

Out of Control! – ID: 10137

By Michele Patrick

Time required

35 minutes

Activity Overview

In this activity, students make XY Line Plots and scatter plots to create run charts and control charts (types of time plots) to determine if a manufacturing process is statistically stable. Students begin by looking for any unusual patterns or trends in variation and then calculate sample means and upper and lower control limits to create control charts which they then interpret.

Concepts

- *Run charts and control charts*
- *Sample means*
- *Upper and lower control limits*

Teacher Preparation

- *This activity is designed to be used for students studying Statistics and Probability.*
- *This activity assumes students are familiar with means, standard deviations, and scatter plots.*
- *If it has already been introduced, review the 68-95-99.7 rule for normal distributions. This will help students better understand the placement of the upper and lower control limits in the control charts.*
- *You may wish to mention that although widely used in manufacturing, control charts can also be used in other situations, such as monitoring a person's heart rate.*
- *The screenshots on pages 2–6 (top) demonstrate expected student results. Refer to the screenshots on pages 6 (bottom) and 7 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter “10137” in the quick search box.***

Classroom Management

- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *The student worksheet helps guide students through the activity and provides a place for students to record their answers.*
- *The TI-Nspire solution document [StatAct33_OutOfControl_Soln_EN.tns](#) shows the expected results of working through the activity.*

TI-Nspire™ Applications


Calculator, Graphs & Geometry, Lists & Spreadsheet, Notes, Data & Statistics

Problem 1 – Run Charts

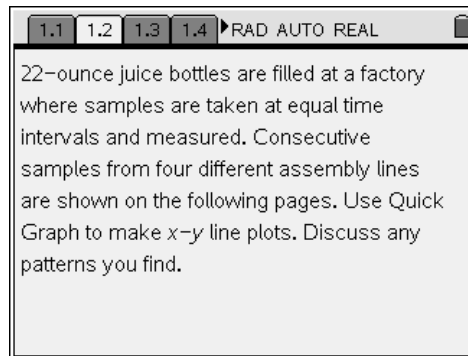
Introduce the concept of quality control and how it is used in business, specifically in manufacturing, via random sampling at regular intervals.

It is important to track changes both at regular intervals and over a period of time, not just at the beginning or end of the work day or work week.

In Problem 1, bottles filled at a factory are supposed to each get 22 ounces of juice. Even if the machinery does not have problems, it is unlikely that each bottle will get precisely 22 ounces. There will be some variation. But when is variation normal and when is it out of control?

Have students move to page 1.3 to see the data for the first assembly line. Have students glance at the volumes and ask if the variation appears acceptable. They will probably have a difficult time answering by just looking at the values. This is why a graph is so much more effective. Have students highlight both columns. To select a column, press **MENU > Actions > Select > Select Column** or press the up arrow when the title cell is active. Once one column is highlighted, hold down the **SHIFT** key () and use the arrows to select the second column.

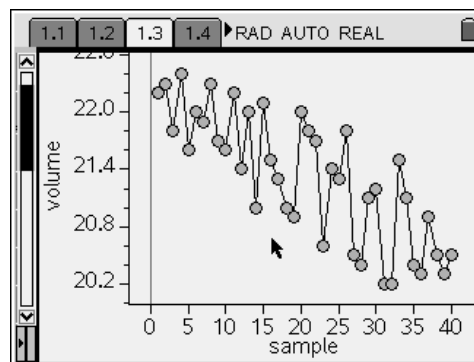
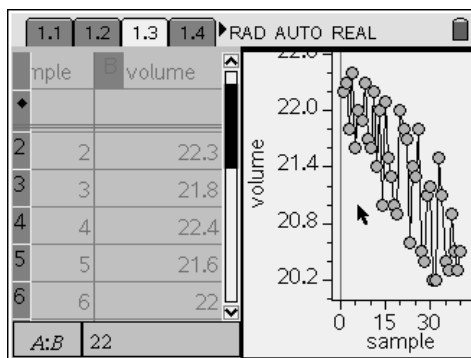
Instruct students to press **MENU > Data > Quick Graph** and then **MENU > Plot Type > XY Line Plot**. Explain that this time plot is called a run chart. To better see it, have students extend the graph to the left by selecting **Tools > Page Layout > Custom Split**. Press the left arrow to move the vertical dividing line to the left.



