This packet contains a copy of the original problem used to create the activity, rationale and explanation behind the "I Notice, I Wonder" focal activity, and some thoughts on why this activity works well with TI-Nspire ${ }^{\text {TM }}$ technology.

All of the problems and activities are samples of the Math Forum's Problems of the Week, paired with activities from the Problem Solving and Communication Activity Series. We are highlighting activities and problems that make good use of TI-Nspire ${ }^{\text {TM }}$ handhelds.

Teachers and/or students are able to electronically access this and similar problems after setting up a login (free) available from the Math Forum @ Drexel. Sign up using the link on the Technology Problems of the Week (tPoW) login page, or use your existing KenKen® or Problems of the Week login-see this page for details: http://mathforum.org/tpow/about.html

## Midpoint Quadrilateral



This problem presents an opportunity for students to think about properties of quadrilaterals, and also to work on confirming observations through geometric reasoning.

If your state has adopted the Common Core State Standards, this alignment might be helpful:

## Geometry: Prove Geometric Theorems

G.CO.11. Prove theorems about parallelograms.

## Mathematical Practices

3. Construct viable arguments and critique the reasoning of others.
4. Look for and make use of structure.

This activity focuses on the strategy: I Notice, I Wonder. The activity encourages students to use dynamic geometry software to notice and wonder, and then suggests the specific strategy of making structure visible to help students find more relationships.

In this activity we use the TI-Nspire ${ }^{\text {TM }}$ software's dynamic geometry software. Students notice and wonder about relationships in an interactive construction. They are encouraged to make and test conjectures and reason about them.

Students can use the TI-Nspire ${ }^{\text {TM }}$ to measure their drawing as well as adding auxiliary lines to make structure more visible.

Do your students like to use their mathematical imaginations? Wonder about math all around them? Discover and invent new patterns? Here are some ways for them to share their ideas and learn about other students' and mathematicians' ideas!

## http://mathforum.org/explorers/

| Are you a Math Explorer? |
| :--- | :--- |
| Do you like to use your mathematical imagination? Wonder about math all around you? Discover and invent new patterns? Here are some ways to share |
| your ideas and learn about other students' and mathematicians' ideas! |
| Paw Problems of the Week |

## The Activity

Key Screen Shots


## Possible Responses

Noticings, Wonderings, and Conjectures

The outer quadrilateral can be lots of weird shapes
The midpoint quadrilateral is more regular
The midpoint quadrilateral is a parallelogram
The midpoint quadrilateral's opposite sides are congruent
The midpoitn quadrilateral's opposite sides are parallel

You can make the midpoint quadrilateral a rectangle
You can also make it a square
The outer quadrilateral looks like a trapezoid when the midpoint quadrilateral is a rectangle
The outer quadrilateral looks like a kite when the midpoint quadrilateral is a square.

Argument for why the midpoint quadrilateral is a parallelogram

When you draw either diagonal of the outer quadrilateral $A B C D$, it splits it into two triangles, $A B D$ and CDB.
Because we have connected the midpoints of adjacent sides, we've also connected the midpoints of adjacent sides of the triangles.
Call the midpoint of $A B, M$ and the midpoint of $A D, N$.
AMN is similar to $A B D$ by SAS~, since AM is half of $A B$, AN is half of $A D$, and angle $A$ is congruent to itself.

Therefore, angle AMN is congruent to angle ABD because corresponding angles in similar triangles are congruent.
Therefore, MN is parallel to BD since corresponding angles congruent imply parallel lines.
The same argument establishes that the line connecting the midpoints of triangle CDB is parallel to BD.
Since the lines connecting both sets of midpoints are parallel to the same line, they are parallel to each other.
Finally, the same series of arguments can be used to show that the other set of opposite sides of the midpoint quadrilateral are parallel to one another, and therefore the midpoint quadrilateral is a parallelogram.

A similar argument could be made establishing that opposite sides are congruent, rather than parallel.

