Midsegments of Triangles

TEACHER NOTES

About the Lesson

In this activity, students will explore the properties of the midsegment, a segment that connects the midpoints of two sides of a triangle. First, students will construct and investigate one midsegment and the relationship of the new small triangle to the original triangle. Then, all three midsegments will be constructed and this figure will be explored. As a result, students will:

- Construct a line segment joining the midpoints of two sides of a triangle and conjecture a relationship between the segment and the third side.
- Prove and apply the Triangle Midsegment Theorem and its converse.

Vocabulary

- midsegment
- midpoint
- parallel

Teacher Preparation and Notes

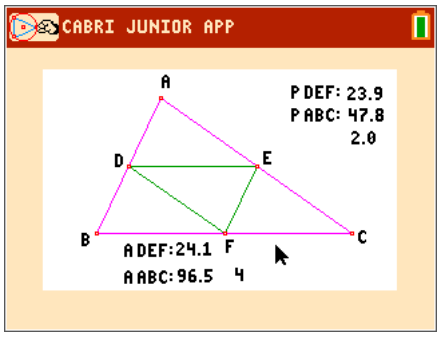
- The midsegment of a triangle is a line segment connecting the midpoints of any two sides of the triangle. It is parallel to and half as long as the third side of the triangle.
- Before beginning this activity, make sure that all students have the Cabri Jr. application installed.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

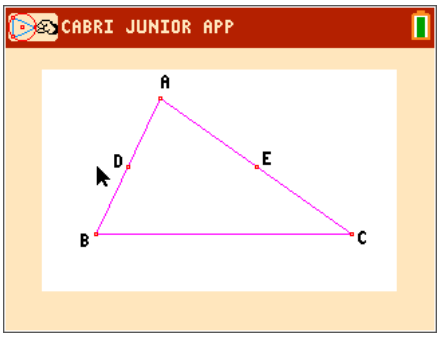
Lesson Files:

- Midsegments_of_Triangles_Student.pdf
- Midsegments_of_Triangles_Student.doc

Problem 1 – One Midsegment

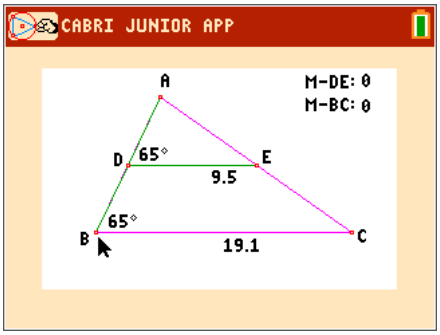
Students begin the activity by creating a *Cabri*^{TM} *Jr.* file to be used again later in the activity. They will construct a triangle and the midpoints of two sides. The triangles that students create do not need to match the triangle shown.

Students should name this file *MIDSEG* and not save over this file. If students accidentally save over the file, they will need to create a new triangle and the midpoints of its sides.



Next, students will construct $\overline{DE}$. They will find the slope and length of $\overline{DE}$ and of $\overline{BC}$ and the measure of $\angle ADE$ and $\angle ABC$. Students can also examine the lengths of other segments in the triangle, as well.

Students should make conjectures about relationships in the figure. Be sure that they drag the vertices of $\triangle ABC$ around the screen to confirm their conjectures.



1. Use the **D. & Length**, **Angle**, and **Slope** tools in the **Measure** menu to explore the properties of the midsegment. Fill in the blanks below.

Length of _____ = _____ Length of _____ = _____

Measure of ∠ _____ = _____ Measure of ∠ _____ = _____

Slope of _____ = _____ Slope of _____ = _____

Sample Answers:

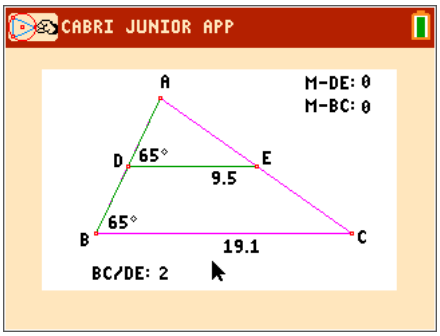
Length of $\overline{BC}$ = 19.1; Measure of $\angle ABC$ = 65°; Slope of $\overline{BC}$ = 0

Length of $\overline{DE}$ = 9.5; Measure of $\angle ADE$ = 65°; Slope of $\overline{DE}$ = 0

2. What conjectures can you make about the midsegment $\overline{DE}$ and its relationship to $\triangle ABC$? Be sure to drag the vertices of $\triangle ABC$ around the screen to confirm your conjectures.

Sample Answer: If *D* is on $\overline{AB}$ and *E* is on $\overline{AC}$, then $\overline{DE}$ is half the length of $\overline{BC}$

Next, students will find the ratio of *BC* to *DE*. Dragging a vertex of △*ABC*, students can record their observations on their worksheets.



Teacher Tip: Ask: What is the relationship between △*ADE* and △*ABC*? Can you prove it?

3. Drag a vertex of △*ABC* and observe the results.

Length of $\overline{BC}$ = _____ Length of $\overline{BC}$ = _____

Length of $\overline{DE}$ = _____ Length of $\overline{DE}$ = _____

Ratio = _____ Ratio = _____

Sample Answers: Length of $\overline{BC}$ = 17.4; Length of $\overline{DE}$ = 8.7; Ratio: 2
Length of $\overline{BC}$ = 16.8; Length of $\overline{DE}$ = 8.4; Ratio: 2

4. Complete the conjectures.

The length of the midsegment is _____.