1.1 1.2 1.3 1.4 ▸ RAD AUTO REAL

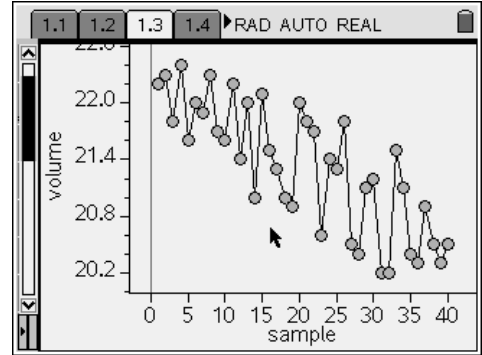
A	sample	B	volume	C	D	E
2	2	22.3				
3	3	21.8				
4	4	22.4				
5	5	21.6				
6	6	22				

A:B 22

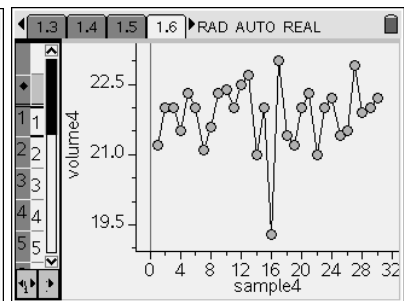
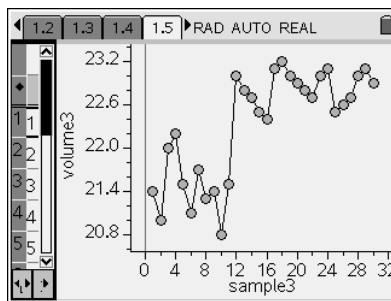
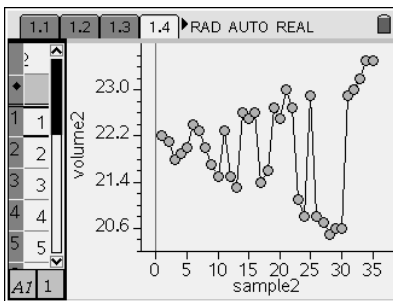


Ask students to describe any patterns or trends they see in the data. They should notice that there is a downward trend.

We don't know what acceptable deviations from the mean are, but something on the assembly line, possibly due to wear and tear, is having an affect on the amount of juice it is dispensing. We will later see how to determine if a value is too high or too low. For now, just discuss that this type of trend indicates variation that is not random.



Have students work independently to create run charts for the second, third, and fourth assembly lines, noting any suspicious patterns or trends. When everyone has finished, discuss their conclusions.



The second control chart shows increasing variation; the third shows an upward shift, and the fourth shows an unusually low value. All of these are signals that something unusual is happening with the process and should be looked into. Another type of trend that signals instability in a process is a cyclical pattern. High values followed by low values followed by high values and so on.

Problem 2 – Control Charts: Weights

In Problem 2, three items are randomly taken at a time and the means of the samples are plotted. Introduce the scenario on page 2.1.

1.4 1.5 1.6 2.1 RAD AUTO REAL

A QC technician takes the weights of three randomly selected cheese wheels packaged every hour, as shown on the next page.

Create a column with the means of each sample. Title it *means*. On the following page, create the scatter plot (sample, means). Connect the points. Does the process appear stable?

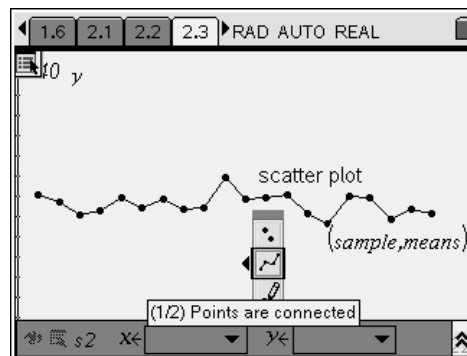
Instruct students to find the sample means on page 2.2. One way to do this is to type the formula **=mean(b1:d1)** in cell E2 and then copy the formula down through cell E20 (**MENU > Data > Fill Down**).

