# TI-*nspire*™

## Derivative of a Function & Discovery of Power Rule
by – Joshua Mize

### Activity overview
*This activity will walk students through investigations of the value of the slope of a tangent line to polynomial curves at various points. An instantaneous scatter plot will be generated so that the students will discover that they can not only investigate derivatives of various polynomials, but that the derivatives themselves are in fact functions also – something that many students don't catch on to right away when beginning Calculus.*

### Concepts
*Slopes of lines*
*Tangent lines to a curve*

### Teacher preparation
Students are expected to have just finished learning limit definition of a derivative and other prerequisite limit skills, including evaluating the limit definition of a derivative for simple functions like $f(x) = x^2$ and $f(x) = x^3$. Additionally, the students are expected to know that the slope of a tangent line to a curve at a point is equivalent to the value of a derivative of the function at that point.

### TI-Nspire Applications
*Graphs & Geometry, Lists and Spreadsheet, Notes*

### Step-by-step directions

The following steps highlight the construction of one of the problems contained within the *Derivative of a Function.tns* file. The steps will be identical for the other polynomials contained within the *Derivative of a Function.tns* file.

Create an introductory slide for each of the polynomials highlighting the steps that the students will be taking during the given problem.

These steps will highlight the construction for a cubic (3$^{rd}$ degree) polynomial.

> **Problem 3**
> **Let f(x) be a 3rd degree polynomial.**
>
> Drag the labeled point along the x–axis and observe the characteristics of the value of the derivative (slope of tangent line).

Graph a polynomial, changing the values of your coefficients so that all of the local maximums and minimums are visible within the viewing screen. Changing the viewing screen itself will also achieve this.
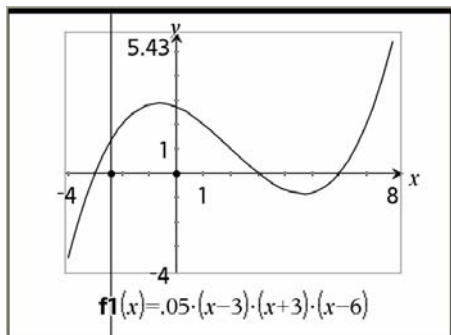
$$f1(x)=.05\cdot(x-3)\cdot(x+3)\cdot(x-6)$$
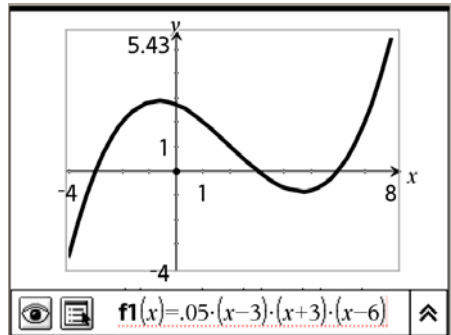
You may use *Ctrl-G* to hide the entry line used to enter the function itself.

$$f1(x)=.05\cdot(x-3)\cdot(x+3)\cdot(x-6)$$
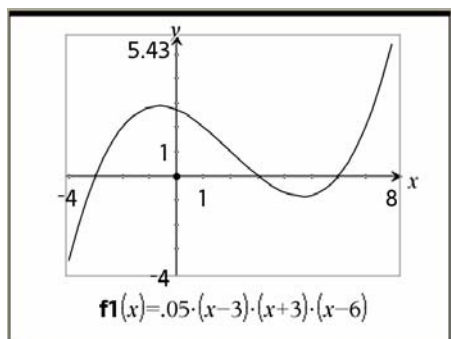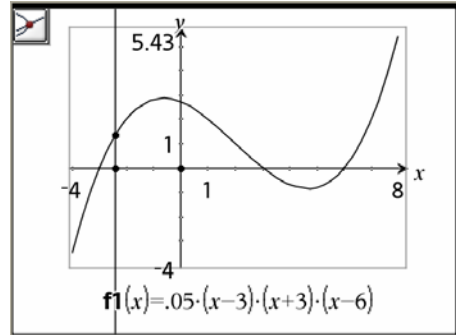
Select the *Point On* tool…Click on the *x*-axis to place the point on the *x*-axis. We'll call this point $(a, 0)$.

