# Energy of Free Oscillations – ID: 12192

By Irina Lyublinskaya

Time required
*45 minutes*

## Topic: Circular and Simple Harmonic Motion

- *Use the equation ke = (1/2)$mv^2$ to solve problems involving mass, velocity, and kinetic energy.*
- *Use the equation pe = (1/2)$kx^2$ to solve problems involving spring–mass systems and elastic potential energy.*
- *Calculate the amount of mechanical energy contained in a system.*
- *Solve problems using the law of conservation of mechanical energy.*

## Activity Overview

*In this activity, students explore energy transformation and energy conservation in simulated spring–mass oscillations. Students derive equations for the elastic potential energy of the spring and the kinetic energy of the oscillating mass and observe changes in both values caused by variations in the position and velocity of the mass. Students then apply these concepts to problem solving.*

## Materials

*To complete this activity, each student or student group will require the following:*

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

## TI-Nspire Applications

*Graphs & Geometry, Lists & Spreadsheet, Data & Statistics, Notes, Calculator*

## Teacher Preparation

*Before carrying out this activity, you should review with students the concepts of free oscillations, period of oscillations for a spring–mass system, the equation of motion for an oscillating mass, and the equations for elastic potential energy and kinetic energy.*

- *The screenshots on pages 2–8 demonstrate expected student results. Refer to the screenshots on pages 9–10 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "12192" in the search box.***

## Classroom Management

- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration during this activity:

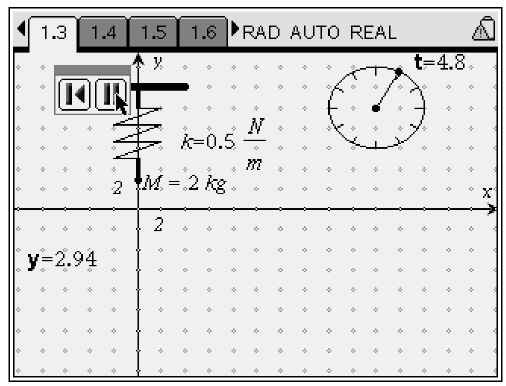
- What is the relationship between the kinetic energy of an oscillating mass and the potential energy of an elastic spring during free oscillations?
- What happens to the total mechanical energy of the system in the absence of any forms of resistance?

The purpose of this activity is to allow students to observe simulated spring–mass oscillations and collect data for the position and velocity of the oscillating mass. Based on these data, students will derive equations for the elastic potential energy of the spring and the kinetic energy of the oscillating mass and derive the law of conservation of mechanical energy.

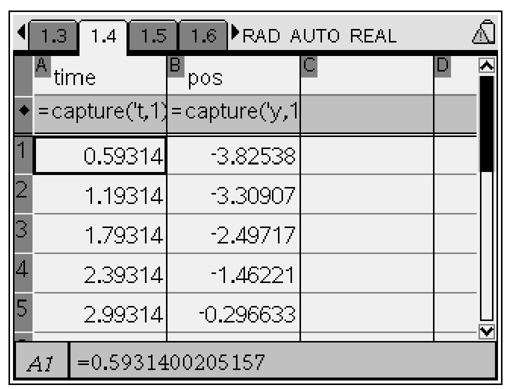
This activity consists of three problems. In the first problem, students collect position and velocity data from a simulation and graph the position and velocity of the oscillating mass. In the second problem, students derive equations for potential and kinetic energy, graph both equations and the total mechanical energy of the system, and confirm the law of conservation of mechanical energy. In the third problem, students apply the law of conservation of mechanical energy to problem solving. They use simulations to check their work.

