Mathematical Methods (CAS) 2002 *Examination* 2 *solutions Q 1 to 3*

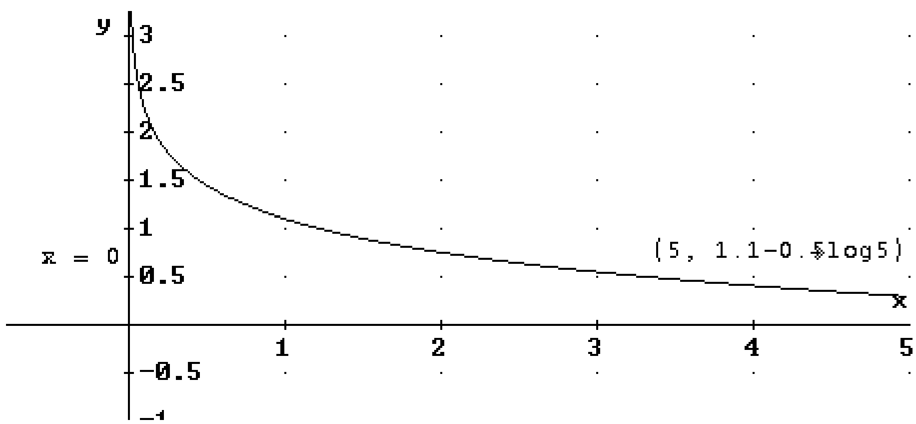
Note: To use *Derive* efficiently, students should be familiar with the 'tick plus equals' and 'tick plus approximately equals' evaluation buttons. These simultaneously 'author' and 'evaluate' expressions exactly and numerically respectively.For example, the 'tick plus equals' or 'author and exact evaluation' button works well for Question 1, while the 'tick plus approximately equals' or 'author and numerical approximation' works well for Question 2, and both are used in Question 3.Students should also be familiar with the use of defined functions, where f(x):= *the rule of the function*, such as in Question 1.

Question 1

#1: $f(x) :\approx 1.1 - 0.5\cdot LOG(x)$

#2: $f(5)$

#3: $$\frac{11}{10} - \frac{LN(5)}{2}$$

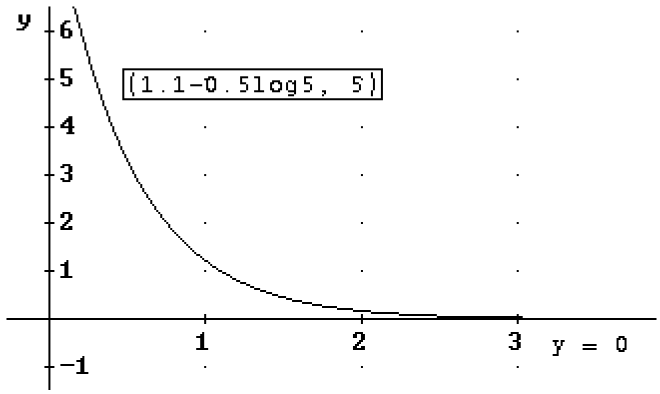


The inverse function of f exists as f is a 1-1 function. Its rule can be found by solving the equation s = f(r) for r:

#4: SOLVE(s = f(r), r)

#5: $$r = e^{11/5 - 2\cdot s}$$

The domain of f inverse is the same as the range of f, [1.1-0.5log5, infinity).



#6: SOLVE([a - b·LOG(1) = 0.5, a - b·LOG(1.5) = 0.3], [a, b])

#7: $$\left[a = \frac{1}{2} \wedge b = \frac{1}{5\cdot LN\left(\frac{3}{2}\right)}\right]$$

These two equations can also be solved readily by inspection: a = 1/2 since log(1) = 0, and substituting this into the second equation gives 0.3 = 1/2-blog(1.5) from which b = 0.2/log(1.5)or 1/5log(1.5)

#8: SOLVE(k·f(x) = t, x)

#9: $$x = e^{11/5 - 2\cdot t/k}$$

This question can be answered easily using the following:

1. f is a decreasing function, therefore its minimum value occurs at its right end domain endpoint(5, 1.1-0.5log(5)).
2. k is a vertical dilation scale factor, and dilates the graph of f *away* from the x axis.
3. for any given positive real value of T, the graph of g(x) = k*f(x) *will* intersect the graph of y = T if the value of T is greater than the minimum value of
g(x) = k*f(x), and hence the equation g(x) = T will have a solution.

