EXPLORATIONS

Chapter 3

Applications of the Derivative

In this chapter, you will explore two common applications of the derivative, optimization and a related rate.

Example 1: Designing a cylinder

Many optimization problems involve volumes and surface areas. This example shows how to solve a classic cylinder problem with the TI-89.

A right circular cylinder with a top has a volume of 355 ml. Determine the dimensions of the cylinder with minimal surface area.

Solution

Define the surface area as a function of the radius. Compute the first derivative, set it equal to zero, and determine the minimum point. You also can obtain the same result on a graph with the minimization commands.

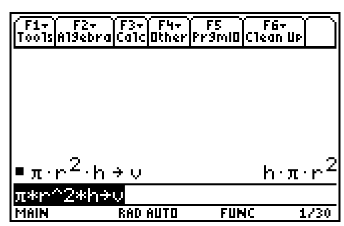
Solving numerically

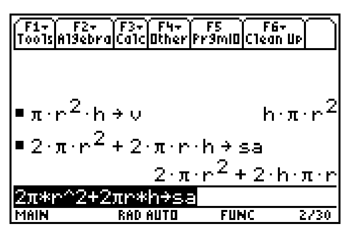
1. Press [2nd] [F6] **Clean Up** and select **2:NewProb** to clear variables and set other defaults.
2. Store an expression for the volume to the variable v.

   [2nd] [π] [×] **R** [^] **2** [×] **H** [STO▸] **V** [ENTER].

   Store an expression for the surface area to the variable sa in a similar manner.

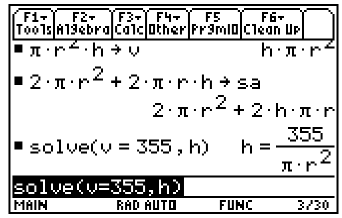
   2 [2nd] [π] [×] **R** [^] **2** [+] **2** [2nd] [π] [×] **R** [×] **H** [STO▸] **SA** [ENTER]





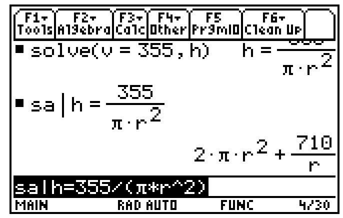
3. Since the volume is constant for this problem, you can use the **solve(** command to solve for h in terms of r.

   [F2] **1:solve(V** [=] **355** [,] **H** [)] [ENTER]



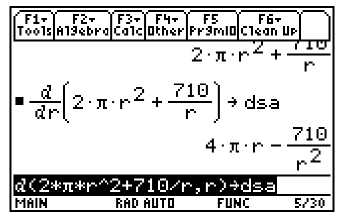
4. Substitute the result from step 3 into the surface area formula to express the surface area as a function of r only.

   SA [|] ⏶ [ENTER] [ENTER]



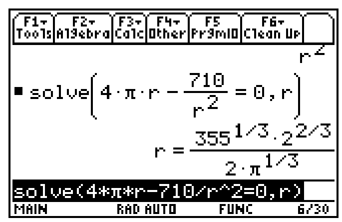
5. Compute and store the derivative of your surface area function.

   [2nd] [d] ⏶ [ENTER] [,] **R** [)] [STO▸] **DSA** [ENTER]

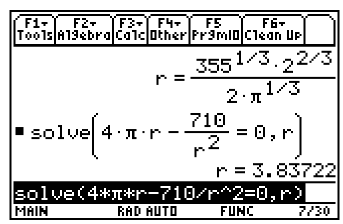


6. Use the **solve(** command to find the value of r when the derivative is zero.

   [F2] **1:solve(** ⏶ [ENTER] [=] **0** [,] **R** [)] [ENTER]

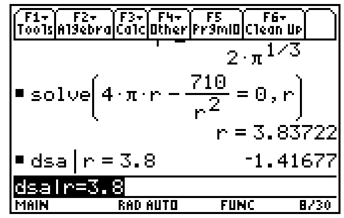


   To see a decimal estimate for the value, press [◆] [ENTER].