## Problem 1 – Exploring the position and velocity of an oscillating mass

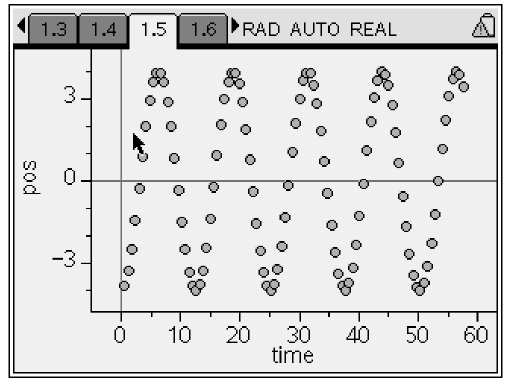
**Step 1:** Students should open the file **PhyAct_12192_Energy_of_Oscillations.tns**, read the first two pages, and then move to page 1.3, which shows a mass attached to a spring suspended vertically. In this simulation, *t* represents the time in seconds and the clock measures up to 60 sec. The variable *y* represents the displacement of the mass from equilibrium, in meters. Students will use the animation to observe the changes in the position of the oscillating mass as time changes.



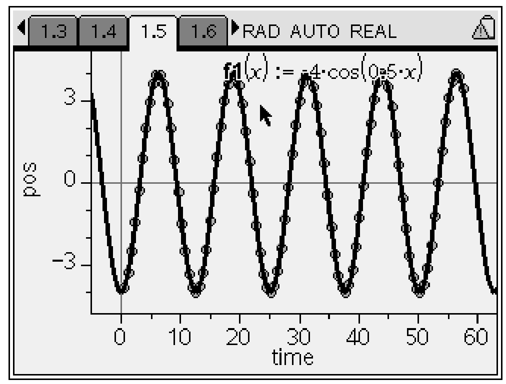

**Step 2:** Students should start the animation and allow it to run for 60 sec. The data for the position and time will be automatically captured in the *Lists & Spreadsheet* application on page 1.4.



**Step 3:** Next, students should use the *Data & Statistics* application on page 1.5 to make a graph of position vs. time for the mass. They should use *time* as the *x*-axis variable and *pos* as the *y*-axis variable. The scatter plot will display position as a function of time.



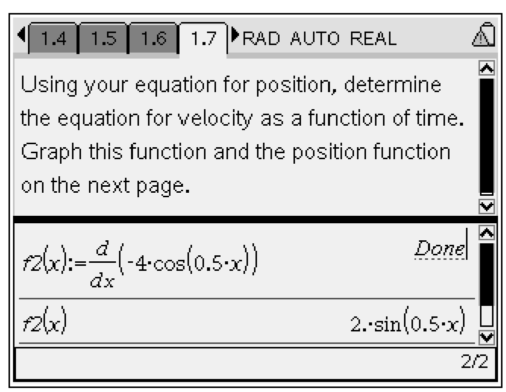

**Step 4:** Next, students should analyze the scatter plot and derive the equation for position as a function of time. Students can derive the equation in one of two ways. They can use the equation that describes the position of an oscillating mass and the information about the physical properties of the system (spring constant, mass, and initial displacement) to derive the equation. Or, they can use the properties of the data (amplitude, frequency, intersection points) to derive the equation mathematically. Either way, they should plot the function they derive on the graph (**Menu > Analyze > Plot Function**) and modify it if necessary until it fits the data. Then, they should answer question 1.



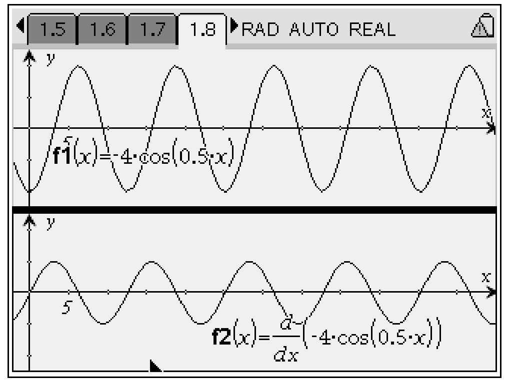

**Q1.** What is the equation describing the relationship between position and time?

**A.** *The equation should be pos = –4·cos(0.5·time). Encourage students to discuss how they derived the equation from the properties of the system or the properties of the graph.*

**Step 5:** Next, students should use the *Calculator* application on page 1.7 to determine the equation for velocity from the position equation. They should use the derivative template to determine the equation for velocity. (The derivative template can be accessed by pressing ctrl.) They should set this equation equal to **f2**(*x*). (The position equation should have been automatically assigned to function **f1**(*x*).) Students can press ctrl tab to move between applications. If students do not have TI-Nspire CAS technology, they can enter the derivative equation directly.



