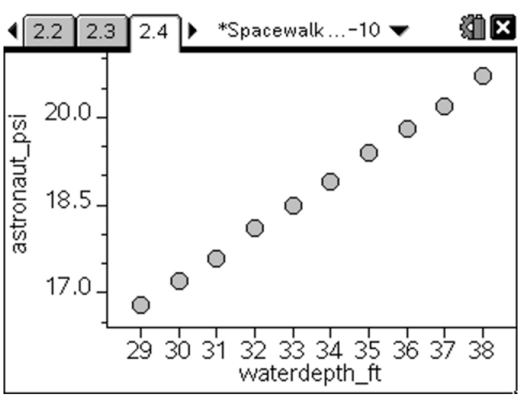
The residual plot shows a scattering of points on either side of the x-axis and therefore indicates a linear association between water depth and the pressure read on the gauges.

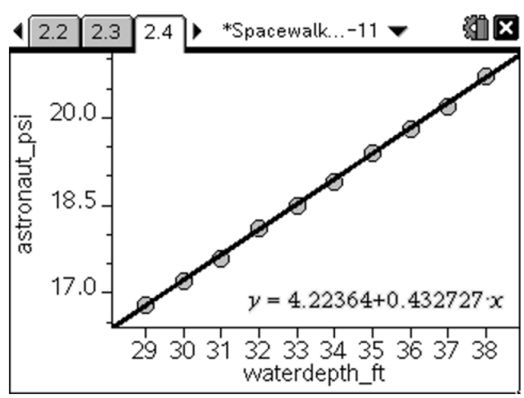
1.14 Is a linear model appropriate for the plotted data? Explain in statistical terms.

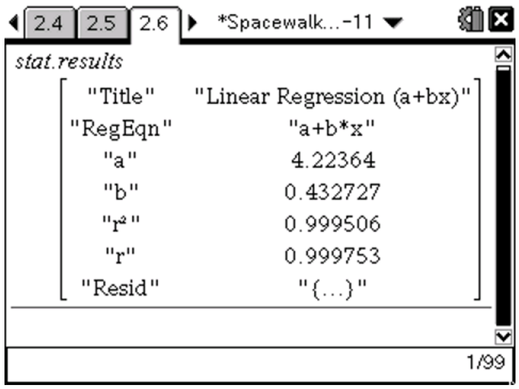
Yes. The data shows a strong, positive, linear relationship between the explanatory and response variable. The data points are very close to the line, with no outliers or influential points.

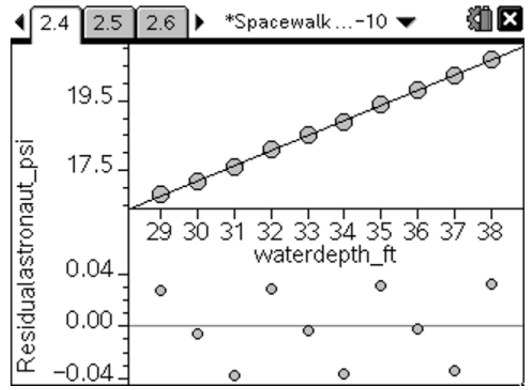
B. Follow the instructions on TI-Nspire pages 2.1-2.3 to plot the gauged pressure experienced by astronauts (psi) vs. the water depth (ft). Determine the correct mathematical model to fit the data and give statistical evidence to support your decision by answering the related embedded questions found in the TI-Nspire document.

Students are given the data about the depth of water and astronaut pressure from Table 1 and are given instructions to find the same type of plots and information as in part A. The following screenshots show the student results.









Embedded Questions:

2.5 Determine the correlation coefficient and interpret its value.

The r value is very close to 1 indicating a very strong, positive, linear relationship between water depth and the pressure felt by the astronaut.

2.7 The plotted data shows what type of association between the depth and the water pressure?

A strong, positive association

2.8 Describe the association based on the residuals of the graph.

The residual plot shows a scattering of points on either side of the x-axis and therefore indicates a linear association between water depth and the pressure read on the gauges.

2.9 Is a linear model appropriate for the plotted data? Explain in statistical terms.

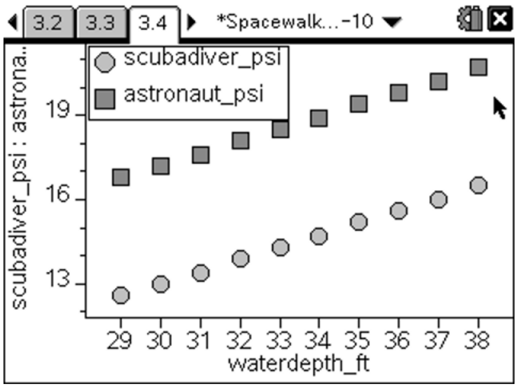
The plotted data shows a strong, positive, linear relationship between the explanatory and response variable. The data points are very close to the line and there are no outliers or influential points.