$$f1(x)=.05\cdot(x-3)\cdot(x+3)\cdot(x-6)$$
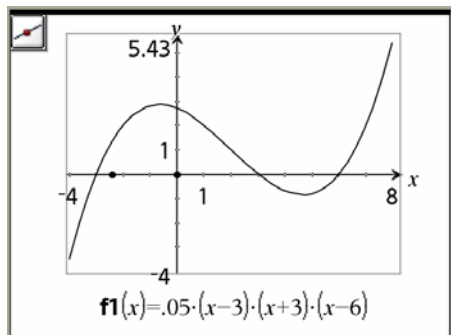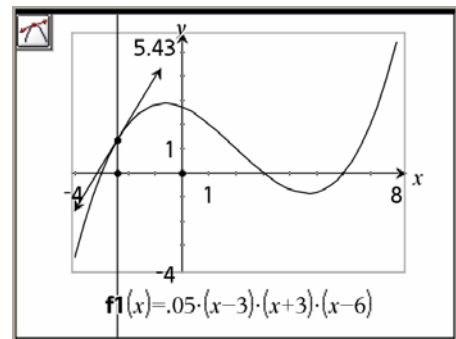
Create a perpendicular to the *x*-axis through the point you just created.

$$f1(x)=.05\cdot(x-3)\cdot(x+3)\cdot(x-6)$$

Create the point of intersection between the perpendicular and the curve.  We'll call this point $(a, f(a))$.

$$f1(x) = .05 \cdot (x-3) \cdot (x+3) \cdot (x-6)$$

Create the tangent to the curve through $(a, f(a))$.

$$f1(x) = .05 \cdot (x-3) \cdot (x+3) \cdot (x-6)$$

Hide the perpendicular line.

$$f1(x) = .05 \cdot (x-3) \cdot (x+3) \cdot (x-6)$$
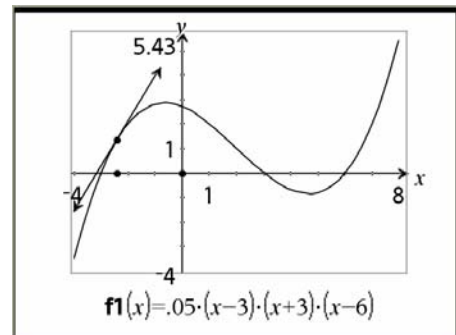
Measure the value of the slope of the tangent line.  Place this measured value somewhere on the screen.

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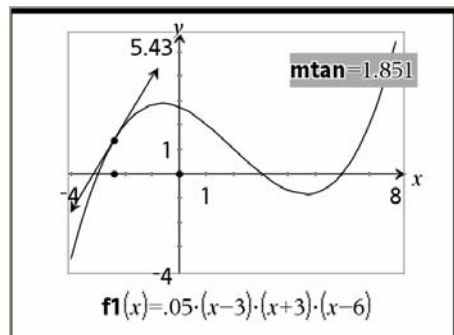$$f1(x) = .05 \cdot (x-3) \cdot (x+3) \cdot (x-6)$$
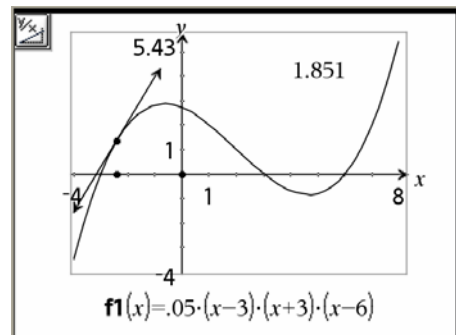
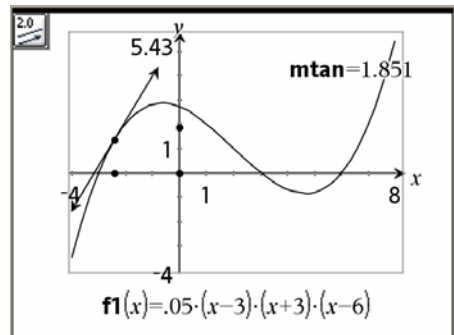Click on the measurement for the slope to select it.
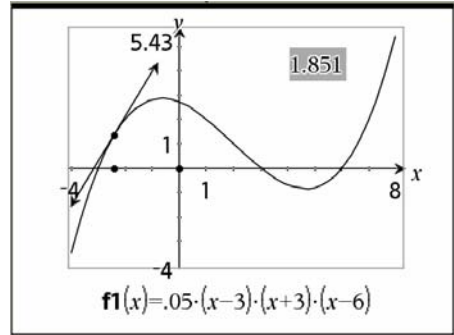
Assign this value to a variable of your choosing.  In this demonstration, it has been assigned to the variable **mtan**.
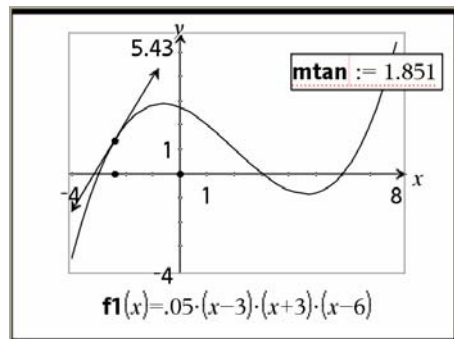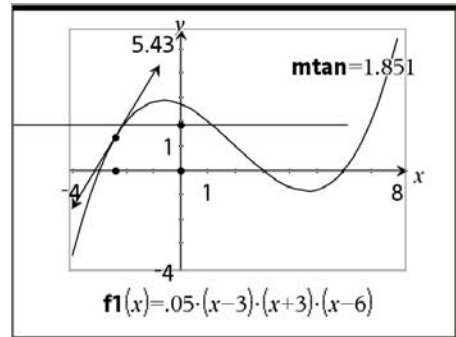
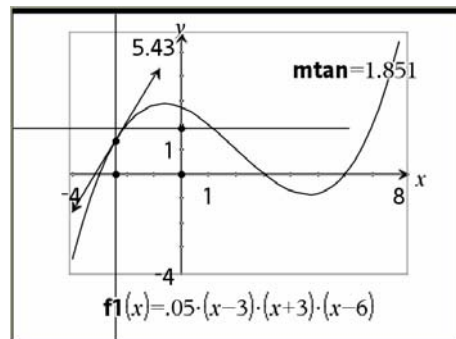Click on the newly assigned variable to select it.