To be able to make a scatter plot on the next page, be sure students title the column. The title of *means* is suggested.

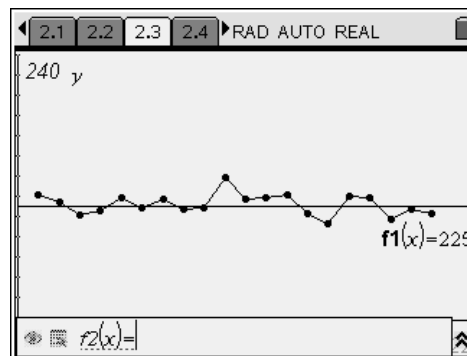
	A sa...	B	C	D	E means
1	1	226.4	225.9	226	226.1
2	2	223.5	226.2	226.6	225.433
3	3	223.9	224	224.8	224.233
4	4	224.3	223.5	225.9	224.567
5	5	226.5	227	224.1	225.867
E1	=mean(b1:d1)				

Have students create the scatter plot on page 2.3. To do this, they must first define the graph type by pressing **MENU > Graph Type > Scatter Plot**. Then select *sample* for the x-variable and *means* for the y-variable.

Once the scatter plot is graphed, press **MENU > Actions > Attributes** to connect the points. The label (*sample, means*) can be moved out of the way, hidden, or deleted. Ask students if they see any suspicious patterns in variation. (no)



Tell students that the mean weight of the cheese wheels is supposed to be 225 grams with a standard deviation of 1.5 grams. To see if the points in the chart are around where they should be, have them graph the function, $y = \text{target mean}$, which is $y = 225$. This horizontal line is called the center line.



To graph the function, students will need to change the Graph Type to **Function**. Based on this diagram, ask students if they think the cheese packaging process is in control. Some may say yes because some points are above the line and some are below. Others may still question how far away from the line a point can be.

Explain that the last items we need to graph are the upper and lower control limits. The formulas are given on page 2.5.

Depending on the background knowledge of your students, you can discuss how the formulas use the Central Limit Theorem and are analogous to finding confidence intervals.

For sample size n ,

Upper control limit: $\mu + 3\frac{\sigma}{\sqrt{n}}$

Lower control limit: $\mu - 3\frac{\sigma}{\sqrt{n}}$

Calculate both control limits on the next page. Then graph them on the control chart on page 2.3.

The upper and lower limits are 227.598 and 222.402 respectively. Have students graph the functions $y = \text{upper control limit}$ and $y = \text{lower control limit}$ on their chart. They can either retype the values in the function rule, or, store them as variables in the *Calculator* application and then type those variable names in the function rule.

To store a value, press $\text{ctrl} + \text{sto} \rightarrow \text{var}$ and then the variable name. Here, **ucl** and **lcl** are used.