**Step 6:** Next, students should move to page 1.8, which contains two *Graphs & Geometry* applications. They should graph position as a function of time, **f1**($x$), on the top graph and velocity as a function of time, **f2**($x$), on the bottom graph. To make the graphs easier to examine, students can hide the function entry lines by pressing ctrl G. (If you wish, you may have students plot the derivative equation directly—i.e., plot the equation **f3**($x$) = $2 \cdot \sin(0.5 \cdot x)$—instead of plotting **f2**.) Once students have graphed the two equations, they should answer questions 2 and 3.




**Q2.** What can you say about the velocity of the oscillating mass when it passes through the equilibrium position?

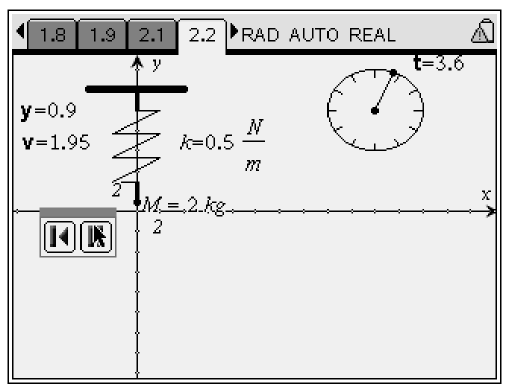
**A.** *The magnitude of velocity is maximum.*

**Q3.** What can you say about the position of the oscillating mass when its velocity is zero?

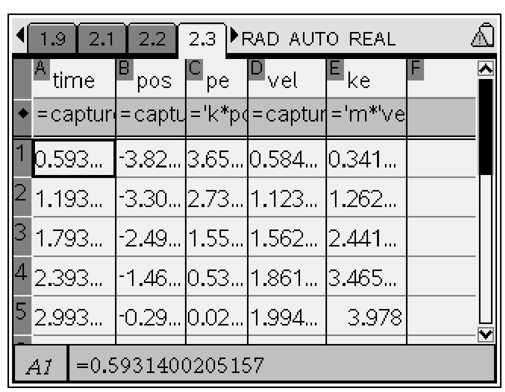
**A.** *The mass is displaced furthest from the equilibrium position.*

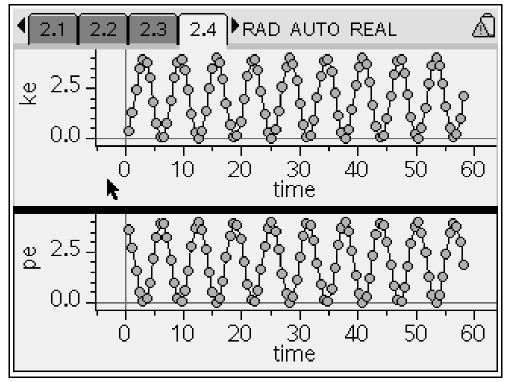
## Problem 2 – Exploring conservation of mechanical energy

**Step 1:** Next, students should read page 2.1 and then move to page 2.2, which contains a simulation similar to that on page 1.3. In this simulation, both the position (**y**) and the velocity (**v**) of the mass are shown. Students should run the animation for 60 sec. The data are collected automatically in the *Lists & Spreadsheet* application on page 2.3. Note: As the TI-Nspire collects more data, it may run more slowly. To increase the speed, you may have students delete the data from problem 1 on page 1.4 before carrying out the animation on page 2.2.)




**Step 2:** Next, students should move to page 2.3 to study the collected data. Page 2.3 also contains formulas calculating the elastic potential energy of the spring (**pe**) and the kinetic energy of the mass (**ke**) as functions of time. These values are calculated using the equations $pe = \frac{1}{2}ky^2$ and $ke = \frac{1}{2}mv^2$.






