Math Objectives

- Students will describe the relationship between the focus and the directrix associated with three types of conics.
- Students will define parabolas, ellipses, and hyperbolas by their focus and directrix.
- Reason abstractly and quantitatively (CCSS Mathematical Practice).
- Look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

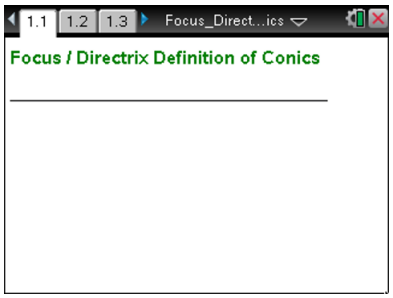
- parabola
- ellipse
- hyperbola
- focus
- directrix

About the Lesson

- This lesson involves observing and describing relationships between the focus and the directrix of each conic: parabolas, ellipses, and hyperbolas.
- As a result, students will:
  - Manipulate the focus and the directrix of a conic to observe the relationships between the focus, the directrix, and the conic.
  - Observe the effect of the relationship between the focus and the directrix on the shape of an ellipse or hyperbola.
  - Define a parabola, an ellipse, and a hyperbola, by their respective focus and directrix.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge.
- Use Quick Poll to assess students' understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:

Student Activity

Focus_Directrix_Definition_of_Conics_Student.pdf

Focus_Directrix_Definition_of_Conics_Student.doc

TI-Nspire document

Focus_Directrix_Definition_of_Conics.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

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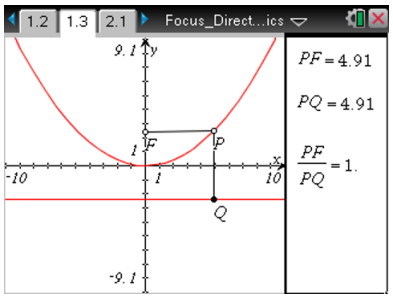

1. The point *F* is called the *focus* of the parabola pictured, and the horizontal line is called the *directrix*.
   a. As you drag the point *F*, what do you notice about the distance between *F* and the x-axis and the distance between the directrix and the x-axis?

   Sample Answers: The distance between the focus and the x-axis and the distance between the directrix and the x-axis are always equal. As *F* moves, the directrix moves accordingly to maintain the equivalence.

   b. Suppose you set the focus to (0, 18). What would the equation of the directrix be? How do you know?

   Sample Answers: The equation of the directrix would be $y = -18$. The focus is 18 units above the x-axis, so the directrix also needs to be 18 units away from the x-axis. This will be a horizontal line 18 units below the x-axis, with equation $y = -18$.

2. Point *P* is an arbitrary point on the parabola.
   a. What do the segments $\overline{PF}$ and $\overline{PQ}$ represent?

   Sample Answers: $\overline{PF}$ is the segment connecting point *P* on the parabola to the focus, *F*. $\overline{PQ}$ is the segment connecting point *P* to the nearest point on the directrix, *Q*.

   b. As you drag point *P* along the parabola, what do you observe about the distance between *P* and the focus and the distance between *P* and the directrix?

   Sample Answers: The two distances are always equal.

   Teacher Tip: Teachers might want to point out to students that *PF* gives the distance from the point *P* to the focus, while *PQ* gives the distance from point *P* to the directrix, where distance to a line is the perpendicular distance.

c. If you change the location of the focus, what happens to the relationship you observed in part b? Explain.

Sample Answers: It remains the same. The distance between the point on the parabola and the focus and the distance between the point on the parabola and the directrix are always equal. The location of the directrix changes in correspondence to any change in the location of the focus.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

3. Based on your observations in Questions 1 and 2, what is the relationship between any point on the parabola and the focus and the directrix? Explain.

Sample Answers: Any point on the parabola is the same distance from the focus as it is from the directrix.

4. Use your response to Question 3 to define a parabola in words based on its focus and directrix.

Sample Answers: A parabola is the locus of points which are equidistant from the focus and the directrix.

