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Are All AP Statistics programs created equal?
Using the chi-squared significance test to compare student success

Statistics Key Topics: Statistical Inference—Chi-Squared Test for Independence

NCTM Principles and Standards
>   Content Standards—Data Analysis and Probability

Develop and evaluate inferences and predictions that are based on data
- Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference.

Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.
- Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each.

Focus: In this lesson students will apply the chi-squared test for independence to a data set of high school AP Statistics Exam scores. They will set up a two way table and calculate the chi-squared test statistic by hand. In addition, students will use matrices on their TI-83/84 Plus calculator to compute the chi-squared statistic and p-values for the given data set.

Prerequisite skills: Students need to be familiar with statistical inference and the mechanics of hypothesis testing.

Materials: TI-83/84 Plus Graphing Calculator

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Motivating Question: Do public high schools in rural, suburban and urban areas have different passing rates for students who sit for Advanced Placement Exams?

To answer this question we will look at a sample of students who took the AP Statistics exam at 3 high schools in Washington State. 42 out of 71 students passed the exam from rural Richland High School. (A score of 3 or higher on the AP exam is considered passing.) 72 out of 103 students passed the exam at suburban Bellevue High School while 61 out of 107 students passed at urban Franklin High School.

Because we are comparing more than two proportions we cannot use a two sample z test. Instead we will use the chi-squared (X^2) test for independence to see if there are significant differences among the proportion of passing rates at the 3 high schools.

1. State the null and alternative hypotheses that we should use. (Remember H_0 is a statement of no effect).

    H_0: H_a:

2. Fill in the following two-way table with the information from above.

	RHS	BHS	FHS	Total
Pass				
No Pass				
Total				

Note: This is a 2 X 3 matrix (two rows X 3 columns) with $(r - 1)(c - 1)$ degrees of freedom).

Use the table from above to answer these questions.

3. What is the total proportion of students who did not pass?

4. What is the proportion of Bellevue High students who did not pass.

5. How many Bellevue High students would we expect to have passed if the passing proportion were the same for all schools?

The X^2 test looks for deviations in the observed data versus what is expected if the null hypothesis is true (the passing rates at all 3 schools are equal). A simple way to calculate the expected count for each of the 6 cells is to compute $\dfrac{\text{row total} \cdot \text{column total}}{\text{table total}}$.

6. Find the expected values and fill in the table below.

	RHS	BHS	FHS
Pass			
No Pass			

7. Calculate the value of the X^2 test statistic using your calculator. $X^2 = \sum \dfrac{(\text{observed-expected})^2}{\text{expected}}$

L1	L2	L3	3
72	64.1	------	
61	66.6		
42	44.2		
31	38.9		
46	40.4		
29	26.8		
------	------	------	

L3 =(L1−L2)²/L2

Put the 6 observed values in L_1 and the 6 expected values in L_2. Enter the equation shown above in L3 to get each of the individual X^2 amounts

From the home screen press [2nd][STAT]Math, 5: Sum(L_3) to get the total X^2 test statistic. Record your sum below.

X^2 test statistic=_____

```
sum(L3
           4.115212897
X²cdf(4.12,100,2
           .1274539699
```

Calculate the p-value. Press [2nd][VARS]7:X^2pdf(. The parameters are lower bound, upper bound, degrees of freedom. The X^2cdf feature calculates the area under the X^2 curve between two point that you define. We want the area to the right of our X^2 value and the upper bound of 100 is far enough out that little to no area lies to its right.

8. Using the p-value from above, what conclusion can you make in regards to answering our initial question about AP passing rates?

Using the X^2 significance test on your calculator.
It can be time consuming to calculate each expected cell count like you did in #6 above. Your TI-83/84 Plus calculator can do all the work for you using matrices and can also compute the X^2 statistic and p-value as well. Here's how.

Enter the observed counts into matrix A. Press [2nd][x^{-1}]Edit, 1: [A]. The dimensions should match the number of rows and columns from the original two way table in #2.

```
MATRIX[A]  2 ×3
[ 72   61   42 ]
[ 31   46   29 ]
```

Perform a X^2 significance test. Press[STAT] TESTS, C: X^2-Test. Matrix A is where the observed values are and choose Matrix B for the calculator to store the expected values (Press [2nd][x^{-1}] Names 2: [B] to navigate there).

```
X²-Test
 Observed:[A]
 Expected:[B]
 Calculate Draw
```

9. Report the X^2 statistic and p-value below. Compare these values to what you computed in #7.

$X^2 =$_____   p-value =_____

10. Verify your expected cell counts from #6 by looking at matrix B. (Press [2nd][x^{-1}] Names 2: [B][ENTER]).

Conclusion: The X^2 test for independence can be used any time you want to compare two or more observed samples to determine if they might reasonably have come from some larger set. In our, example we wished to see if the AP scores from schools in rural, suburban and urban settings varied significantly. Because our p-value was large (.1275), our data set did not give strong enough evidence to suggest that AP scores differed. More than one time in ten we could have gotten results like this if in fact the null hypothesis was true—the passing rates are the same at all schools.