## Activity 4

## Pencil Me In

Think about a simple electrical circuit consisting of a battery and a light bulb. The battery, or voltage source, is like a pump, pushing electrons around a closed path and causing the bulb to glow. The flow of electrons is called current. The bulb provides resistance to electron flow; the bigger the bulb, the larger the voltage source required to maintain a steady current.

## Introduction

In this experiment, the light bulb will be replaced by a segment of graphite. Instead of using bigger or smaller bulbs, you will vary the length of the graphite segment to change the resistance. A constant current will be maintained while the resistance is changed. Using a CBL ${ }^{\text {TM }}$ or CBL $2^{\text {TM }}$, you will measure voltages across increasing lengths of graphite, then use this data to build a mathematical model relating resistor length and voltage.

## Equipment Required

- TI InterActive! ${ }^{\text {TM }}$ software
- CBL or CBL 2 with a compatible TI calculator
- TI voltage probe
- Two alligator clip jumper leads (RadioShack ${ }^{\circledR}$ \#278-1156C)
- A 9-volt battery
- Pair of scissors
- Graphite pencil (No. 2 pencil)


## Setup

| CBL | CBL 2 |
| :--- | :--- | :--- |
| 1.Use a pair of scissors to cut out the box <br> on the last page of this activity. With a <br> graphite pencil, completely darken the <br> rectangle within this box. | 1.Use a pair of scissors to cut out the box <br> on the last page of this activity. With a <br> graphite pencil, completely darken the <br> rectangle within this box. |
| 2.Use the alligator clip jumper leads to <br> connect the positive lead of the battery <br> to the $\mathbf{0} \mathbf{~ c m ~ ( c e n t i m e t e r ) ~ m a r k ~ o n ~ t h e ~}$ <br> rectangle, and the negative lead of the <br> battery to the $\mathbf{1 2} \mathbf{~ c m ~ m a r k . ~}$ | 2.Use the alligator clip jumper leads to <br> connect the positive lead of the battery <br> to the $\mathbf{0} \mathbf{~ c m ~ ( c e n t i m e t e r ) ~ m a r k ~ o n ~ t h e ~}$ <br> rectangle, and the negative lead of the <br> battery to the $\mathbf{1 2} \mathbf{c m}$ mark. |


| CBL |
| :---: |
| 3. Plug the voltage probe into the CH 1 port | of the $\mathrm{CBL}^{\text {TM }}$. In this experiment, you will use the CBL as a stand-alone device to measure voltage.

4. Press the ON/HALT button, then press the MODE key to put the CBL in multimeter mode.
5. Press the CH VIEW button repeatedly until the CH 1 channel, with V appears.
6. Connect the black CBL/CBL 2 voltage lead to the $\mathbf{0} \mathbf{c m}$ mark.

CBL 2
3. Plug the voltage probe into the $\mathbf{C H} 1$ port of the CBL $2^{\text {TM }}$.
4. Connect the CBL 2 to a compatible TI calculator.
5. Run the DATAMATE program. For details on how to load and use the DATAMATE program, see the CBL 2 guidebook.
6. Connect the black CBL/CBL 2 voltage lead to the $\mathbf{0} \mathbf{c m}$ mark.


## Collecting and Recording the Data

1. Start TI InterActive! ${ }^{\text {TM }}$ The software opens to a new, blank document.
2. Title your new document Pencil Me In and add your name and the date. Click the Save button 国 to save and name your document.
3. Click the List button
 to open the Data Editor.
4. Click the empty cell at the top of list L1. Type the first segment length, $\mathbf{0}$, and then press the down-arrow key to move to the next cell. Continue until you have entered all of the segment lengths into L1.
5. With the experiment set up as shown above and the CBL/CBL 2 taking voltage readings, position the red CBL/CBL 2 voltage lead on the $\mathbf{2 c m}$ mark of the graphite segment.
6. Click the empty cell at the top of list $\mathbf{L 2}$. Type the first recorded voltage in the top cell and then press the down-arrow key to move to the next cell.
7. Repeat this procedure until you have entered all of the voltage readings into L2.

Note: Be sure to disconnect the leads from the battery when you are finished collecting the data so that the battery does not overheat.

## Working With the Data

1. Click the Scatter Plot button , then click the Stat Plots tab. In the uppermost text box, type L1 to specify it as the list containing the $x$-coordinates. Press the Tab key to move to the second text box, and type $\mathbf{L 2}$ to specify the list containing the $y$-coordinates.

2. Press Enter, then click the Zoom Statistics button $-=$. The viewing boundaries adjust automatically to show all the plotted data. Your segment length versus voltage plot should appear as a straight line passing through the origin.

If you are not satisfied with your results, start again and collect a new set of data.
If you are satisfied with your data, make a sketch of the segment length versus voltage data that you collected on one of the blank grids in the Appendix. Label the horizontal and vertical axes on your sketch.
3. Click the Save to Document button
$\square$
ए to save the graph in your TI InterActive! ${ }^{\text {TM }}$ document.

## Analysis and Questions

1. When the relationship can be expressed in the form $y=k x$, the variables $x$ and $y$ are said to be directly proportional or vary directly, and $k$ is called the constant of variation. In order to find a direct variation model for segment length and voltage, you will need to find the value of $k$, the constant of variation. You will use the guess-and-check method. Click the $f(x)$ tab in the upper left corner of the Functions window. Start with an initial guess of $k=1$. Type $f(\mathbf{x}):=1^{*} \mathbf{x}$ in the uppermost text box of the $f(x)$ tab. Press Enter to superimpose the graph on the plotted data.