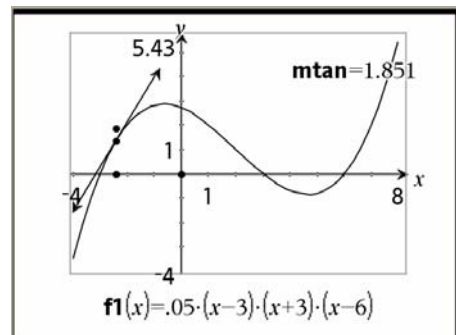Transfer the value of this measurement to the *y*-axis creating a new point $\left(0, \textbf{\textit{mtan}}\right)$.

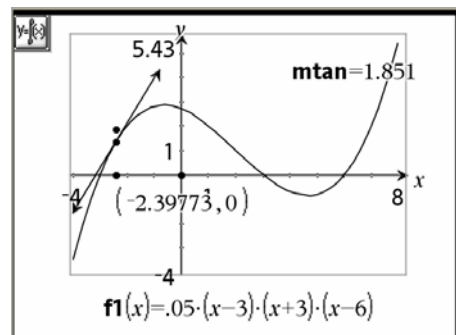Create a perpendicular to the *y*-axis through the point $(0, \mathbf{mtan})$.

Create a perpendicular to the *x*-axis through the point $(a, 0)$.

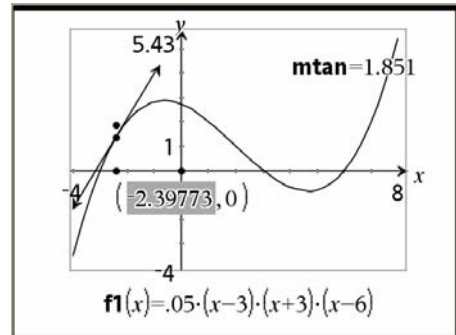Create a point of intersection of the two perpendiculars and then hide the perpendiculars. The coordinates of this point will be $(a, \mathbf{mtan})$.
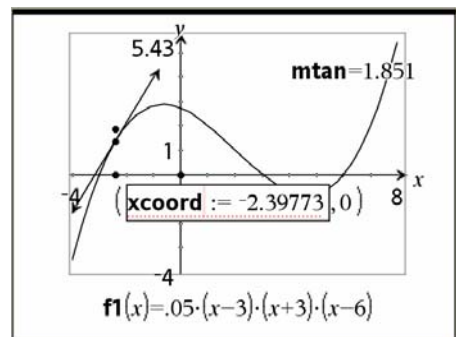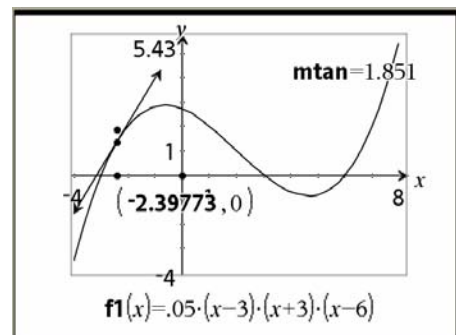
Measure the coordinates of $(a, 0)$.