The slope of the midsegment is _____.

Answers: The length of the midsegment is half the length of the side of the triangle it does not intersect. The slope of the midsegment is parallel to the side it does not intersect.

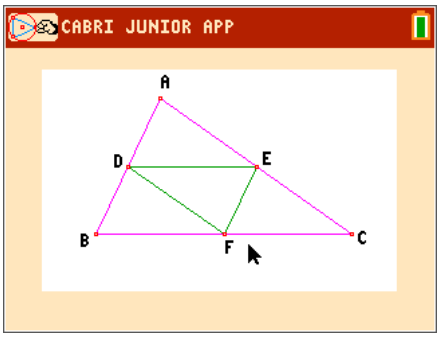
5. What is the relationship between △*ADE* and △*ABC*? How would you prove it?

Sample Answer: △*ADE* and △*ABC* are similar triangles. Because $\overline{BC} \parallel \overline{DE}$, ∠*AED* and ∠*ACB* are congruent and ∠*ADE* and ∠*ABC* are congruent. ∠*BAC* and ∠*DAE* are congruent. *BC* = 2*DE*. Therefore △*ADE* and △*ABC* are similar because all angles are congruent and corresponding sides have the same ratio.

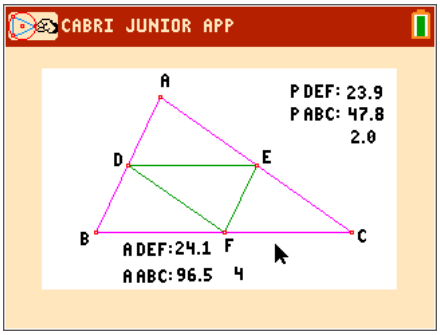
Problem 2 – Three Midsegments

Students will open the file *MIDSEG* from Problem 1 or create a new triangle similar to before.

Students should find the midpoint of the third side. Next, students should construct △*DEF*. Make sure that students see that this triangle is made up of midsegments of △*ABC*.



Next, students will find the perimeter and area for △*ABC* and △*DEF*. They will then find the ratio of the perimeters and areas. Remind students to find the ratio of △*ABC* to △*DEF*, so that they have a number larger than one.



Teacher Tip: Ask: What is the relationship between △DEF and △ABC? Can you prove it? What is the relationship between △DEF and △ADE? Can you prove it?

6. Use the **D. & Length** and **Area** tools in the **Measure** menu to measure the perimeter and area of △*DEF* and △*ABC*.

Perimeter of △ABC = _____ Area of △ABC = _____

Perimeter of △DEF = _____ Area of △DEF = _____

Sample Answers: Perimeter of △*ABC* = 47.8; Area of △*ABC* = 96.5
Perimeter of △*DEF* = 23.9; Area of △*DEF* = 24.1

7. Drag a vertex of △*ABC* and observe the results.

Ratio of Perimeters = _____

Ratio of Areas = _____

Answer: Ratio of Perimeters = 2; Ratio of Areas = 4

8. What is the relationship between $\triangle DEF$ and $\triangle ABC$? How would you prove it?

Sample Answer: $\triangle DEF$ and $\triangle ABC$ are similar. $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{DE}$, and $\overline{AC} \cong \overline{DF}$, therefore $AB = 2EF$, $BC = 2DE$, and $AC = 2DF$ because of the Triangle Midsegment Rule. Therefore $\triangle DEF$ and $\triangle ABC$ are similar because of the side-side-side rule.

9. What is the relationship between $\triangle DEF$ and $\triangle ADE$? How would you prove it?

Answer: $\triangle DEF$ and $\triangle ADE$ are congruent. $\angle ADE$ and $\angle DEF$ are alternate interior angles so they are congruent. $\angle AED$ and $\angle EDF$ are alternate interior angles so they are congruent. DE = DE. Therefore, $\triangle DEF$ and $\triangle ADE$ are congruent because of the angle-side-angle rule.

Apply the Math

10. If $DE = 6.2$ inches, and $AB = 11.4$ inches, find the lengths of $\overline{BC}$ and $\overline{EF}$.

Sample Answer: Length of $\overline{BC} = 12.4$; length of $\overline{EF} = 5.7$

11. If the perimeter of $\triangle ABC$ is 32 cm, find the perimeter of $\triangle DEF$.

Sample Answer: Perimeter of $\triangle DEF = 16$

12. If the area of $\triangle DEF$ is 8.6 cm^2, find the areas of $\triangle ABC$, $\triangle ADE$ and $\triangle BDF$.

Sample Answer: Area of $\triangle ABC = 34.4$ cm^2, area of $\triangle ADE = 8.6$ cm^2, area of $\triangle BDF$ is 8.6 cm^2

13. If $m\angle AED = 27°$ and $m\angle A = 64°$, find as many other angle measures as possible.

Sample Answers: $m\angle A = 64°$, $m\angle B = 89°$, $m\angle C = 27°$, $m\angle ADE = 89°$, $m\angle AED = 27°$, $m\angle EDF = 27°$, $m\angle DEF = 89°$, $m\angle BDF = 64°$, $m\angle DFE = 64°$, $m\angle CEF = 64°$, $m\angle EFC = 89°$, $m\angle BFD = 27°$