**Concepts**

The purpose of this document is to introduce the accumulation, or area-so-far, function, and to help discover graphically the connection between differential and integral calculus. The first part of the Fundamental Theorem of Calculus involves a function defined by



where  is a continuous function on  and  varies between  and  The function  depends only on the value of and as  varies  also varies. Therefore,  is a function of 

On Page 1.2, the user can manipulate the values of  and  The value of  is dynamically computed and displayed, and the geometric interpretation of accumulated area, or area-so-far, is also shown as a shaded region on the graph. There are five points on the graph that can be moved vertically to change the definition of the piecewise defined linear function 

Page 2.2 presents a similar graph and calculations: the graph of a piecewise defined linear function  and the value in the left pane. The bottom pane displays a complete graph of  The values of and can be changed by grabbing and moving along the horizontal axis, or by using the sliders in the left pane. The dynamic connection between the two graphs and the vertical alignment allows the user to discover the relationship between the functions and 

**Course and Exam Description Unit**

Section 6.4: The Fundamental Theorem of Calculus and Definite Integrals

**Calculator Files**

PWL\_Definite\_Integral\_Function.tns

**Using the Document**

PWL\_Definite\_Integral\_Function.tns: On page 1.2, a function  is presented as a piecewise defined linear graph. The vertices that connect the linear pieces of the graph can be moved up or down (in integer steps) by grabbing any marked point on the graph and dragging to another location. The values of  and  can also be changed by grabbing the corresponding point and dragging along the horizontal axis. These values can also be manipulated by using the sliders in the left pane. For a fixed value  the value of  is displayed in the bottom pane.

On page 2.2, a function  is presented as a piecewise defined linear graph in the top pane. The vertices can be moved up or down, and the values of  and  can be changed on the graph or by using the sliders. The graph of the function  is displayed in the bottom pane.

Page 1.1

|  |  |
| --- | --- |
|  | In Problem 1, the integral of a piecewise defined linear function  is considered. The vertices that connect the linear pieces of the graph of  can be moved vertically, in integer steps, by grabbing any marked point on the graph and dragging to another location. The slider arrows can be used to change the limits of integration,  and  The user can also grab and move the points on the graph representing  and  |

Page 1.2

|  |  |
| --- | --- |
|  | The graph of a function  for  is shown in the top right pane. The vertices that connect the linear pieces of the graph can be moved up or down (in integer steps) by grabbing the point and dragging to another location. The slider arrows in the left pane can be used to change the values of  and  in steps of 0.1. The user can also grab and move the points on the graph representing  and  in the top pane. The value of , for the current values of  and , is given in the bottom pane. This value updates dynamically, that is, as  or  changes. The shaded region in the graph is a geometric interpretation of the value of   |

Page 1.3 (top)

|  |  |
| --- | --- |
|  | This page contains the definitions for the functions  and  The calculator variablesrepresent the  of the points on the graph of  The user does not need to alter these definitions. However, one might consider deriving this definition for  |

Page 1.3 (middle)

|  |  |
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|  | In order to speed up the calculations (and the display in Problem 2), the function  is defined analytically. The function  is used in the definition of  |

Page 1.3 (bottom)

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|  | This is the analytical definition for the function  The user does not need to change any of these functions. However, they are provided in case the user would like to add more nodes or enhance this file in some other way. |

Page 2.1

|  |  |
| --- | --- |
|  | In Problem 2, the integral of a piecewise defined linear function  is considered, and the graph of the function  is also displayed. The vertices that connect the linear pieces of the graph of  can be moved vertically by grabbing any marked point on the graph and dragging to another location. The slider arrows can be used to change the limits of integration,  and  The user can also grab and move the points on the graph representing  and  The graph of the function  is displayed dynamically. |