Click on the value of the *x*-coordinate to select it.



Assign this newly selected value to a variable. For this demonstration it has been assigned to the variable **xcoord**.



The *x*-coordinate is now in bold type to indicate that it has been assigned to a variable.
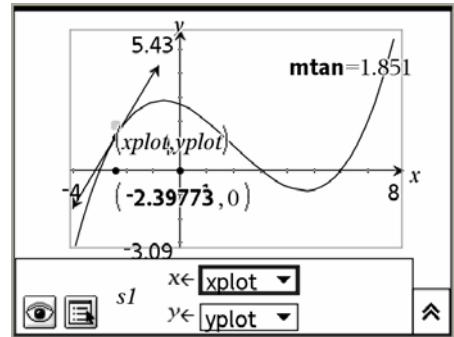


Insert a *Lists & Spreadsheet* page. Title column A **xplot** and column B **yplot**.
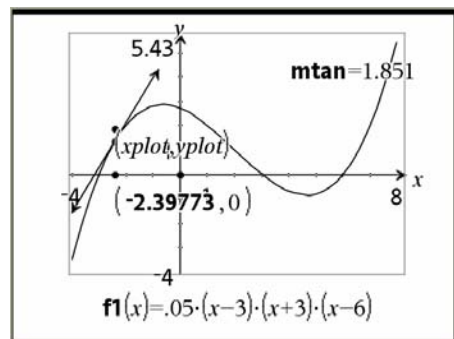
On the formula line for each column, select *Data Capture > Automated Data Capture*, and replace the variables (**var**) with **xcoord** for **xplot** and **mtan** for **yplot**.

Go back to the graph screen and create a scatter plot. Select **xplot** for the
x-variable and **yplot** for the y-variable.

Use *Ctrl-G* again to hide the entry line.

Hide the labels for the scatter plot, the coordinates for $(a,0)$, and the value of **mtan**.
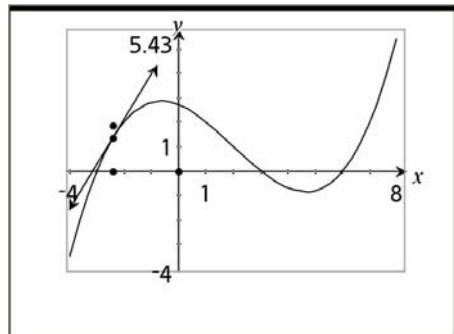
Change the point $(a,0)$ so that it is now a empty circle…this will make it stand out from the other points. Additionally, for this demonstration, this point will be labeled with "*Drag me!*" to indicate to the user which point needs to be moved.

The following screen shots highlight the results of the automated scatter plot when the point on the *x*-axis is moved back-and-forth.



On the List & Spreadsheet page, highlighting the formula cell in each column and hitting enter twice will clear the data in the respective column and allow the problem to be completed again by another user.
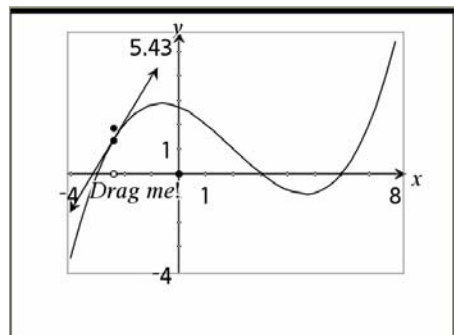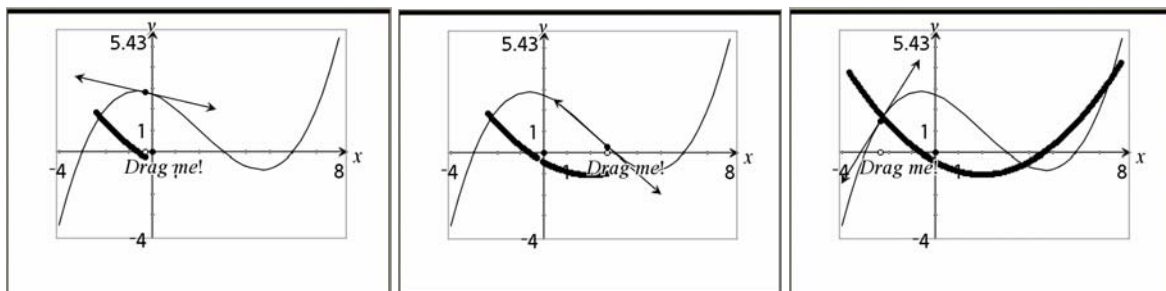