It is unlikely that your first guess for the value of $k$ produced a model that matched the data closely. Click in the text box of the $f(x)$ tab again and edit the direct variation equation, replacing the old value, $k=1$, with your new guess for $k$. Press Enter to update the graph. Repeat the guess-and-check procedure until you find a $k$-value that models the data well and record it in the space below:

$$
k=
$$

2. TI InterActive! ${ }^{\text {TM }}$ lets you check the value of $k$ you just found by calculating the line of best fit.
a. Click the Save to Document button and return to the Data Editor.
b. Click Statistical Regressions 漚。
c. Click the down arrow $\boldsymbol{\rightharpoonup}$ next to Calculation Type, scroll down the list and click on Linear Regression ( $\mathbf{a x}+\mathbf{b}$ ). In the text box labeled $\mathbf{X}$ List, type L1; in the box labeled $\mathbf{Y}$ List, type L2.
d. Click Calculate to find the regression equation, $y=a x+b$, and its variables. Record the regression equation values of $a$ and $b$ below:

|  | $y=a x+\boldsymbol{b}$ |
| :--- | :--- |
| $a=$ |  |
| $b=$ |  |

e. Click Save Results. TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document.
f. Close the Data Editor.

How does the value of $a$ in the linear regression equation compare with the $k$-value you found by guess-and-check?

What should the $b$-value from the regression equation be? Explain.
3. Drag the graph to position it below the Statistical Regressions result if necessary, and then double-click on the graph in the TI InterActive! document to refresh the Graph window. In the second text box of the $f(x)$ tab, type $f(x):=$ regeq( $\mathbf{x}$ ) and press Enter. TI InterActive! graphs the equation that was created as the Statistical Regressions result.
Which equation seems to fit the data better? Which equation is a better direct variation model? Why?
4. Use the direct variation modeling equation you found in this activity to predict the voltage reading corresponding to a 5 cm long graphite segment. Record this value in the table below as the predicted voltage reading.

| Voltage Readings |  |
| :---: | :---: |
| Predicted | Actual |
|  |  |

Test your prediction by positioning the red $\mathrm{CBL}^{\text {TM }}$ voltage lead at the 5 cm mark on the graphite segment you used in this experiment. Be sure the CBL is in multimeter mode (or, if you are using the CBL $2^{\mathrm{TM}}$, be sure that you have executed the DATAMATE program). Record this value in the table above as the actual voltage reading. How do these results compare?
5. Save and print your TI InterActive! document.

## Extensions

- Repeat the experiment, but this time start with an 8 cm segment instead of a 12 cm segment of shaded with graphite paper. Take voltage readings every 2 cm . Note the value of the constant of variation, $k$, between voltage and segment length. Repeat for a 20 cm segment of graphite paper (take readings every 5 cm ). Record the $k$-value. Does there appear to be a relationship between the total length of the shaded segment and the voltage of the battery?

Note: To control experimental conditions, try to keep the width and thickness of the graphite shading constant in both cases.

- Use the bottom of a coffee can and a graphite pencil to make a circle on a blank sheet of paper, leaving a 1 cm gap between the starting and ending points of the circle. Color over the circle's outline with the pencil (but maintain the gap), making the width of the circle's outline about $1 / 2 \mathrm{~cm}$. Use a ruler or measuring tape to mark off $2 \mathrm{~cm}, 4 \mathrm{~cm}$, and 8 cm lengths on the circle measured from the gap. Attach the jumper cables from the battery to starting and ending points of the circle. Be sure these two cables are not in contact with each other.

Set up the experiment as before and record voltage readings for $2 \mathrm{~cm}, 4 \mathrm{~cm}$, and 8 cm lengths. Develop a direct variation modeling equation for this experiment. Measure the voltage reading across the entire circle (with the red CBL voltage lead positioned at the end point of the circle). Use this reading together with the direct variation modeling equation to predict the circle's circumference. Measure the diameter of the circle, then compute the circumference using the formula $\mathrm{C}=\pi \mathrm{d}$. How does this result compare with your prediction?

> \&< Cut along dotted line


## Teacher Notes

## Activity 4: Pencil Me In



## Math Concepts

- $\mathrm{CBL}^{\text {™ }} / \mathrm{CBL} 2^{\text {TM }}$
- Direct Variation
- Linear Function


## Activity Notes

- The segment that is colored in with pencil must be very dark and consistent to get good data.
- Remind students to disconnect the battery leads as soon as they finish collecting data so that the battery does not become overheated. This will extend the life of the battery.


## Sample Data

| Segment Length <br> (cm) | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Voltage Reading <br> (volts) | 0 | 1.04 | 2.13 | 3.55 | 5.14 | 6.63 | 8.72 |

## Analysis and Questions - Key

1. $k=0.68$.
2. $\quad a=0.721, b=-0.436$. The $a$ - and $k$-values are in close agreement; the $b$-value should equal zero.
3. Both fit the data well. The regression equation is not a direct variation equation since the $y$-intercept is non-zero.
4. Predicted $=3.40 ;$ actual $=2.93$. They are in reasonably close agreement.
