NUMB3RS Activity: What Are You Implying?
Episode: "Backscatter"

Topic: Explicit and implicit functions **Grade Level:** 11 - 12
Objective: Graphing and analyzing explicit and implicit functions
Time: 30 minutes
Materials: TI-83/84 Plus for the activity. Download a program or use a computer with Computer Algebra Software (CAS), for extensions only

Introduction
In "Backscatter," the topic of explicit and implicit functions is discussed as Charlie is lecturing to a Calculus class. Unfortunately, his lecture is interrupted when two members of the Russian Mob enter his class and sit in the back row.

Discuss with Students
Your students have been using explicit functions ever since they began to make a graph by plotting points obtained from an equation. For the explicit function $y = x^2$, it is possible to find a value of y from a given value of x in the domain of the function directly without having to rearrange the equation or do anything else. The essential idea is that y can be found explicitly from x. There is nothing hidden from view nor is there anything implied about what one needs to do to find x.

If students have studied conic sections, they will know that not all relationships between y and x are described through an explicit equation. For a given value of x in the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, the corresponding value of y can only be found by doing some basic algebra. As your students solve for y they will see that there are actually two real number values of y for any x in the interval $-4 < x < 4$. We are dealing with a relation, not a function. It is commonplace to say that the equation of the ellipse defines y as an implicit function of x.

Another familiar example of an implicit function is $xy = 1$. In this case and the previous one it is possible to solve for y in terms of x to obtain an explicit function. Because it is possible to solve for y in terms of x for these examples, a good question to ask your students is why bother studying implicit functions? Why not use only explicit functions? Often the implicit version is more elegant, and it provides more up-front information. But in many instances it is often impossible to solve for y in terms of x. It may surprise your students to know there are many examples where it simply is not possible to isolate y in terms of x no matter how much mathematics you know. One such example is $x^4 + xy + y^4 = 3$. This equation along with others will be discussed in the activity.

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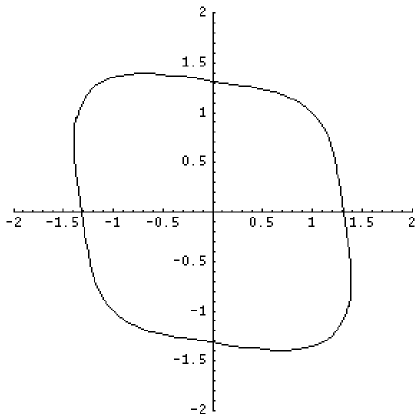
Student Page Answers:

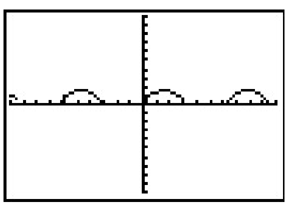
1a. The following points are on the graph: $\left(\sqrt[4]{3}, 0\right)$, $\left(-\sqrt[4]{3}, 0\right)$, $\left(0, \sqrt[4]{3}\right)$, $\left(0, -\sqrt[4]{3}\right)$, (1, 1), (–1, –1)

1b. (i) true, (ii) true, (iii) true, (iv) true, (v) true, (vi) false

1c.

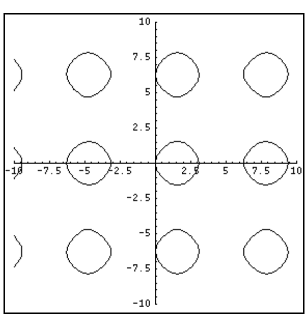


2a.



2b. The period is 2π.

2c. Note that if (a, b) is a point on sin(x) + cos(y) = 1, then so are the following points for all integral values of k: (a, $-b$), ($a + 2\pi k$, b), (a, $b + 2\pi k$), and ($a + 2\pi k$, $b + 2\pi k$). The complete graph of sin(x) + cos(y) = 1 for $|x| < 10$ and $|y| < 10$ is shown below.



2d. Answers vary. One possible answer is the circle $x^2 + y^2 = 1$.

Note: The graphs in 1c and 2c were generated using the utility on this Web site:
http://www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=mathcom&s1=graphs&s2=equations&s3=basic

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Name: ________________________________ Date: ______________

NUMB3RS Activity: What Are You Implying?

In "Backscatter,” the topic of explicit and implicit functions is discussed as Charlie is lecturing to a class. Unfortunately, his lecture is interrupted when two members of the Russian Mob enter his class and sit in the back row. This activity will help you learn more about explicit and implicit functions.