Finally, back on the graph screen, you can split the screen if desired to provide the directions to the user on the same screen without having the flip back-and-forth with a notes page.
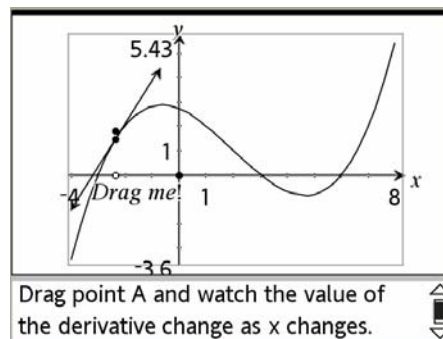


*Step-by-step activity directions with screenshots, sample data, etc. as needed. Screenshots should be created using the TI-Nspire handheld and resized to 70% for best visibility.*

*Steps/problems/exercises go here*               *Screenshots go here @ 70%*

## Activity extensions

- *A similar activity can be adopted to graphically discover the derivatives of functions like sin(x), cos(x), $e^x$, and ln(x).*

# The Derivative as a Function & Discovery of the Power Rule for Differentiation

Turn on your TI-Nspire handheld device and open the *Derivative as a Function* file. This file will ask you to investigate derivatives of 5 different polynomials: linear, quadratic, cubic, quartic, and quintic. Please be sure to make careful notes on what you observe in each problem as it will be helpful as it pertains to class discussion.

**Recall** that you can find a tangent line to a curve $f(x)$ at any point $x = a$. The slopes of these tangent lines tell you the value of the derivative of the function at that particular point... $f'(a)$.

Now move the point labeled "*Drag me!*" along the *x*-axis and notice that as you move the point, the location of the tangent line moves also. The extra point plotted is measuring the value of $f'(a)$ for that specific location.

Now move on to investigate problems 1-5 on the TI-Nspire handheld device. For each of the problems, please move the labeled point along the *x*-axis and make observations about the characteristics of the resulting scatter plot. The automatically generated scatter plot will represent the value of the derivative at that particular point. In other words, each point on the scatter plot will have the coordinates $(a, f'(a))$.

**Problem 1** -- Please make notes of all observations related to the value of the derivative for problem 1 in the space below. When you are finished, please move on to problem 2.

**Problem 2** -- Please make notes of all observations related to the value of the derivative for problem 2 in the space below. When you are finished, please move on to problem 3.

**Problem 3** -- Please make notes of all observations related to the value of the derivative for problem 3 in the space below. When you are finished, please move on to problem 4.

**Problem 4** -- Please make notes of all observations related to the value of the derivative for problem 4 in the space below.  When you are finished, please move on to problem 5.

**Problem 5** -- Please make notes of all observations related to the value of the derivative for problem 5 in the space below.  When you are finished, please move on to the table at the top of the next page.

| When $f(x)$ is… | $f'(x)$ is … |
|---|---|
| linear | |
| quadratic | |
| cubic | |
| quartic | |
| quintic | |
| Use the information in this table to generalize what we will refer to as the **Power Rule for Differentiation**. | |
| $a \cdot x^n + ...$ | |

**Use the data in the table above to predict characteristics of the derivative of a 17th degree polynomial.**

**Extension:**  Go back to each of the five polynomials on your TI-Nspire handheld device and change some of the values for the coefficients.  Be careful to observe and changes in the characteristics of your derivative scatter plot.  Make any such observations below.

**Note:**  *One important observation you hopefully made was that the derivative of the functions given to you are in fact functions themselves.  This is not something that is noticed all the time.*