

Sum Rectangles

ID: 12099

Time Required
15 minutes

Activity Overview

In this activity, students will graphically and numerically explore Riemann sums and develop an understanding of summation notation for adding these rectangles.

Topic: Riemann sums

- *Graphically exploring Riemann sums*
- *Summation notation*

Teacher Preparation and Notes

- *The file `area.89p` is a program file. It contains several approximation methods using Riemann sums, the Trapezoidal Rule, and Simpson's Rule. This program should be sent to student graphing calculators prior to beginning this activity*
- ***To download the calculator program file (.89p file) and student worksheet, go to education.ti.com/exchange and enter "12099" in the keyword search box.***

Associated Materials

- *SumRectangles_Student.doc*
- *main.area.89p*

Suggested Related Activities

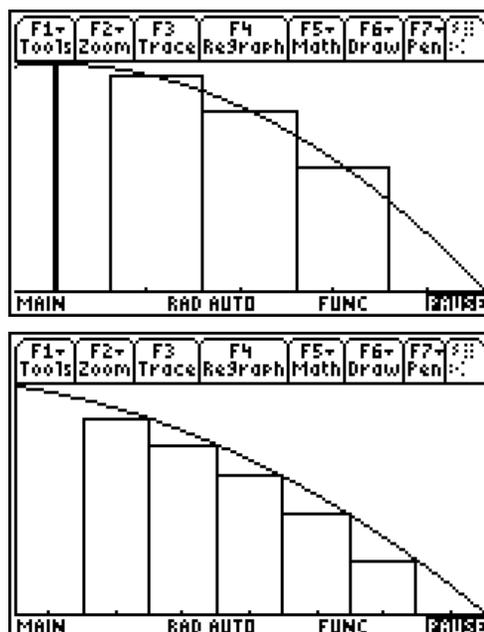
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Exploring the Area Under a Curve (TI-Nspire CAS technology) — 8268*
- *Riemann's Sums (TI-Nspire CAS technology) — 9578*

Part 1 – Graphical Riemann Sums

Students will use the program **area** to complete this part of the activity. To run the program enter **area()** on the Home screen. To use the program, select to view the approximation either graphically or numerically. Next, enter the equation that you are examining and press **ENTER**. Enter the minimum value, maximum value, and the number of steps. Then, select the approximation method and press **ENTER**. The approximation will be displayed on the screen. Then, select to enter another function, do another approximation or to exit the program.

Many questions can be asked and observations made. This section can be extended by asking about other functions. Students should graph other functions that have different concavity or slope to complete Exercise 3.



Students get a visual of what Riemann sums are. If only a few rectangles are graphed students may be able to better see and understand why the midpoint is an over approximation in some situations and an under approximation at other times.

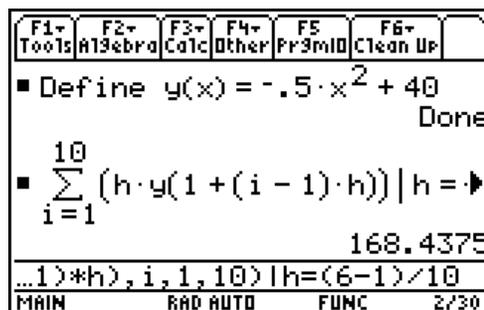
Student Solutions

1. The midpoint approximation is between the left and right endpoint approximations. The left is too big and the right too small.
2. The Riemann sums converge (to the definite integral) as $n \rightarrow \infty$. With thinner widths, the rectangles approach the true area.
3. (a) over, (b) under, (c) over, (d) under. For example, when the function is decreasing and concave down, the function curves more steeply for the second half of the rectangle than the first. Therefore, since there is more rectangle above than below the graph, it over estimates the area.

Part 2 – Summation notation

In Part 2, students examine the summation notation. By reading and answering questions on the student worksheet, students will better understand the sigma notation.

To make entering the summation easier, encourage students to define the function before entering the sum. To enter a summation on the HOME screen, students should select **F3:Calc > 4:Σ(sum**. The syntax for the summation function is as follows:
 $\Sigma(\text{expression, variable, low, high})$



Student Solutions

- $\Delta x = h = 1$
- $y(a) + y(a + 1) + y(a + 2) + y(a + 3) + y(a + 4)$
- The right Riemann sums from a to b with 5 subintervals, looks like $y(a + 1) + y(a + 2) + y(a + 3) + y(a + 4) + y(a + 5)$. Note $a + 5 = b$ since $\Delta x = 1$
- $$\sum_{i=1}^n (h \cdot y(a + (i - 1)h)) \mid h = \frac{6 - 1}{n}$$

When $n = 10$, Left Riemann sum = 168.4375

When $n = 20$, Left Riemann sum = 166.328125

When $n = 50$, Left Riemann sum = 165.0375

When $n = 100$, Left Riemann sum = 164.603125

Extension – Area Programs

Students will use the **area** program to compare area approximation methods.

Student Solutions

- | | | |
|---------------|---------------|----------------|
| $n = 10$ | $n = 50$ | $n = 100$ |
| mid = 71.5625 | mid = 71.6625 | mid = 71.6656 |
| trap = 71.875 | trap = 71.675 | trap = 71.6688 |
- For $y = x^2$, an increasing concave up function in this domain, the midpoint is approaching $71 \frac{2}{3}$ from the left and the trapezoid method is approaching the value of the definite integral from the right. Midpoint gives a better approximation.