You have been using explicit functions ever since you learned how to make a graph by plotting points obtained from an equation, such as $y = x^2$. In this case, it is possible to find a value of y from a given value of x directly, without having to rearrange the equation or do anything else. The essential idea is that y can be found explicitly from x. There is nothing hidden from view nor is there anything implied about what one needs to do to find x.

As you may know from your study of conic sections, not all relationships between y and x are described through an explicit equation. For a given value of x in the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, the value of y can only be found by doing some basic algebra. For $x = 0$, you will see that there are actually two values of y and so we are not dealing with a function, but with a relation. It is commonplace to say that the equation of the ellipse defines y as an implicit function of x.

Another example of an implicit function is $xy = 1$. In this case and the previous one, it is possible to solve for y in terms of x to obtain an explicit function, so why bother studying implicit functions? Why not use only explicit functions? The implicit version is often more elegant and it provides more up-front information. But many times it is impossible to solve for y in terms of x. There are, surprisingly, many examples where it simply is not possible to isolate y in terms of x no matter how much mathematics you know. One such example is $x^4 + xy + y^4 = 3$. This equation along with others will be discussed in the activity.

1. Even though it is impossible to solve for y in terms of x in the equation $x^4 + xy + y^4 = 3$, it is possible to find several points on this curve.

a. Begin by finding the intercepts. Then study the coefficients of the three terms and the constant term as well as the underlying symmetry of the equation and try to obtain other points by inspection. How many points can you find?

b. Decide if each of the following statements is true or false.

(i) If the point (a, b) lies on the curve $x^4 + xy + y^4 = 3$, then so does the point $(–a, –b)$.

(ii) If the point $(a, –b)$ lies on the curve $x^4 + xy + y^4 = 3$, then so does the point $(–a, b)$.

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(iii) If the point $(–a, b)$ lies on the curve $x^4 + xy + y^4 = 3$, then so does the point $(a, –b)$.

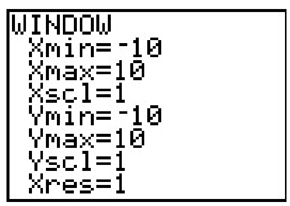
(iv) If the point (a, b) lies on the curve $x^4 + xy + y^4 = 3$, then so does the point (b, a).

(v) If the point (a, b) lies on the curve $x^4 + xy + y^4 = 3$, then so does the point $(–b, –a)$.

(vi) If the point $\left(\frac{1}{a}, \frac{1}{b}\right)$ lies on the curve $x^4 + xy + y^4 = 3$, then so does the point (a, b).

c. Use the results of parts a and b to sketch the graph of $x^4 + xy + y^4 = 3$.

2. a. Try to graph the implicit function $\sin(x) + \cos(y) = 1$. Solve for y in terms of x and use a TI-83/84 Plus calculator to make the graph. Use the WINDOW settings shown below. (Note: Make sure that your calculator is set to Radian mode.)



b. What is the period of this graph?

c. Verify that the point $(0, 2\pi)$ satisfies the relation $\sin(x) + \cos(y) = 1$ but does not appear in the graph you created in part a. Find other points (there are an infinite number of them) that satisfy the relation but do not appear on the graph. Use these additional points to complete the graph of $\sin(x) + \cos(y) = 1$.

d. Give another example of an implicit function where solving for y in terms of x yields an incomplete graph.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

For the Student

Because the TI-83/84 Plus cannot graph implicit functions, you can use CAS software packages (such as Derive) to create the following graphs. Or, you could use the graphing utility available at the following Web site:

Math.com Calculators and Tools
http://www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=mathcom&s1=graphs&s2=equations&s3=basic

1. Graph $\sin(x) + \cos(y) = k$ for $k = 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9$, and 2.0. How do the graphs change from one case to the next? Before making the graphs, try to predict what will happen.

2. Graph $\sin(x) + \cos(y) = k$ for $k = 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1$, and 0.0. How do the graphs change from one case to the next? Before making the graphs, try to predict what will happen.

Related Topic

One particularly interesting type of implicit function is called a “super ellipse.” Information about the super ellipse can be found at the following Web sites:

- Super Ellipse, from Wikipedia
  http://en.wikipedia.org/wiki/Super_ellipse
- A Slice Form Super Egg
  http://www.mathsyear2000.co.uk/explorer/slice/superegg.shtml
- Ovals and Egg Curves
  http://www.mathematische-basteleien.de/eggcurves.htm
- Piet Hein: Dining Table “Super Ellipse”
  http://www.klassik.dk/1/722/2217/hein-piet.html

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