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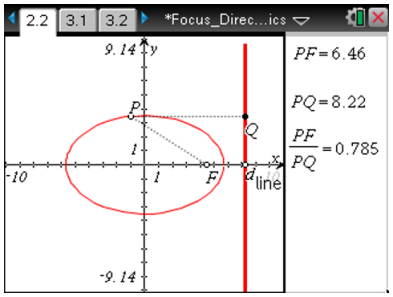

5. Point *F* is the *focus* of the ellipse and the vertical line is the *directrix*.
   a. Drag point *F* along the positive x-axis. Does the directrix change with the focus as in the case of the parabola? What relationship do you observe between the focus and the directrix?

Sample Answers: As long as the *x*-coordinate of the focus is less than the *x*-coordinate of any point on the directrix, the directrix will not change location. However, when the *x*-coordinate of the focus is greater than or equal to the *x*-coordinate of points on the directrix, no ellipse will be formed.

b. Set the focus to the point (3, 0). What happens to the shape of the ellipse as you drag point *d* to change the location of the directrix? What is the relationship between the focus and the directrix and the shape of the ellipse?

Sample Answers: As the directrix comes closer to the focus, the ellipse becomes more oblong, with the height of the ellipse decreasing. As the directrix moves farther away from the focus, the ellipse becomes rounder, with the height of the ellipse increasing.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 2 at the end of this lesson.

6. Set the focus to point (3, 0) and *d* to (6, 0). Point *P* is an arbitrary point on the ellipse.
   a. What do the segments $\overline{PF}$ and $\overline{PQ}$ represent?

Sample Answers: $\overline{PF}$ is the segment connecting point *P* on the ellipse to the focus. $\overline{PQ}$ is the segment connecting point *P* on the ellipse to the nearest point on the directrix, *Q*.

   b. As you drag point *P* along the ellipse, what do you observe about the relationship between *PF* and *PQ*?

Sample Answers: While the lengths *PF* and *PQ* change as point *P* moves around the ellipse, the ratio of the two lengths, $\frac{PF}{PQ}$, remains constant.

Teacher Tip: Teachers might want to draw student attention to the ratio of *PF* and *PQ* shown at the bottom of the screen, and instruct them to observe the effects that moving *P* have on the ratio.

   c. Relocate the focus and/or the directrix, and drag point *P* along the new ellipse. What happens to the relationship between *PF* and *PQ*?

Sample Answers: Relocating the focus and the directrix might result in a change in the ratio of the two lengths, but, for fixed focus and directrix, as *P* moves around the ellipse, the ratio remains constant.

7. Based on your observations in Questions 5 and 6, what must be the relationship between any point on an ellipse and the focus and the directrix of the ellipse.

   Sample Answers: For any point on the ellipse, the ratio of the distance between the point and the focus and the distance between the point and the directrix is a constant value less than 1.

8. Use your response to Question 8 to define an ellipse in words by its focus and its directrix.

   Sample Answers: An ellipse is the locus of points whose distances to the focus and to the directrix are in a fixed ratio less than 1.

   Teacher Tip: Students might not conclude that the ratio must be less than one. It is not necessary for the teacher to address this issue during this question, as it will be revisited at the end of the activity.

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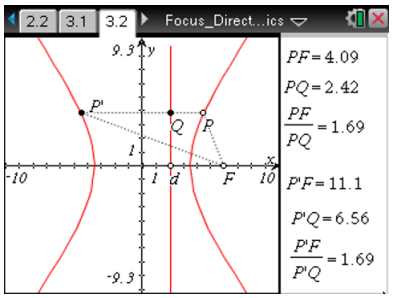

9. Point *F* is the *focus* of the hyperbola and the vertical line is the *directrix.*
   a. Drag point *F* along the positive x-axis. Does the directrix change with the focus as in the case of the parabola? What relationship do you observe between the focus and the directrix?

   Sample Answers: As the focus changes, the directrix remains fixed. However, if the focus has an *x*-coordinate less than the *x*-coordinate of points on the directrix, no hyperbola is formed.

   b. Set the focus to the point (6, 0). What happens to the shape of the hyperbola as you drag point *d* to change the location of the directrix? What is the relationship between the focus and the directrix and the shape of the hyperbola?