**Step 3:** Next, students should move to page 2.4, which contains two *Data & Statistics* applications. Students should graph the kinetic energy of the oscillating mass (**ke**) vs. time on the top graph. They should graph the elastic potential energy of the spring (**pe**) vs. time on the bottom graph. To make the trends in the data easier to see, students can connect the data points in each graph (**Menu > Plot Properties > Connect Data Points**). (Note: If students run the simulation for more than 60 sec, their plots will have diagonal lines connecting the first and last data points. If this occurs, students can either delete the data and rerun the simulation or simply ignore the diagonal lines on the screen.) Once they have made the two graphs, students should answer questions 4 and 5.

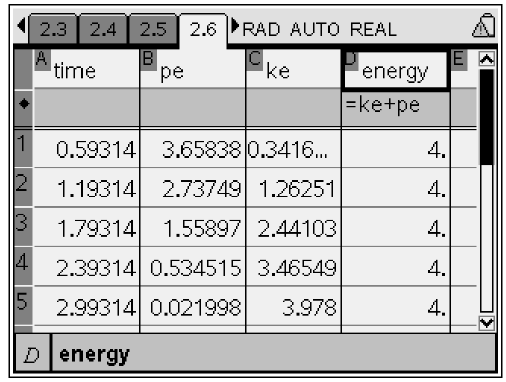
**Q4.** What statement can you make about the relationship between the kinetic energy of the mass and the potential energy of the spring? How does this relate to energy transformation?

**A.** *The kinetic energy of the mass is transformed into the potential energy of the spring. When kinetic energy reaches its maximum value, potential energy is zero; when potential energy reaches its maximum value, kinetic energy is zero.*

**Q5.** What prediction can you make about the total mechanical energy of the system?

**A.** *Students' answers will vary. If students interpret the graphs correctly, they should predict that the total mechanical energy, which is the sum of the kinetic energy of the oscillating mass and the elastic potential energy of the spring, is constant in the absence of air resistance and friction.*

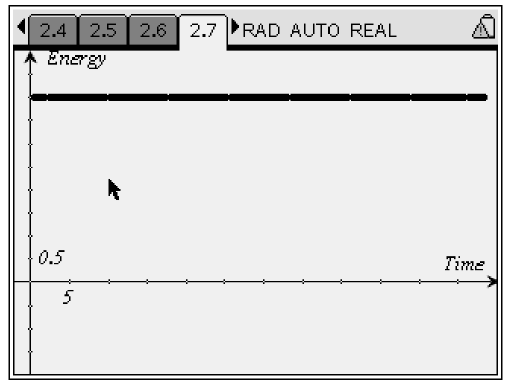
**Step 4:** To test whether the relationships identified in questions 4 and 5 are correct, students should move to page 2.6. This page contains a *Lists & Spreadsheet* application in which the total mechanical energy of the system is calculated at each moment of time. Students should examine the data.



| | A time | B pe | C ke | D energy |
|---|---|---|---|---|
| ◆ | | | | =ke+pe |
| 1 | 0.59314 | 3.65838 | 0.3416... | 4. |
| 2 | 1.19314 | 2.73749 | 1.26251 | 4. |
| 3 | 1.79314 | 1.55897 | 2.44103 | 4. |
| 4 | 2.39314 | 0.534515 | 3.46549 | 4. |
| 5 | 2.99314 | 0.021998 | 3.978 | 4. |



**Step 5:** Next, students should move to page 2.7, which shows a scatter plot of total energy as a function of time. Students should study the graph and then answer questions 6 and 7.




**Q6.** Do the data on the previous two pages support the prediction you made in question 5? If not, identify any errors in your reasoning.

**A.** *Students' answers will vary.*

**Q7.** Use the equations for position, velocity, kinetic energy, and elastic potential energy to prove that the total mechanical energy of the system is constant over time.

