Name ___________________

Class ___________________

Problem 1 – Envelope Construction

Drag point *P* around the circle on page 1.3.

Display the locus of the perpendicular line as *P* travels along the circle. Select the **Locus** tool from the Construction menu, click on the line, and then click on point *P*.

- What shape does this generate?

The point *F* is a special type of fixed point that can be used to generate an ellipse. Ellipses have two such fixed points, called foci (singular: focus). You will now explore the other focus.

First, hide the locus you created earlier. Then reflect point *F* over the *y*-axis using the **Reflection** tool from the Transformation menu. Label the image point *F'*.

Now draw segment *F'P*, and construct a line perpendicular to segment *F'P* through point *P*.

Display the locus of the new perpendicular as *P* travels along the circle.

- What shape does this generate?

- How do the two loci you constructed compare?

Drag *F* and observe the ellipse.

- Describe the relationship between an ellipse and its foci.

- Describe the relationship between the two foci of an ellipse.

Problem 2 – String and Pins Construction

An **ellipse** is defined as the set of points in a plane such that the sum of the distances from two fixed points (foci) in that plane is constant.

You can use this definition as another way to construct an ellipse.

On page 2.3, points *f1* and *f2* will be the foci of the ellipse. The values of *d1* and *d2*, determined by the slider, will be the distances from *f1* and *f2* (respectively) to the point on the ellipse.

Calculate *d3*, the sum of *d1* and *d2*. Then drag the slider.

- What is the effect on *d1*, *d2*, and *d3*?

Now construct two circles using the **Compass** tool from the Construction menu, one with center *f1* and radius *d1* and another with center *f2* and radius *d2*. Mark the intersections of the two circles. Draw four line segments connecting these points to points *f1* and *f2*.

- Hide the circles and drag the slider. Then display loci of the each intersection point as the slider travels along the segment. What shape is formed by the loci?

Problem 3 – Semimajor and Semiminor Axes

Page 3.2 shows another ellipse. Press play to start the animation.

As the point travels around the curve, its distance from the center of the ellipse changes. The minimum distance is called the **semiminor axis** and the maximum distance is called the **semimajor axis**.

- Where is *P* when these minimum and maximum distances occur?

Problem 4 – Deriving the Equation of an Ellipse

Page 4.2 shows an ellipse centered at the origin. The foci are also shown. The lengths of the segments connecting (x, y) to the foci are *d1* and *d2*. Let c be the distance from the center to each focus. Let a be the distance from the center of the ellipse to the vertex (a, 0).

Follow the steps on pages 4.3 through 4.7 to derive a general equation for an ellipse centered at the origin.

1. What is the equation for the distance from (–a, 0) to (a, 0) in terms of a, *d1*, and *d2*?

2. What are the expressions for *d1* and *d2*?
Remember, the distance formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

3. Substitute these expressions into the equation from Step 1.

4. Remove the radicals in the equation by isolating a radical on one side, squaring both sides, and simplifying. Repeat until no radicals remain.

5. Factor out *x,* then simplify.

6. Divide both sides by $a^2 - c^2$.

7. Drag (x, y) so that $x = 0$. What can you conclude about *d1* and *d2*?

8. Use the information from above to rewrite the equation from Step 4.

9. Let b be the distance from the center to co-vertex (0, b). Use the Pythagorean Theorem to write an expression for b^2 in terms of a and c.

10. Use the expression you just found to rewrite the equation in Step 6.