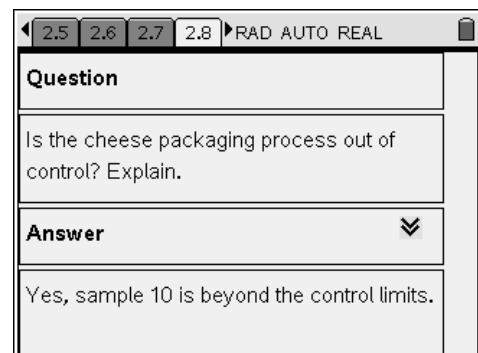
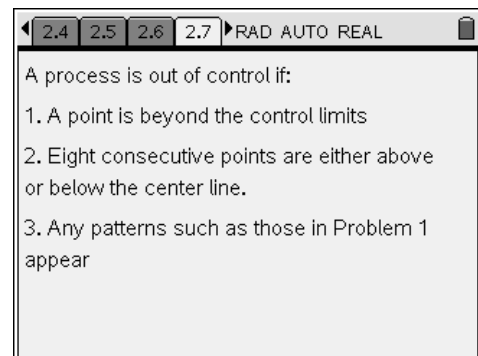
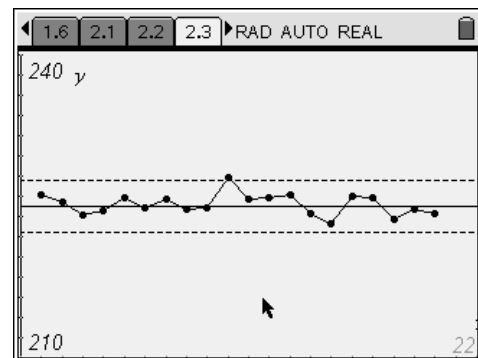
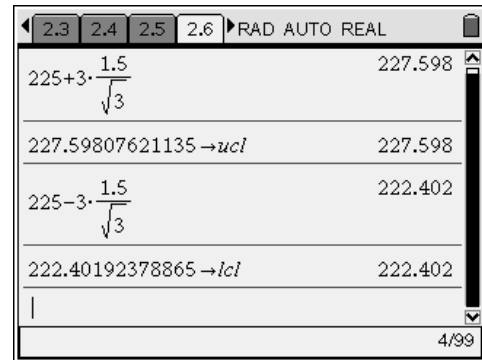
To better distinguish among the lines, students can make the upper and lower limits dashed (**MENU > Actions > Attributes**).

Explain that this chart is called a control chart. Because other sample characteristics can be graphed with center lines and upper and lower limits (such as ranges, proportions, and standard deviations), it is also known as the \bar{x} chart.

Use page 2.7 to begin a discussion on signals that show that a process is out of control.

These are the most standard criteria used. Some companies may look for additional criteria, such as a certain number of consecutive points increasing (or decreasing). Mention the risk of type I and II errors while determining if a process is out of control. These criteria may need to be tweaked for the specific process being studied depending on the possible sources of error.

Ask students if their control chart indicates statistical instability based on these criteria. They should say yes, as there is a sample mean above the upper control limit.



Problem 3 – Control Charts: Temperature

Introduce the scenario on page 3.1 and then allow students to work on creating the control chart independently. As before, they will need to find the sample means and calculate the upper and lower limits.

Walk around and assist students as needed. Students should conclude that the process is not stable because of the unusual number of consecutive points above the center line. Also, the points appear to be the start of a cyclical up and down cycle.

2.7 2.8 3.1 3.2 ▶ RAD AUTO REAL

A substance produced at a factory should have an average temperature of 4.5°C. Past experience shows that when everything is working correctly, there is a standard deviation of 0.6°C.

Two samples are taken every 15 minutes as shown on page 3.2. Graph the control chart with its center line and control limits.

2.8 3.1 3.2 3.3 ▶ RAD AUTO REAL

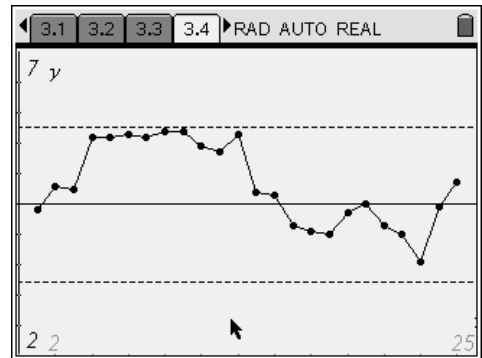
$$4.5 + 3 \cdot \frac{.6}{\sqrt{2}} \quad 5.77279$$

5.7727922061358 → ucl 5.77279

$$4.5 - 3 \cdot \frac{.6}{\sqrt{2}} \quad 3.22721$$

3.2272077938642 → lcl 3.22721

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Out of Control! – ID: 10137

(Student)TI-Nspire File: *StatAct33_OutOfControl_EN.tns*

1.1 1.2 1.3 1.4 ▶ RAD AUTO REAL

OUT OF CONTROL!

