Mathematical Methods (CAS) 2002 Examination 1 part 1 sample solutions Q 1 to 7

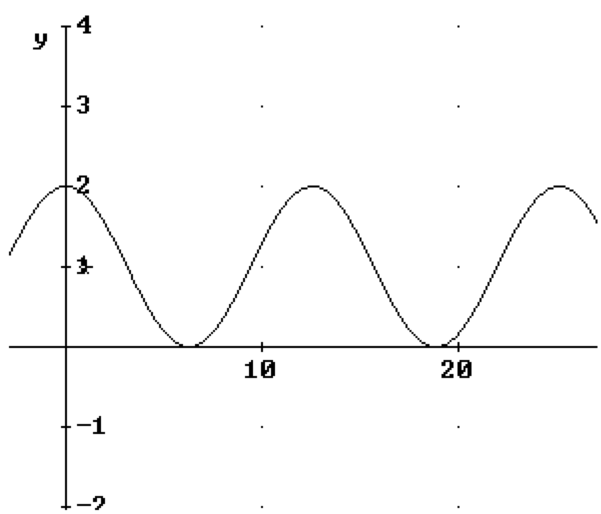
Note: To use *Derive* efficiently, students should be familiar with the 'tick plus equals' and 'tick plus approximately equals' evaluation buttons. These simultaneously 'author' and 'evaluate' expressions exactly and numerically respectively.For example, the 'tick plus equals' or 'author and exact evaluation' button works well for Question 14, while the 'tick plus approximately equals' or 'author and numerical approximation' works well for Question 26. Students should also be familiar with the use of defined functions in the form f(x):= *rule of the function*, such as in the sample solutions for Question 10.

Some questions are conceptual in nature, that is, technology will not be of assistance, for example, Question 7. For other questions, such as Question 1, the facility of *Derive* to quickly produce scaled graphs, and the like, means that such problems could be tackled by inspection of each alternative, although this is not a recommended approach.In many cases students will reason what is likely to be the answer, and then confirm this with *Derive*.

Question 1

As no horizontal translation is involved, the graph is a transformed cos graph. The period is 4π, so the graph is an A cos(x/2)+ B graph. The graph has been translated vertically up 1 unit so B = 1, and the graph is a 'right way up' cos graph so A = 1, hence the correct answer is E. Check by drawing the graph, 8π is about 24 units and a bit and 2π is about 6 units and a bit:

#1: $1 + \text{COS}\left(\dfrac{x}{2}\right)$

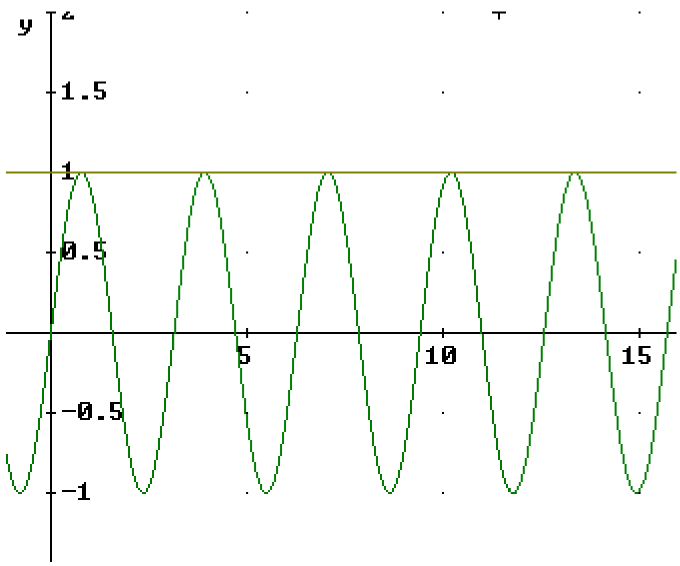


Question 2

This question can be answered without computation. The graph of sin (2x) is the graph of sin(x) dilated horizontally by a factor of 1/2. Thus while sin(x) has maxima of 1 at π/2, 2π+π/2 = 5π/2, 4π+π/2 = 9π/2 etc, sin(2x) will have maxima of 1 at π/4, 5π/4, 9π/4 etc. Neither A, B nor C provide values sufficiently large for the sum of solutions from 0 to 4π, and C does not define a unique exact value (which must exist), hence the answer is E. Given that 4π is larger than 12, this reasoning can be checked visually:

#2: $\mathrm{SIN}(2\cdot x)$

#3: 1



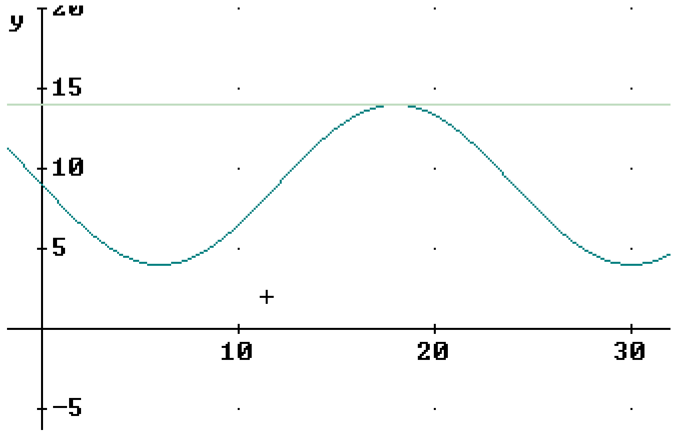
Question 3

This question is best tackled graphically. The graph of y = 14 meets the graph of y = 9–5sin(πt/12) near 18 = 6 pm (best of the available alternatives), hence the answer is D.

It can also be tackled conceptually.The model has a period of 24 hours. The graph of y = 9–5sin(πt/12) will have its maximim value when sin(πt/12) has the value -1. This will occur when πt/12 = 3π/2 or t = 18 hours = 6 pm.This can readily be checked from the graph:

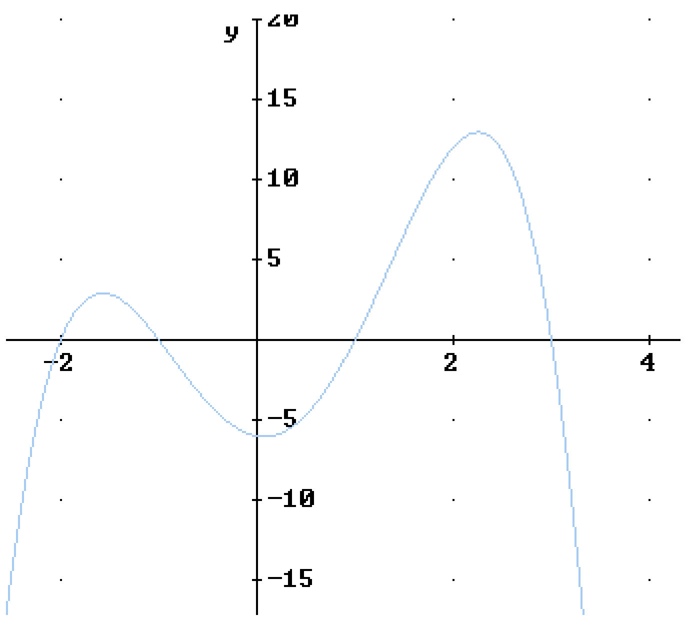
#4: $9 - 5\cdot\mathrm{SIN}\left(\frac{\pi\cdot t}{12}\right)$

#5: 14



Question 4

This question could be done quickly using ***Derive*** by checking each alternative, however, from the shape of the graph it is clearly a negative quartic with factors corresponding to roots from left to right of (x+2), (x+1), (x−1) and (x−3). Thus, the required form would be y = −(x+2)(x+1)(x−1)(x−3). There is no alternative directly in this form, however B and C can be eliminated since they represent prositive quartics. A has an incorrect factor (x−2) so either D or E is correct. the (3−x) factor incorporates a negative coefficient, that is (3−x) = −(x−3), so E is correct. D has a 'double negative' and represents a positive quartic. The answer can be checked graphically:



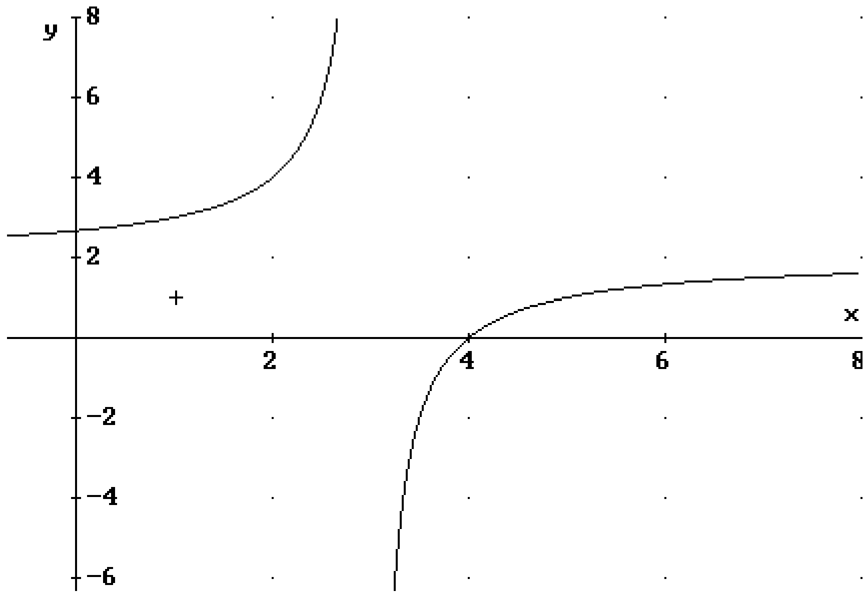
#6: $(x + 2)\cdot(x + 1)\cdot(x - 1)\cdot(3 - x)$

Question 5

A vertical asymptote at x = 3 gives b = −3, so B and C are the only possible correct alternatives. The shape of the graph indicates a 'negative' hyperbola graph, so C can be selected as the correct answer, since the coefficient of a in B is positive.

The answer can be checked by plotting y = -2/(x-3) + 2:

#7: $$-\frac{2}{x-3}+2$$



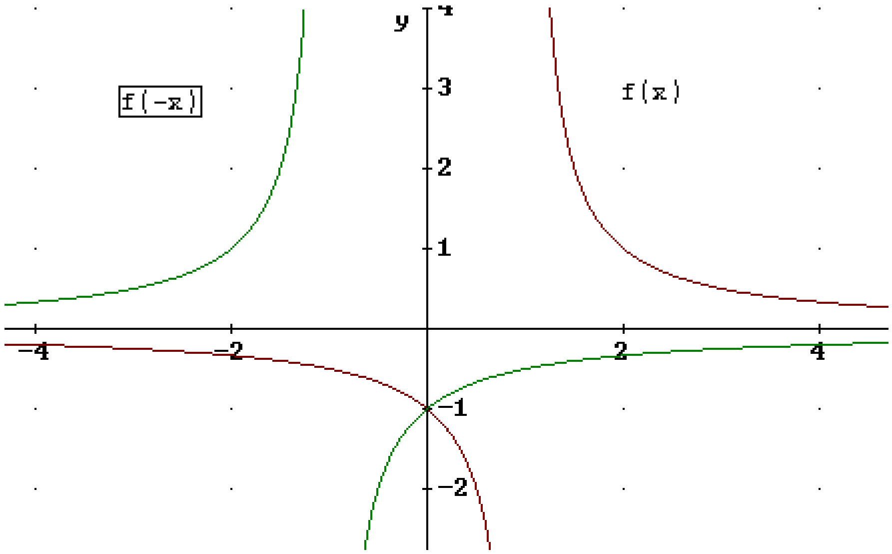
Question 6

This is a conceptual question, students need to recognise that the graph of y = f(-x) - and its key features such as the vertical asymptote - will correspond to the graph of y =f(x) - and its vertical asymptote - reflected in the vertical, or y, axis. Thus, A is the corrcet alternative.

An alternative, but cumbersome and less efficient approach (and one which is not aleways possible) would be to attempt to model the curve by a specific defined function rule y = f(x) and then observe the graph of f(-x) and then select the best corresponding alternative. in this case care needs to be taken to focus on the relevant features of the graphs.

#8: $$f(x) := \frac{1}{x-1}$$

#9: f(-x)



Question 7

This is a purely conceptual question,students needs to recognise that the graph of the inverse function is obtained by reflection in the line y = x (which they may wish to draw in on the paper to assist themselves), **noting that the diagrams are drawn to a 1 to 1 scale** (even though the scale has not been specified). The correct alternative A can, in this case, be identified by consideration of the linear segment alone.