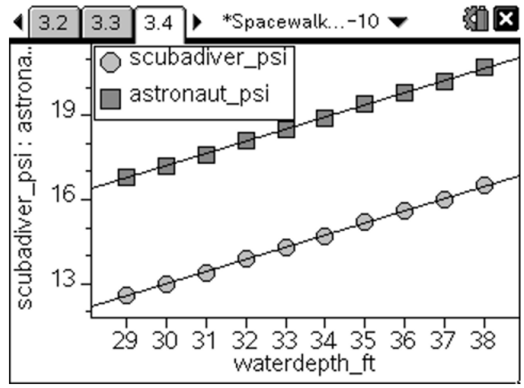
C. Follow the instructions on TI-Nspire pages 3.1-3.3 to create a plot showing both sets of response variables on the same axes. Then answer the related embedded questions found in the TI-Nspire document.

Students are given the data from Table 1 and the following instructions:

- *On the next page, press **menu** and add a **Data & Statistics** page.*
- *Use **waterdepth_ft** as the explanatory variable and plot both sets of response variables. First select **scubadiver_psi**, then press **ctrl**, **menu**, and select **Add Y Variable** to plot **astronaut_psi**.*
- *Select the appropriate regression model to plot.*

The following screenshots show the student results.





Embedded Questions:

3.5 What transformation value changes the value of the pressure felt by astronauts and scuba divers? Explain how you determined that value.

The pressure values shift upwards by 4.2 psi, which is the vertical shift of each data point or a change of 4.2 on the entire linear regression equation.

3.6 What do you notice about both sets of data? What accounts for the difference between the graph of the astronauts' data and the graph of the scuba divers' data?

The data points appear to shift up or down the same amount. This is because the astronauts wear pressurized suits and thus feel more pressure than scuba divers.

Note to Educator: Astronauts' suits are pressurized at 4.2 psi both in the NBL and in space when astronauts perform spacewalks.

3.7 Predict the pressure experienced by an astronaut in the NBL at a water depth of 45 feet.

You cannot predict the pressure at 45 feet because it is outside the domain of the graph. The NBL is only 40 ft deep. This is considered an example of extrapolation.

Scoring Guide

Parts A, B, and C are scored as Essentially Correct (E), Partially Correct (P), or Incorrect (I).

Question	Score	Description
A	*Essentially Correct (E)*	All graphs are labeled and interpreted correctly, all questions are answered correctly, and all statistical information is complete and accurate.
	Partially Correct (P)	Graphs are labeled and interpreted correctly, all questions are answered correctly, but some statistical information is incomplete or inaccurate.
	Incorrect (I)	Graphs are not labeled correctly, some questions are answered correctly, but most statistical information is incomplete or inaccurate.
B	*Essentially Correct (E)*	All graphs are labeled and interpreted correctly, all questions are answered correctly, and all statistical information is complete and accurate.
	Partially Correct (P)	Graphs are labeled and interpreted correctly, all questions are answered correctly, but some statistical information is incomplete or inaccurate.
	Incorrect (I)	Graphs are not labeled correctly, some questions are answered correctly, but most statistical information is incomplete or inaccurate.
C	*Essentially Correct (E)*	All graphs are labeled and interpreted correctly, all questions are answered correctly and all statistical information is complete and accurate.
	Partially Correct (P)	Graphs are labeled and interpreted correctly, all questions are answered correctly, but some statistical information is incomplete or inaccurate.
	Incorrect (I)	Graphs are not labeled correctly, some questions are answered correctly, but most statistical information is incomplete or inaccurate.

Point Distribution

Suggested 4 points total to be given as follows:

4 pts	**Complete Response**	All three parts essentially correct.
3 pts	**Substantial Response**	Two parts essentially correct and one part partially correct.
2 pts	**Developing Response**	Two parts essentially correct and no part partially correct. *OR* One part essentially correct and one or two parts partially correct. *OR* Three parts partially correct.
1 pt	**Minimal Response**	One part essentially correct and no parts partially correct. *OR* No parts essentially correct and two parts partially correct.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school AP Statistics instructors.

NASA Expert

Samuel Strauss, DO, MPH – Flight Surgeon, Kelsey-Seybold Clinic, Neutral Buoyancy Laboratory, NASA Johnson Space Center, Houston, TX

AP Statistics Instructor

Sharon Cichocki – Texas Instruments T^3 (Teachers Teaching with Technology™) National Instructor, Hamburg High School, Hamburg, NY