

Balloon

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Abstract: This activity is an application of differentiation. Students inflate a balloon and observe the relationship between the rate its volume is changing and the rate points on its surface are getting closer to each other. They then use the symbolic capacity of their calculator and calculus to determine the exact rate of change.

NCTM Principles and Standards:

Algebra standards

- a) Understand patterns, relations, and functions
- b) generalize patterns using explicitly defined and recursively defined functions;
- c) analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- d) use symbolic algebra to represent and explain mathematical relationships;
- e) judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
- f) draw reasonable conclusions about a situation being modeled.

Problem Solving Standard build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof Standard

- a) recognize reasoning and proof as fundamental aspects of mathematics;
- b) make and investigate mathematical conjectures;
- c) develop and evaluate mathematical arguments and proofs;
- d) select and use various types of reasoning and methods of proof.

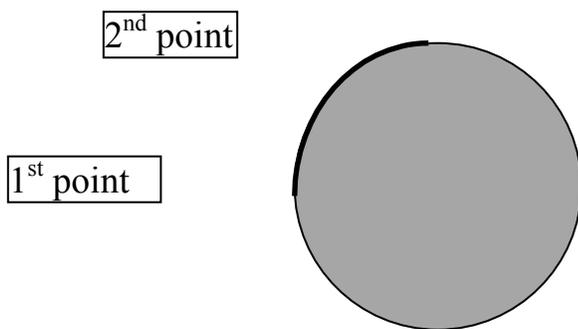
Representation Standard : use representations to model and interpret physical, social, and phenomena.

Key topic: Applications of Derivative - Related Rates: determining the rate of change.

Degree of Difficulty: Elementary to moderate

Needed Materials: Balloons, TI-89 calculator

Situation: Take a balloon and inflate it so that the diameter is approximately 6 inches. If we assume that the balloon is spherical, we can measure its circumference by wrapping a measuring tape around it. Divide the circumference by 2π to find the radius. Here, we'll assume that the radius is 3 inches. Now, draw two marks on the balloon and measure the distance between them with the measuring tape. If you let air out of the balloon, how quickly are the two points getting close to each other? Let's assume that you let air out at the constant rate of $4 \text{ in}^3/\text{minute}$ and that the two points were 5 inches from each other initially.



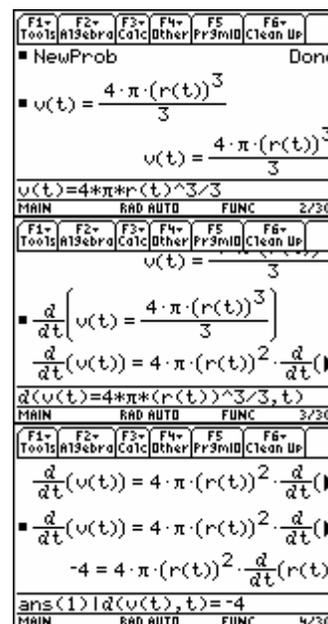
The two points on the balloon lie on an arc of a circle of length 5 whose radius is initially 3. As the balloon shrinks, the length of the arc will change at a rate proportional to the radius. So all we need to do is find the rate of change of the radius and then multiply that by $5/3$ - the ratio of the initial arc length to the initial radius.

The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Both V and r are functions of time, so we need to enter the formula into the calculator to reflect that fact:

Differentiate both sides of this equation with respect to t :

Evaluate this expression to reflect that the volume is changing at the rate of $4 \text{ in}^3/\text{minute}$



Evaluate this expression when $r = 3$:

Solve the equation for $\frac{dr}{dt}$

Find the rate the distance between the points is changing by multiplying $\frac{dr}{dt}$ by $5/3$:

The amount of change is about $1/20$ of an inch per minute. This shouldn't be too surprising because the volume of the balloon initially is 113 in^3 and a loss of $4 \text{ in}^3/\text{minute}$ is relatively small

F1	F2	F3	F4	F5	F6
Tools	A13eBrj	Co1c	Other	Pr3mID	Clean Up

$$-4 = 4 \cdot \pi \cdot (r(t))^2 \cdot \frac{d}{dt}(r(t))$$

$$\blacksquare -4 = 4 \cdot \pi \cdot (r(t))^2 \cdot \frac{d}{dt}(r(t)) \quad | \quad \blacktriangleright$$

$$-4 = 36 \cdot \pi \cdot \frac{d}{dt}(r(t))$$

ans(1)|r(t)=3

MAIN	RAD	AUTO	FUNC	5/20
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F1	F2	F3	F4	F5	F6
Tools	A13eBrj	Co1c	Other	Pr3mID	Clean Up

$$-4 = 36 \cdot \pi \cdot \frac{d}{dt}(r(t))$$

$$\blacksquare \frac{-4}{36 \cdot \pi}$$

$$\frac{-1}{9 \cdot \pi} = \frac{d}{dt}(r(t))$$

ans(1)/(36π)

MAIN	RAD	AUTO	FUNC	6/20
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F1	F2	F3	F4	F5	F6
Tools	A13eBrj	Co1c	Other	Pr3mID	Clean Up

$$\left[\frac{-1}{9 \cdot \pi} = \frac{d}{dt}(r(t)) \right] \cdot 5$$

$$\blacksquare \frac{-5}{27 \cdot \pi} = \frac{5 \cdot \frac{d}{dt}(r(t))}{3}$$

ans(1)*5/3

MAIN	RAD	AUTO	FUNC	7/20
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F1	F2	F3	F4	F5	F6
Tools	A13eBrj	Co1c	Other	Pr3mID	Clean Up

$$\frac{-5}{27 \cdot \pi} = \frac{5 \cdot \frac{d}{dt}(r(t))}{3}$$

$$\blacksquare \frac{-5}{27 \cdot \pi} \quad - .0589463$$

ans(1)/(27*π)

MAIN	RAD	AUTO	FUNC	8/20
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