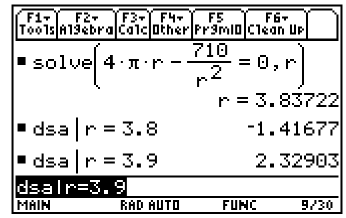
7. To test that the point is a minimum, you can compute the value of the derivative at a point on each side. First, test a point on the left.

   DSA [|] **R** [=] **3.8** [ENTER]



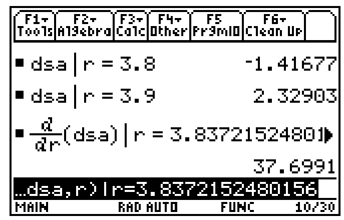
   Now test a point on the right.

   [◀] [←] **9** [ENTER]

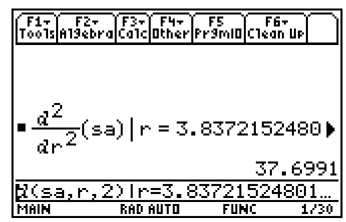


   The derivative values imply that the curve is falling and then rising, so your value is a minimum point.

8. Alternately, you can use the second derivative test to test that the point is a minimum. There are two ways to compute the second derivative. Since **dsa** is the derivative of **sa**, you can differentiate **dsa** and evaluate it at the value of r computed as the critical point in step 6.

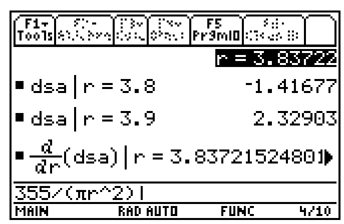


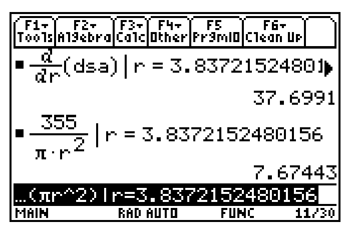
   You also can compute a second derivative of **sa** at the same critical point. Recall that a second derivative is computed when a 2 is used as the third argument of the differentiate command.



9. Compute the height of the cylinder using the value of r from step 6.

   355 [÷] [(] [2nd] [π] **R** [^] **2** [)] [|] [▲] (11 times) [ENTER] [ENTER]

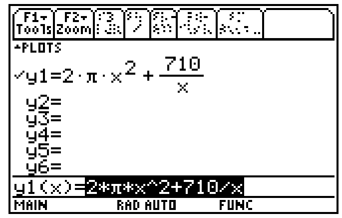




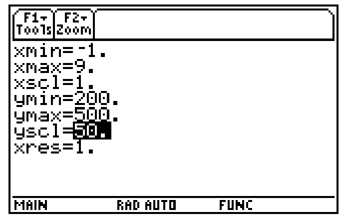
Solving graphically

1. Press [◆] [Y=] to display the Y= Editor. Press [CLEAR] as necessary to delete any functions. Define the function (the surface area formula from step 4 of *Solving Numerically*).

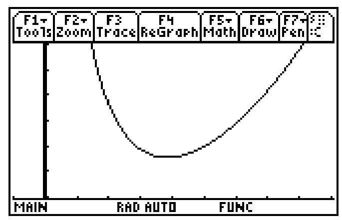
   2 [2nd] [π] **X** [^] **2** [+] **710** [÷] **X** [ENTER]



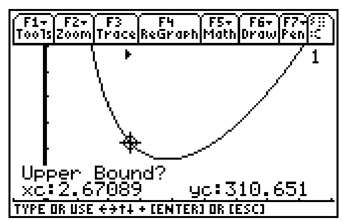
2. Press [◆] [WINDOW] and set the Window variable values as shown.

