Applications of Parabolas 1.0

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Student Activity

7 8 **9** **10** **11** 12

TI-Nspire™ Investigation Student 50 min

Applications of Parabolas

Teacher Notes:

The purpose of this activity is to extend student understanding around *how* a parabola focuses all types of waves, sound, light or otherwise. The video tutorial briefly jumps into some physics showing how sound might be considered as a wave and similarly with electromagnetic waves. Waves are then considered as a set of points on a straight line; this allows us to do some mathematics to calculate what happens to each point on the wave.

Each point on the wave is reflected from the parabola back through the focal point. Students calculate the distance from the line to the curve (substitution) then from the curve through the focal point (distance between two points).

Regardless of the coordinates chosen for the point on the line, the distance is the same, this is a natural result of the construction and definition of a parabola. This is not trivial. If the points on the wave travelled different distances to pass through the focal point the wave would be completely jumbled. Sounds would be unrecognisable, light would be different colours, and the radio waves collected from billions of light years away would be completely useless!

In addition to gaining a deeper insight to parabolas and their applications, this activity is designed to highlight how different areas of mathematics overlap, in this case, the distance between two points and some simple geometry. This purposeful review provides a great opportunity to include some spaced retrieval!

Fun Fact:

The 500m diameter radio telescope in China can be dilated to change the focal length! That is one very big and expensive mathematical example of dilating a parabola!

Australian Curriculum Standards

AC9M9A03
Find the gradient of a line segment, the midpoint of the line interval and the distance between 2 distinct points on the Cartesian plane.

AC9M10A04
Use mathematical modelling to solve applied problems … choosing to apply linear, quadratic or exponential models; interpret solutions in terms of the situation; evaluate and modify models as necessary and report assumptions, methods and findings.

AC9M10SP01
Apply deductive reasoning to proofs involving shapes in the plane and use theorems to solve spatial problems.

Lesson Notes

Students will need access to the "Parabola Applications 1" file. This file can be downloaded from the Texas Instruments Australia website. To transfer the file from a computer to a students' calculator, use the TI-Nspire teacher software or the web-browser tool: https://nspireconnect.ti.com/nsc/

Instructions on how to transfer files using the web-browser tool: https://youtu.be/IXgHSvQpBh4

Author: P. Fox

Calculator Instructions: Applications of Parabolas

Watch the video on Parabola Applications 1.0. to help answer the question:

Where are we ever going to use this in the real world?

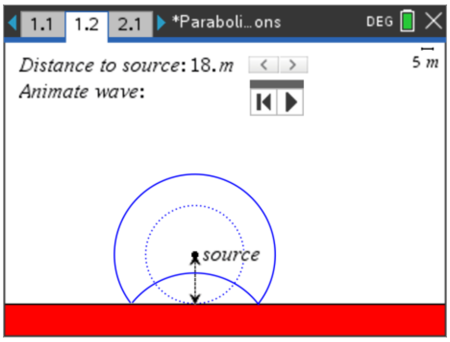
Check out the world's largest parabola measuring 500m in diameter! This behemoth of a telescope is listening to the universe in search of intelligent life beyond our solar system?

While it is not specifically looking for potential NEOs (Near Earth Objects), it would certainly be able to detect one!

https://youtu.be/Qg5RHJ0uUYY

Search CNEOs – Centre for Near Earth Objects to find out more satellites (parabolic reflectors) are being used to help save the world!

Open the TI-Nspire file: Parabolic Applications 1



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A wave is emitted from the source and travels uniformly outwards in all directions. This could be a sound, light or radio wave. In the diagram opposite part of the wave has hit the wall and reflected.

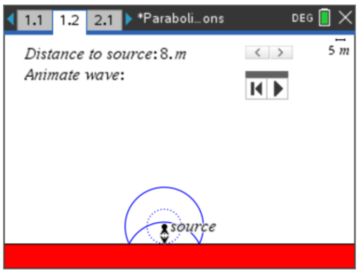
Adjust the distance to the source (further from the wall) and notice what happens to the 'curvature' of the wave.

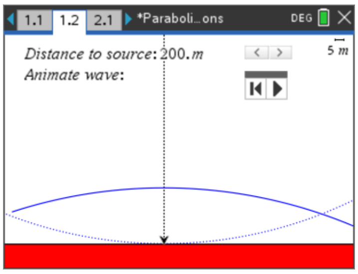
Note: Animations take a **lot** longer to come into the picture when the source is a long way away.