**A.** *The position of the mass as a function of time is given by* $y(t) = y_0 cos(\omega t)$. *The equation for velocity as a function of time is* $v(t) = -\sqrt{\frac{k}{m}} y_0 sin(\omega t)$ *(because* $v(t) = -\omega y_0 sin(\omega t)$, *and* $\omega = \sqrt{\frac{k}{m}}$ *). Kinetic energy is given by the equation* $ke = \frac{1}{2} mv^2$, *and potential energy is given by* $pe = \frac{1}{2} ky^2$. *Combining these equations yields the equation for total mechanical energy of the system:*

$E = \frac{1}{2} k[y_0 cos(\omega t)]^2 + \frac{1}{2} m\left(\sqrt{\frac{k}{m}} y_0 sin(\omega t)\right)^2$. *This equation can be simplified as follows:*

$E = \frac{1}{2} ky_0^2 [cos^2(\omega t) + sin^2(\omega t)] = \frac{1}{2} ky_0^2$. *Therefore, the total mechanical energy of the system is independent of time. If you wish, you may have students solve this equation to verify that the total mechanical energy of the system in the simulation is 4 J:*

$E = \frac{1}{2} ky_0^2 = \frac{1}{2}(0.5\ N/m) 16\ m^2 = 4\ J$.

## Problem 3 – Problem solving

**Step 1:** Next, students should read page 3.1, and then move to page 3.2 to solve the first problem.

**Q8.** A 0.321 kg mass is attached to a spring with a spring constant of 12.3 N/m. The mass is displaced 0.256 m from equilibrium and released. What is the total mechanical energy of the spring? What is the potential energy when the spring is 0.128 m from equilibrium? What is the kinetic energy at this point? What is the speed of the mass at this point?

**A.** *The total mechanical energy of the system is given by* $E = \frac{1}{2}ky_0^2$. *Therefore, the total mechanical energy of the system is* $E = \frac{1}{2}ky_0^2 = \frac{1}{2}(12.3\ \text{N/m})(0.256\ \text{m})^2 = 0.403\ \text{J}$. *Potential energy at any point is given by* $pe = \frac{1}{2}ky^2$. *Therefore, the potential energy at 0.128 m is* $pe = \frac{1}{2}ky^2 = \frac{1}{2}(12.3\ \text{N/m})(0.128\ \text{m})^2 = 0.101\ \text{J}$. *Kinetic energy can be calculated by difference: ke = E – pe = 0.403 J – 0.101 J = 0.302 J. Kinetic energy is given by* $ke = \frac{1}{2}mv^2$. *Therefore, velocity can be calculated using*

$$v = \sqrt{\frac{2ke}{m}} = \sqrt{\frac{(2)(0.302\ \text{J})}{0.321\ \text{kg}}} = 1.37\ \text{m/s}$$

*. Alternatively, students can use the equation for total energy,* $E = \frac{1}{2}ky_0^2 = \frac{1}{2}ky^2 + \frac{1}{2}mv^2$, *and solve directly for velocity: Thus, velocity is given by* $v = \sqrt{\frac{k}{m}(y_0^2 - y^2)}$. *Substituting the given values yields*

$$v = \sqrt{\frac{12.3}{0.321}(0.256^2 - 0.128^2)} = 1.37\ \text{m/s}$$

*. Students can use the simulation on page 3.4 to check their answers. Note that the precision of the animation may not allow students to produce the exact displacement stated in the problem. Therefore, they should use the animation as an approximate check only.*