   Sample Answers: As the directrix moves farther away from the focus, the hyperbola becomes wider. As the directrix moves closer to the focus, the hyperbola becomes more narrow. In general, the closer the focus and the directrix are, the more narrow the hyperbola formed.

10. Set the focus to the point (6, 0) and *d* to (3, 0). Point *P* is an arbitrary point on the hyperbola, and point *P*' is the corresponding point on the other branch of the hyperbola.
    a. What do the segments $\overline{PF}$, $\overline{PQ}$, $\overline{P'F}$, and $\overline{P'Q}$ represent?

Sample Answers: $\overline{PF}$ is the segment connecting point *P* to the focus; $\overline{PQ}$ is the segment connecting point *P* to the nearest point on the directrix, *Q*, $\overline{P'F}$ is the segment connecting point *P*' to the focus, and $\overline{P'Q}$ is the segment connecting *P*' to the nearest point on the directrix, *Q*.

    b. As you drag point *P* along the hyperbola, what do you observe about the relationship between *PF* and *PQ*? Between *P'F* and *P'Q*?

Sample Answers: While the lengths *PF* and *PQ* change as point *P* moves along the hyperbola, the ratio of the two lengths, $\frac{PF}{PQ}$, remains constant. The same is true for *P'F* and *P'Q*.

> **Teacher Tip:** Teachers might want to draw student attention to the ratio of *PF* and *PQ* and the ratio of *P'F* and *P'Q* shown at the bottom of the screen, and instruct them to observe the effects that moving *P* have on the ratio.

    c. Relocate the focus and/or directrix and move *P* along the hyperbola. What effect does this have on the relationships you observed in part b?

Sample Answers: Relocating the focus and the directrix might result in a change in the ratio of *PF* and *PQ*; but, for fixed focus and directrix, as *P* moves along the hyperbola, the ratio is constant. The same is true for *P'F* and *P'Q*, and that ratio is equivalent to the ratio of *PF* to *PQ*.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 3 at the end of this lesson.

11. Based on your observations in Questions 9 and 10, what must be the relationship between any point on the hyperbola (include both branches) and the focus and the directrix of the hyperbola?

Sample Answers: For any point on the hyperbola, the ratio of the distance between the point and the focus and the distance between the point and the directrix is a constant value greater than 1.

12. Use your response to Question 11 to define a hyperbola in words based on its focus and directrix.

Sample Answers: A hyperbola is the locus of points whose distances to the focus and to the directrix are in a fixed ratio greater than 1.

13. Revisit your definitions in Questions 4, 8, and 12. Are your definitions for the three conics sufficiently distinct that, given a definition you could say with certainty which conic you are describing? If so, explain why. If not, modify your definitions to make them sufficiently distinct.

Sample Answers: In order for the definitions to be distinct, the possible values of the ratios must be defined:

- A parabola is the locus of points equidistant to the focus and the directrix
- An ellipse is the locus of points whose distances to the focus and the directrix are in a fixed ratio less than 1.
- A hyperbola is the locus of points whose distances to the focus and the directrix are in a fixed ratio greater than 1.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The focus / directrix definition of a parabola.
- The focus / directrix definition of an ellipse.
- The focus / directrix definition of a hyperbola.
- The effect of the relationship between the focus and the directrix on the shape of an ellipse or hyperbola.

Assessment

1. Give students graphs of foci and directrices and have students determine the resulting conics.
2. Give students ratios of distances between a point on the conic and the focus and the directrix and have students determine the type of conic.

TI-Nspire Navigator

Note 1 Name of Feature: Screen Capture
A *Screen Capture* can be used to dynamically share students' observations regarding the distances between points on the parabola and the focus and the directrix.

Note 2 Name of Feature: Screen Capture
A *Screen Capture* can be used to dynamically share students' observations regarding the relationship between the shape of the ellipse and the location of the focus and the directrix.

Note 3 Name of Feature: Screen Capture
A *Screen Capture* can be used to dynamically share students' observations regarding the ratios of distances between points on the hyperbola and the focus and the directrix.