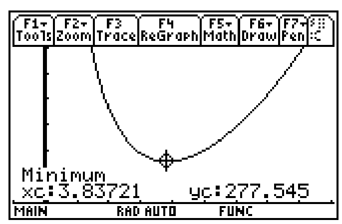


3. Press [◆] [GRAPH] to graph the function.



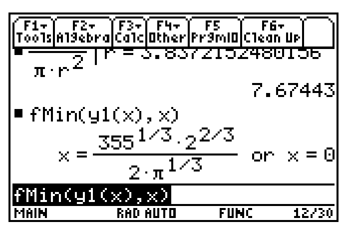
4. To compute the minimum point, press [F5] **Math** and select **3:Minimum**. Now use (◀), or type a value to the left of the minimum point, and press [ENTER]. Press (▶) or type a value for the right bound. The coordinates of the minimum point are displayed.



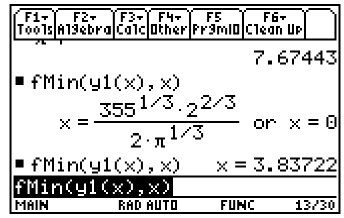


5. To compute a minimum on the Home screen without using derivatives, press [HOME] to return to the Home screen. Enter the command:

   [F3] **6:Min(Y1** [(] **X** [)] [,] **X** [)] [ENTER]



To see a decimal estimate for the value, press (▲) [ENTER].

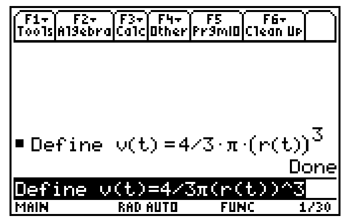


Example 2: Related rates

Many related rates examples investigate how the rates of change of two quantities are related. This classic sphere problem is done by considering both the volume and the radius as functions of time.

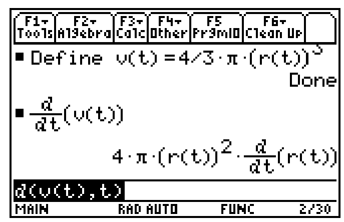
A spherical balloon is being inflated so that the radius is increasing at a steady rate of 2 cm/sec. Find the rate of change of the volume at any time t, and at the time when the radius is 8 cm.

1. Press [2nd] [F6] **Clean Up** and select **2:NewProb** to clear variables and set other defaults.



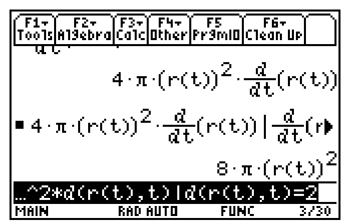
2. To define a volume function, press [F4] **Other** and select **1:Define**. Enter the volume function as shown. Note that the radius is actually a function *r(t)* and that two sets of parentheses are needed to cube the radius.
3. Compute the derivative.

   [2nd] [d] **V** [(] **T** [)] [,] **T** [)] [ENTER]



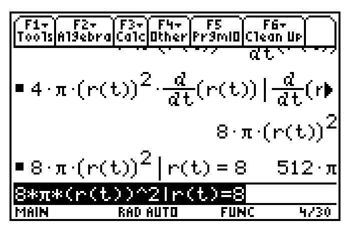
4. Substitute 2 for $\dfrac{dr}{dt}$ in the previous result to find the volume for t.

   (▲) [ENTER] [|] [2nd] [d] **R** [(] **T** [)] [,] **T** [)] [=] **2** [ENTER]



5. Use the previous result to find the volume when the radius is 8 cm.

   (▲) [ENTER] [|] **R** [(] **T** [)] [=] **8** [ENTER]



Exercises

Exercises 1 to 4 involve a right circular cylinder with no top that is constructed from 100 square cm of material.

1. Determine the volume as a function of the radius.
2. Compute the derivative of the volume function.
3. Use the derivative to determine the dimensions of the cylinder with maximum volume.
4. Compute the dimensions of the cylinder with maximum volume directly, without use of the derivative.

Exercises 5 and 6 involve a spherical iceball that is melting in such a way that the volume decreases at the rate of 6 cm^3 / sec.

5. Compute the derivative $\frac{dr}{dt}$ at any time t.
6. Compute the derivative $\frac{dr}{dt}$ at the time when the radius is 1 cm.