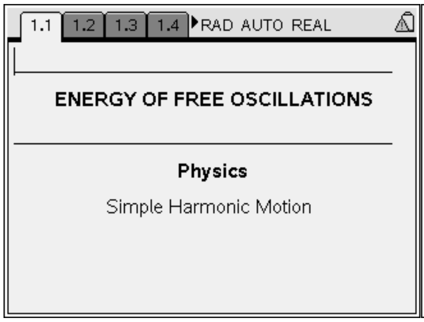
 

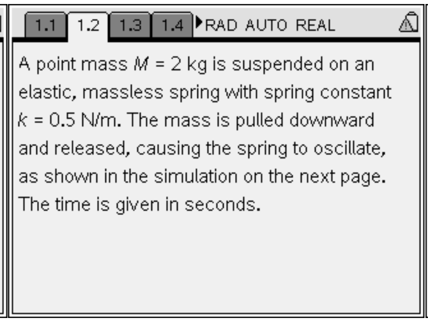
**Q9.** A 500 g block on a spring is pulled 20 cm and then released. The subsequent oscillations are measured to have a period ($T$) of 0.80 sec. At what position (or positions) is the speed of the block 1.0 m/s? (Hint: Remember that $\omega = \frac{2\pi}{T}$ and $k = m\omega^2$.)

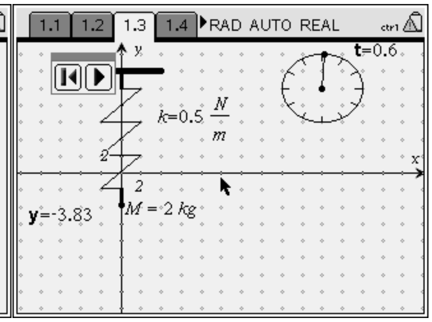
**A.** *$\omega = \frac{2\pi}{T} = 7.85\ rad/s$. From earlier, the equation for the total mechanical energy of the system is given by $E = \frac{1}{2}ky_0^2 = \frac{1}{2}ky^2 + \frac{1}{2}mv^2$, where $k = m\omega^2$. Thus, $y = \pm\sqrt{y_0^2 - \frac{v^2}{\omega^2}}$. Substituting the given values yields $y = \pm\sqrt{0.2^2 - \frac{1.0^2}{7.85^2}} = \pm 0.154\ m$. Again, students can use the simulation on page 3.4 to check their answers. Alternatively, students can again solve the problem in steps. They would first calculate* k*, then calculate the total energy of the system, then calculate kinetic energy, then calculate potential energy, and finally calculate the position of the mass at the given speed.*

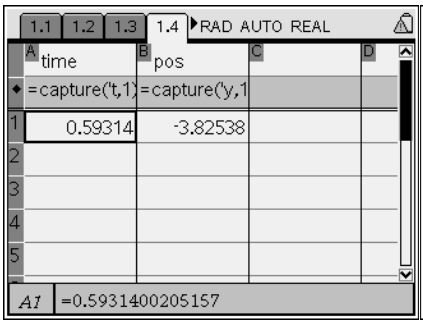
# Energy of Free Oscillations – ID: 12192