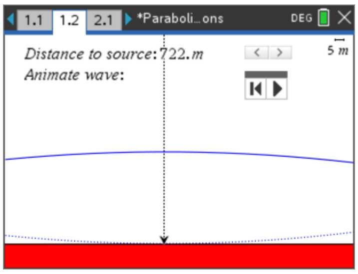
Question: 1.

What happens to the curvature of the wave as the source is moved further from the wall (surface)?

Answer: With the distance to source set at 200+m the wavefront appears straighter than when the source is positioned at 18m and even straighter again when the source moves to 722m. [see below]







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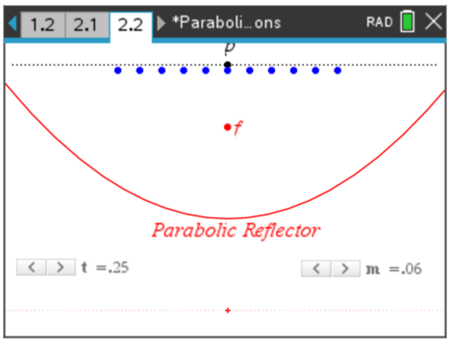

The shape of the parabolic reflector can be changed using slider: '**m**'.

A straight wave front passing through P can be thought of as a series of points. Press slider '**t**' repeatedly, or animate it, to see how each point on the wavefront is reflected when it hits the parabolic reflector.

Author: P. Fox

Question: 2.

What happens to all the points after they strike the parabolic reflector?

Answer: All the points reflect through the focus. The points reach the focus at the same time!

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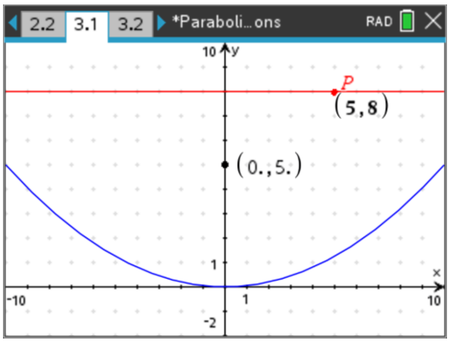
A parabola with equation: $y = 0.05x^2$ has been graphed. The focal point is shown, so too a wave, represented by the line $y = 8$.

Point P is shown with the coordinates: (5, 8). This point can be moved vertically to gain an estimate of how far it is from the line to the curve. Point P moves parallel to the y axis, in the same direction as the wave.



Question: 3.

Point P currently has the coordinates: (5, 8). How far, vertically, is point P from the parabola?

Answer: Current y value: 8. When point P lands on the parabola: $y = 0.05(5)^2 = 1.25$. 8 – 1.25 = 6.75

Question: 4.

When point P strikes the surface of the parabolic reflector, how far will it be from the focal point (0, 5)?

Answer: Using the previous coordinates (5, 1.25), point P will be: $d = \sqrt{(5-0)^2 + (5-1.25)^2}$ = 6.25

Question: 5.

What is the total distance travelled by point P (on the wave) as it travels from $y = 8$ and finally reaches the focal point?

Answer: Sum of the previous two answers: 6.25 + 6.75 = 13

Question: 6.

Write down the coordinates of three other points on the wave at: $y = 8$. Determine each of the distances travelled as they move along the wave, reflect from the parabola and finally reach the focal point. Comment on the result.

Answer: The answer will be the same for each point: Total distance = 13 units. All points on the wave front will travel the same distance on their journey to the parabolic reflector and finally through the focal point.
Comment: If the distances were different, in the case of sound, the sound would be incoherent as it passes through the focal point.

Question: 7.

The wave front is moved to a new location: $y = 6$. Write down the coordinates of 3 new points on this wave front. Determine the distance each of your points will travel as they travel along the wave, reflect from the parabolic mirror and then pass through the focal point. Comment on the result.

Answer: The answer will be the same. Each point will travel a total distance of 11 units. As before, the points all travel the same distance (11 units) as they reflect from the parabola and pass through the focal point.
Comment: As above, they must travel the same distance otherwise the resulting wave at the focal point would no longer be 'coherent'.

