Half Life

ID: 9287

Time required
30 minutes

Topic: Exponential Functions

- *Graph exponential functions of the form $f(x) = Aa^x$ where A and a are positive real numbers and $a \neq 1$.*
- *Evaluate the exponential function $f(x) = Aa^x$ for any value of x.*
- *Calculate the exponential line of best fit to model bivariate data and use it to predict a value of one variable corresponding to a value of the other.*

Activity Overview

Students will explore exponential decay through an experiment and use the gathered data to generate an exponential regression equation. Students will then repeat the process with a data set and forecast future results.

Teacher Preparation

This investigation offers students an experimental way to generate excellent data with pennies to produce an exponential regression graph and equation.

- *Student's should have already done linear regression and discussed what the Pearson correlation coefficient, r, represents.*

Classroom Management

- *This activity is intended to be* ***teacher-led****. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in this document; be sure to cover all the material necessary for students' total comprehension.*
- *The student worksheet is intended to guide students through the main ideas of the activity, while providing more detailed instruction on how they are to perform specific actions using the tools of the calculator. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on separate sheets of paper, or just use the questions posed to engage a class discussion.*

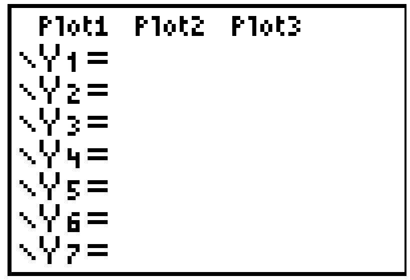
TI-84 Plus Applications

none

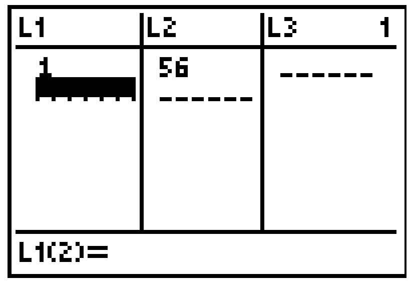
Follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.

Problem 1 – Dropping Pennies

Before beginning, clear any functions from the ! menu.



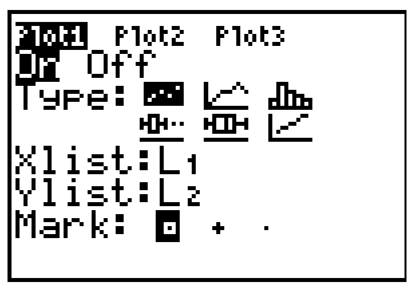
Your teacher will give you a quantity of pennies. To begin the experiment, record the initial settings (trial 1). Open the **List Editor** by pressing S e. Enter the number of trial, 1, in **L1** and enter the number of pennies you currently have in **L2**.



Pick up the pennies and drop them on your desk. Remove any pennies showing tails and record the results (under the trial column enter 2 and under the amount column enter the number of remaining pennies you currently have.)

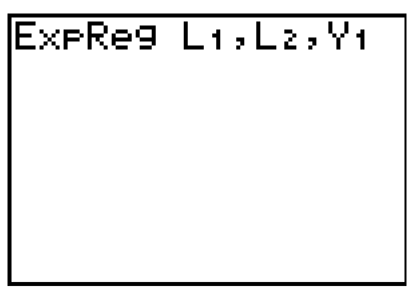
Continue this procedure until there are no more pennies remaining. Do not record the last trial when there are no pennies remaining.

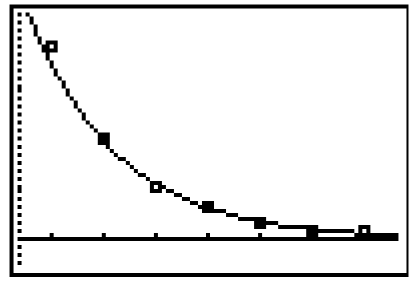
Next, make a scatter plot of the data. Open **Plot1** by pressing ` ! e. Turn the plot on, choose scatter as the type, **L1** for the **XList** and **L2** for the **YList**.



Adjust the window settings. The *x*-axis should run from 0 to an even number greater than the number of trials. The *y*-axis should run from 0 to an even number greater than the number of pennies that you started with. Then press % to view the scatter plot.

Now find an exponential model for the data. Return to the home screen by pressing ` M. The command shown will find a function for the number of pennies vs. the number of trials and store it in **Y1**. The command is found in the **Stat > Calc** menu.





To view the graph of the exponential regression equation together with the scatter plot, press %.

This is an exponential decay graph of rate $\frac{1}{2}$ (it looses around $\frac{1}{2}$ of its amount each time period.)

Compare your data and equation to those around you.

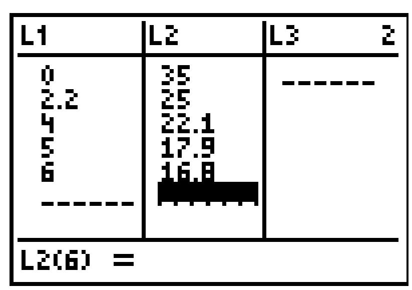
- Are they similar or different?
- Why is this?

Problem 2 – Radioactive decay

Before beginning, clear all data from the lists using the **ClrAllLists** command (found in the catalog).

Many times predictions must be made without a complete data set. To explore this, suppose there are 35 grams of radioactive material and the amount of it remaining over time has been recorded as follows:

(days, amount), (0, 35), (2.2, 25), (4, 22.1), (5, 17.9), (6, 16.8)



Enter this data into **L1** and **L2**, produce the scatter plot and exponential regression equation using problem 1 as an example. Store the exponential regression equation in **Y1**.

The true power of regression is to forecast future results. To predict how much will be left after 15 days, go to the home screen, store 15 as **X**, and evaluate **Y1**.

- Try other values of **X** and determine how many days it will take for the material to be less than 1 gram.
- Will there ever be 0 grams remaining?

Exercises

Clear out **Y1** in the ! screen, remove the last three data points from the lists in problem 1, and compute the exponential regression equation.

- Is this the same as the original equation?

Go to the homescreen and forecast the decayed data points using this new model.

- Does the equation correctly predict the results or just values that are close?

While this activity focused on exponential decay, it could also be used for exponential growth. Go to the **List Editor** and enter the following world population (in millions) into **L1** and **L2**.

(1995, 5691), (1996, 5769), (1999, 6003), (2001, 6157), (2003, 6311).

- Compute the regression equation and predict the population of the world for the year 2008.
- Predict when the world's population will reach 100,000 million (1 trillion).
- Is this possible?

Solutions – Student Worksheet Exercises

1. **a.** Answers may vary. Sample: No, but close. The rounded answer is $45.07*(.54)^{x}$.

b. Values are close, but not exactly the same because this is a prediction. Actual data predictions will vary for each student.

2. **a.** Regression equation is approximately $3.45*(1.01)^{x}$.

b. Prediction is 6566.12 million. The actual answer is 6605 million.

c. The population of the world would reach 100,000 million in the year 2222. It could be possible, but the world can only hold a finite number of people.