(Student)TI-Nspire File: *PhyAct_12192_Energy_of_Oscillations.tns*

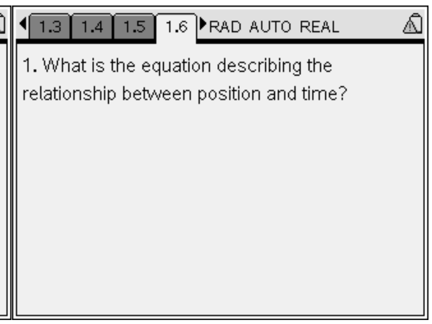


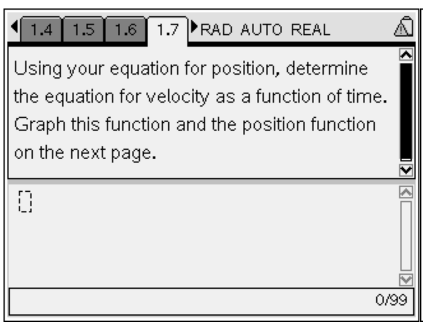



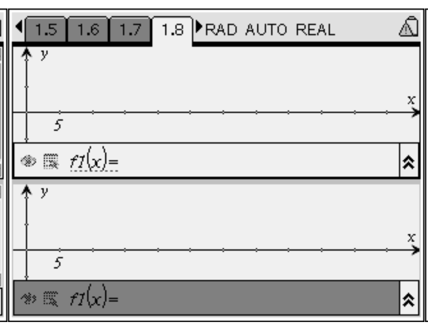



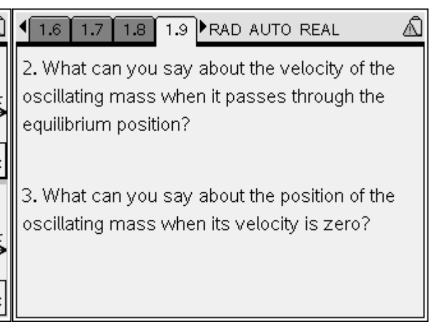



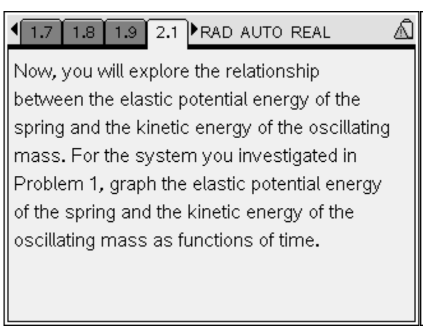


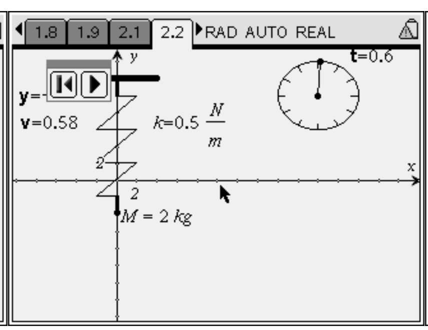


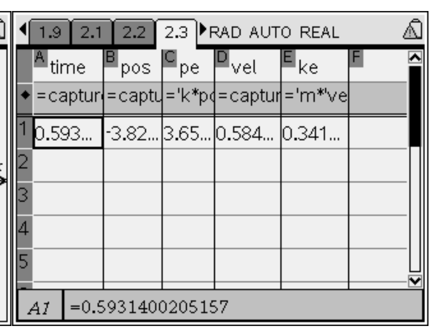



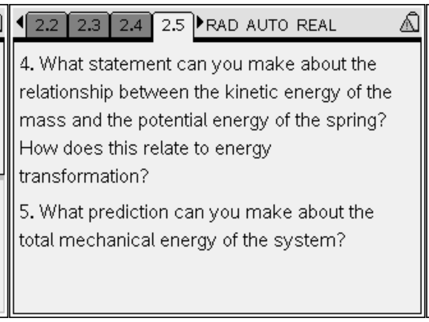



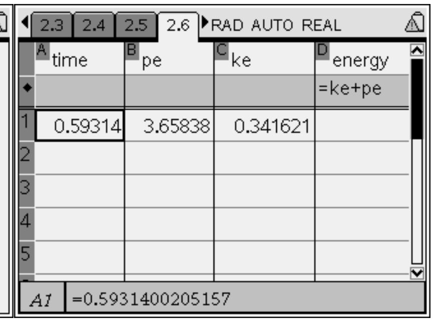





2.2 2.3 2.4 2.5 RAD AUTO REAL

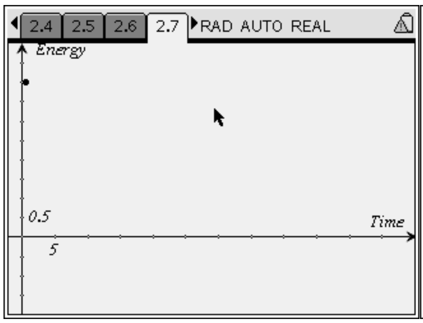
4. What statement can you make about the relationship between the kinetic energy of the mass and the potential energy of the spring? How does this relate to energy transformation?