Question: 8.

Change the curvature of the parabola so that it has the equation: $y = 0.1x^2$, the focal point will move accordingly. Shift the wave front so that it is represented by the line: $y = 5$. Write down the coordinates of 3 new points on this wave front. Determine the distance each of your points will travel as the reflect from the parabolic mirror and pass back through the focal point.

Answer: The answer will be the same for each point. The new total distance will be: 7.5 units.

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Extension:

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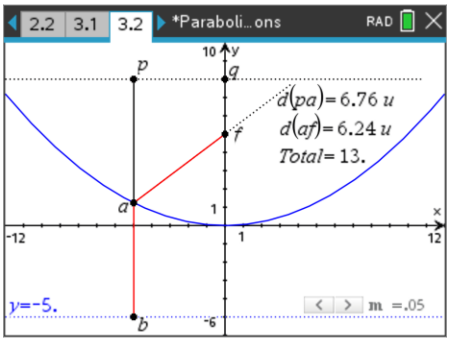

- The line PQ is parallel to the directrix (line passing through b).
- Point Q can be moved up or down along the y axis.
- Point P can be moved along the line PQ.
- The curvature of the parabola can be changed with slider (m).

Question: 9.

Move point P along the line PQ. What do you notice about the total distance: PA + AF?

Answer: The distance remains constant.

Question: 10.

Move point Q. Once again, move point P along the line PQ. What do you notice this time?
Note: Only consider situations where '*a*' is below the wave front[1].

Answer: The distance is different than the previous question, but the total distance remains constant.

Question: 11.

Recall the definition of a parabola, use this to help explain your observations in the previous questions.

Answer: Distance PB is constant since the line passing through b is parallel to line PQ. The definition of a parabola is the set of points equidistant from a line (directrix) and a point (focus). This means that distance AB is the same as distance AF; therefore PA + AF = PA + AB = PB. The journey from point P, through A onto F remains constant, therefore particles on the wave, travelling at the same speed, will cover the same distance in the amount of time!

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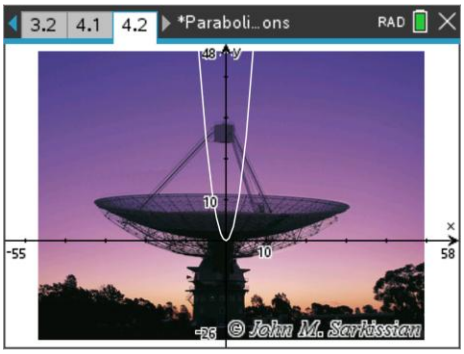

The image in the Graphs application is of the Radio Telescope at Parkes (Australia). The window settings have been scaled to suit the image.

Move the mouse over the graph until it says "graph f1" and shows a double-sided arrow: ⤡

Drag (dilate) the graph until it models the curvature of the telescope, then press:

menu > **Analyse Graph** > **Analyse Conics** > **Foci**

The focal point of the parabola will appear.

Question: 12.

Based on your equation to model the curve of the dish, determine the approximate distance between the dish and the focal point.

Answer: Students should obtain a distance between: 25m and 27m. Actual distance = 26.24m

[1] This is a limitation of the construction, not the geometry.

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Question: 13.

The dish has a diameter of 64 metres, assuming all the energy from a reflected wave arrives at the focal point, with a collection area of radius 5cm. Determine the increase in the intensity.

Answer:

Area of dish = $3200^2\pi \approx 1.024\pi \times 10^7$ cm^2 .

Focal area: 25π

$\therefore$ Intensity (energy) increases by a factor of 4.096×10^5.

The strength of the signal would be amplified by a factor of almost 0.5 million times!

Question: 14.

The giant dish in China has a diameter of 500 metres, assuming all the energy from a reflected wave arrives at the focal point. If the collection area at the focus has a radius of 10cm. Determine the increase in intensity.

Answer:

Area of dish = $25000^2 \times \pi \approx 6.25\pi \times 10^8$ cm^2

Focal area = 100π

$\therefore$ Intensity (energy) increases by a factor of 6.25×10^6.

The strength of the signal would be amplified by a factor of more 6 million! Naturally this factor would be even greater if the focal area is smaller, this comes down to how accurately the structure of this size can be built.

Author: P. Fox