Page 2.2

|  |  |
| --- | --- |
|  | The graph of a function  for  is shown in the top right pane. The vertices that connect the linear pieces of the graph can be moved up or down (in integer steps) by grabbing the point and dragging to another location. The slider arrows in the left pane can be used to change the values of  and  in steps of 0.1. The user can also grab and move the points on the graph representing  and  in the top pane. In the bottom pane, the user can grab and move the point representing  The value of , for the current values of  and , is given in the left pane. This value updates dynamically, that is, as  or  changes. The shaded region in the graph is a geometric interpretation of the value of  The graph of  is displayed in the bottom right pane. This graph changes dynamically as  or  change. |

**Page 2.3**

This page contains definitions for the functions and similar to those on Page 1.3. The user does not need to alter any of these definitions. This page is provided for consideration and added discussion, and in case the user would like to enhance this calculator file in some way.

**Suggested Applications and Extensions**

**Page 1.2**

Use the default function  to answer Questions 1-8. Remember that  is a function of  (for a fixed value of ). The values of  and  can be manipulated, the value  is displayed in the bottom pane, and the shaded region in the top pane represents the accumulated net area bounded by the graph of  and the horizontal axis from  to 

1. Find the domain and range of the function 
2. Use the graph to explain geometrically how to find 
3. On what intervals is  increasing? Decreasing?
4. Find  Explain this answer since the shaded region representing  is above the horizontal axis.
5. On the interval where does  have an absolute maximum value? Explain the behavior of the function  at and around this value. 
6. On the interval where does  have an absolute mimimum value? Does this contradict the Extreme Value Theorem? Why or why not?
7. Let  Explain how the values in Question 1 change.
8. Let  Explain how the values in Question 1 change.
9. Let  Move the points to construct a piecewise defined linear function such that the maximum value of  occurs at  Let  Where does the absolute maximum occur now?
10. Let  Move the points to construct a non-zero piecewise defined linear function such that  Let  Explain the relationship among the shaded regions in the graph of  to the left of 
11. Let  Move the points to construct a non-zero piecewise defined linear function such that the function  is increasing on the interval  In words, describe any special characteristic of your function  Does this suggest another relationship between  and  If so, explain.
12. Let  Is it possible to construct a non-zero piecewise defined linear function such that  If not, why not? If so, construct one such function.

**Page 2.2**

Use the default function  to answer Questions 1-9. Remember that  is a function of  (for a fixed value of ). The values of  and  can be manipulated, the value  is displayed in the left pane, and the shaded region in the top pane represents the accumulated net area bounded by the graph of  and the horizontal axis from  to  The bottom pane displays a complete graph of the function 

1. Find the domain and range of 
2. Find the value  Explain how to determine this value geometrically.
3. Find the value  Explain this answer geometrically.
4. On what intervals is  increasing? Decreasing? What are the values of  on each of these intervals?
5. For what values of  does  have a relative minimum value? Relative maximum value?
6. Where does  attain its absolute maximum value? Absolute minimum value?
7. On what intervals is  concave up? Concave down? Explain the behavior of the function  on each of these intervals.
8. Find any points of inflection on the graph of  Explain the behavior of the graph of at each corresponding 
9. Use your answers to questions 1-8 to suggest a relationship between and 
10. Let  Move the points to construct a non-zero piecewise defined linear function such that the function  is increasing on the interval  In words, describe any special characteristic of your function 
11. Let  Move the points to construct a non-zero piecewise defined linear function such that the graph of the function  has two relative maximum points and two relative minimum points.
12. Let  Is it possible to construct a non-zero piecewise defined function such that the graph of the function  has three relative maximum points? If not, why not? If so, then construct one such graph.
13. Let  Move the points to construct a non-zero piecewise defined linear function such that the graph of the function  is concave down over its entire domain.
14. For your graph constructed in Question 13, explain what happens to the graph of  as  changes.
15. Let  Move the points to construct a non-zero piecewise defined linear even function. Is the function  even, odd, or neither?
16. Let  Move the points to construct a non-zero piecewise defined linear odd function. Is the function  even, odd, or neither?