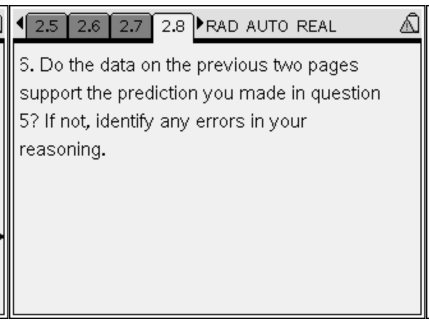
5. What prediction can you make about the total mechanical energy of the system?


2.3 2.4 2.5 2.6 RAD AUTO REAL

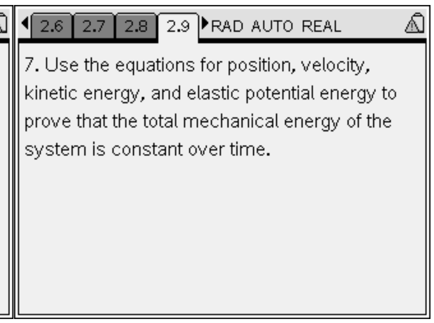
| | A time | B pe | C ke | D energy |
|---|---|---|---|---|
| ◆ | | | | =ke+pe |
| 1 | 0.59314 | 3.65838 | 0.341621 | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |

A1 =0.5931400205157

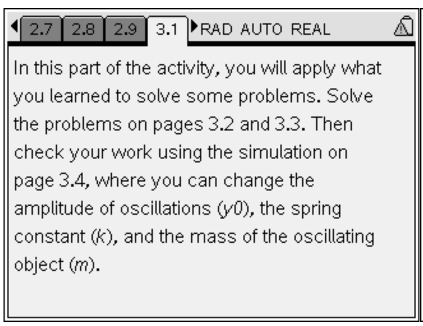




2.5 2.6 2.7 2.8 RAD AUTO REAL

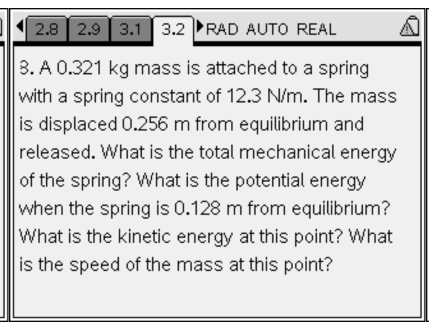
6. Do the data on the previous two pages support the prediction you made in question 5? If not, identify any errors in your reasoning.


2.6 2.7 2.8 2.9 RAD AUTO REAL

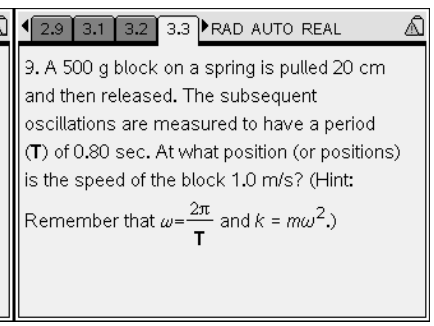
7. Use the equations for position, velocity, kinetic energy, and elastic potential energy to prove that the total mechanical energy of the system is constant over time.


2.7 2.8 2.9 3.1 RAD AUTO REAL

In this part of the activity, you will apply what you learned to solve some problems. Solve the problems on pages 3.2 and 3.3. Then check your work using the simulation on page 3.4, where you can change the amplitude of oscillations ($y0$), the spring constant ($k$), and the mass of the oscillating object ($m$).


2.8 2.9 3.1 3.2 RAD AUTO REAL

8. A 0.321 kg mass is attached to a spring with a spring constant of 12.3 N/m. The mass is displaced 0.256 m from equilibrium and released. What is the total mechanical energy of the spring? What is the potential energy when the spring is 0.128 m from equilibrium? What is the kinetic energy at this point? What is the speed of the mass at this point?


2.9 3.1 3.2 3.3 RAD AUTO REAL

9. A 500 g block on a spring is pulled 20 cm and then released. The subsequent oscillations are measured to have a period (**T**) of 0.80 sec. At what position (or positions) is the speed of the block 1.0 m/s? (Hint: Remember that $\omega = \frac{2\pi}{\mathbf{T}}$ and $k = m\omega^2$.)