Thus, for a solution k(1.1-0.5log(5)must be less than or equal to T, hence the largest value of k for which this will occur is k(1.1-0.5log(5))= T or
k = T/(1.1-0.5log(5)).

Question 2

#10: $$f(x) := \frac{2\cdot x}{a^2}$$

#11: $\text{SOLVE}\left(\int_0^a x\cdot f(x)\,dx = 150,\ a\right)$

#12: $a = 225$

#13: $\int_0^{200} \frac{2\cdot x}{225^2}\,dx$

#14: $\frac{64}{81}$

#15: 0.7901234567

or 0.790 correct to 3 decimal places.

#16: NORMAL(16, 20, 2)

#17: 0.02275013194

or 0.023 correct to 3 decimal places.

Part c is a conceptual question: given Pr(J < 19) = Pr(J > 28) = 0.08, by symmetry the mean μ =(28+19)/2 = 23.5. To find the value of the standard deviation, σ,use
Pr(J < 28) = 0.92 and the value of μ=23.5:

#18: NSOLVE(NORMAL(28, 23.5, σ) = 0.92, σ)

#19: σ = 3.202683851

Part d concerns total probability and conditional probability for i and ii respectively, using the known proportions and basic arithmetic:

#20: $0.137\cdot0.2 + 0.5\cdot0.8$

#21: 0.4274

#22: $\frac{0.5\cdot0.8}{0.137\cdot0.2 + 0.5\cdot0.8}$

#23: 0.9358914365

Question 3

#24: $2\cdot x^4 - x^3 - 5\cdot x^2 + 3\cdot x$

#25: $x\cdot(x^2 + x - 1)\cdot(2\cdot x - 3)$

The values of the a = 1, b = 1 and c = –1 can be identified from the second factor. Alternatively, these can be readily obtained for equations coefficients.

#26: $2\cdot x^4 - x^3 - 5\cdot x^2 + 3\cdot x = 0$

#27: SOLVE($2\cdot x^4 - x^3 - 5\cdot x^2 + 3\cdot x = 0$, x, Real)

#28: $x = -\frac{\sqrt{5}}{2} - \frac{1}{2} \vee x = \frac{\sqrt{5}}{2} - \frac{1}{2} \vee x = \frac{3}{2} \vee x = 0$

For the remaining parts it is convenient to define the following function rule (this could have also been done from the beginning, using q(x) as a defined function with the original rules and p(x) as a defined fucntion with rule p(x) = 0.5*q(x):

#29: $p(x) :\!= 0.5\cdot(2\cdot x^4 - x^3 - 5\cdot x^2 + 3\cdot x)$

#30: $p'(1)$

#31: -1

#32: $p(1)$

#33: $-\frac{1}{2}$

so the rule of the normal is readily found by hand as y – –1/2 = 1(x – 1) or y = x –3/2, or derive can be used to obtain this from Solve(y – –1/2 = 1(x – 10, y).

For b.ii:

#34: SOLVE$\left(p(x) = x - \frac{3}{2}, x\right)$

#35: $x = \frac{3}{2} \vee x = -1 \vee x = 1$

with reference to the graph given, the x value of the point B is x = –1.The gradient of the tangent at this point is:

#36: $p'(-1)$

#37: $$1$$

and the corresponding y value is:

#38: $p(-1)$

#39: $$-\frac{5}{2}$$

Thus, the tangent at B has the rule y - -5/2 =1(x--1) or y = x - 3/2, which is the same rule as for the normal at A.

#40: $p(x) - \left(x - \frac{3}{2}\right)$

#41: $\int_{-1}^{1} \left(p(x) - \left(x - \frac{3}{2}\right)\right) dx$

#42: $$1.733333333$$

or 1.73 square units correct to 2 decimal places.