Statistics

Run Charts, Control Charts,
Means, and Control Limits

1.1 1.2 1.3 1.4 ▶ RAD AUTO REAL

22-ounce juice bottles are filled at a factory where samples are taken at equal time intervals and measured. Consecutive samples from four different assembly lines are shown on the following pages. Use Quick Graph to make x-y line plots. Discuss any patterns you find.

1.1 1.2 1.3 1.4 ▶ RAD AUTO REAL

A	sample	B	volume	C	D	E
1	1	22.2				
2	2	22.3				
3	3	21.8				
4	4	22.4				
5	5	21.6				

A7 1

1.1 1.2 1.3 1.4 ▶ RAD AUTO REAL

A	sample2	B	volume2	C	D	E
1	1	22.2				
2	2	22.1				
3	3	21.8				
4	4	21.9				
5	5	22				

A7 1

1.2 1.3 1.4 1.5 ▶ RAD AUTO REAL

A	sample3	B	volume3	C	D	E
1	1	21.4				
2	2	21				
3	3	22				
4	4	22.2				
5	5	21.5				

A7 1

1.3 1.4 1.5 1.6 ▶ RAD AUTO REAL

A	sample4	B	volume4	C	D	E
1	1	21.2				
2	2	22				
3	3	22				
4	4	21.5				
5	5	22.3				

A7 1

1.4 1.5 1.6 2.1 ▸RAD AUTO REAL

A QC technician takes the weights of three randomly selected cheese wheels packaged every hour, as shown on the next page.

Create a column with the means of each sample. Title it *means*. On the following page, create the scatter plot (sample, means). Connect the points. Does the process appear stable?

1.5 1.6 2.1 2.2 ▸RAD AUTO REAL

A	sa...	B	C	D	E
1	1	226.4	225.9	226	
2	2	223.5	226.2	226.6	
3	3	223.9	224	224.8	
4	4	224.3	223.5	225.9	
5	5	226.5	227	224.1	

A1 1

1.6 2.1 2.2 2.3 ▸RAD AUTO REAL

240 y

$f1(x)=$

2.1 2.2 2.3 2.4 ▸RAD AUTO REAL

The cheese wheels should have a mean weight, μ , of 225 grams and standard deviation, σ , of 1.5 grams.

Graph the center line $y = 225$ on the chart. At this point, do you think the cheese packaging process is in control?

2.2 2.3 2.4 2.5 ▸RAD AUTO REAL

For sample size n ,

Upper control limit: $\mu + 3\frac{\sigma}{\sqrt{n}}$

Lower control limit: $\mu - 3\frac{\sigma}{\sqrt{n}}$

Calculate both control limits on the next page. Then graph them on the control chart on page 2.3.

2.3 2.4 2.5 2.6 ▸RAD AUTO REAL

1

0/99

2.4 2.5 2.6 2.7 ▸RAD AUTO REAL

A process is out of control if:

1. A point is beyond the control limits
2. Eight consecutive points are either above or below the center line.
3. Any patterns such as those in Problem 1 appear

2.5 2.6 2.7 2.8 ▸RAD AUTO REAL

Question

Is the cheese packaging process out of control? Explain.

Answer

2.6 2.7 2.8 3.1 ▸RAD AUTO REAL

A substance produced at a factory should have an average temperature of 4.5°C. Past experience shows that when everything is working correctly, there is a standard deviation of 0.6°C.

Two samples are taken every 15 minutes as shown on page 3.2. Graph the control chart with its center line and control limits.

2.7 2.8 3.1 3.2 ▸RAD AUTO REAL

A	sa...	B	C	D	E	F	G
1	1	3.9	4.9				
2	2	5	4.6				
3	3	6.1	3.4				
4	4	5.2	6				
5	5	4.3	6.9				

A1 1

2.8 3.1 3.2 3.3 ▸RAD AUTO REAL

1

0/99

3.1 3.2 3.3 3.4 ▸RAD AUTO REAL

7 y

$f1(x)=$

3.2 3.3 3.4 3.5 ▸RAD AUTO REAL

Question

Are there indications of an unstable process? Explain.

Answer