Teacher Notes

Activity 11

Investigating the Derivatives of Some Common Functions

Objectives

- Develop the idea of the derivative as a function
- Gather evidence toward some common derivative formulas
- Use numerical and graphical investigations to form conjectures

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 75 minutes

Abstract

In this activity, students will investigate the derivatives of sine, cosine, natural log, and natural exponential functions by examining the symmetric difference quotient at many points using the table capabilities of the graphing handheld. This activity promotes the transition to thinking about the derivative as a function by approximating the derivative at a point action repeatedly, building up a derivative approximation function over a discrete domain. This activity has 18 questions, including a question asking students to write a summary at the conclusion of the activity.

Management Tips and Hints

Prerequisites

Students should be able to graph and modify (zoom, trace, and so on) graphs of functions and generate tables on the graphing handheld.

This activity should be done after the derivative at a point has been studied.

Evidence of Learning

Students should be able to investigate other derivative formulas using the same technique.

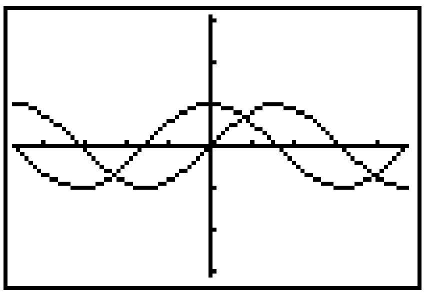
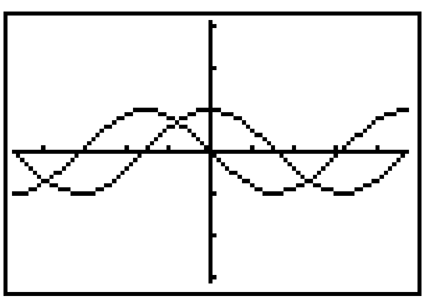
Common Student Errors/Misconceptions

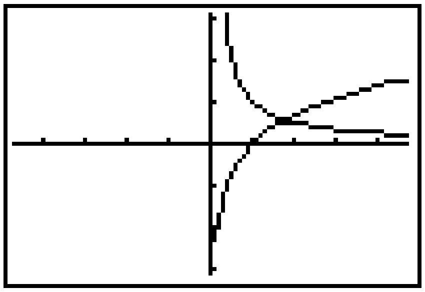
- Students may have to be reminded that the symmetric difference quotient merely approximates the derivative at a point.
- Students may have a difficult time predicting the derivative function on the basis of numerical evidence (especially the first time, as in Question **3**). Questions **1** and **2** are meant to provide clues to help them predict, and students may amend their predictions on the basis of graphical evidence.
- Looking at numerical evidence before graphing has two purposes. First, an important goal of this activity is to move students toward thinking about the derivative as a function. The strategy of building up a function by taking (or approximating) derivatives point-by-point on a discrete domain—much like plotting points when students are initially introduced to functions—is more directly applied numerically than graphically. Second, the discovery of the derivative of ln(x) is rather dramatic when viewed numerically.

Extensions

Explore the command **nDeriv** by making a table with sin(x), the symmetric difference quotient of sin(x) (using Y0), and nDeriv(sin(X), X, X). Ask students what they think the **nDeriv** command does.

Activity Solutions

1. Max: 1; Min: -0.9991
2. Between 1.5 and 1.6 $\left(\approx \frac{\pi}{2}\right)$; between 4.7 and 4.8 $\left(\approx \frac{3\pi}{2}\right)$; and between 7.8 and 7.9 $\left(\approx \frac{5\pi}{2}\right)$
3. $f'(x) = \cos(x)$
4. 
5. The agreement is generally about six decimal places.
6. $f'(x) = {}^{-}\sin(x)$
7. Notice that the symmetric difference quotient appears to be near 0 when $x = 0$ and that it decreases until x is between 1.5 and 1.6 $\left(\approx \frac{\pi}{2}\right)$. The symmetric difference quotient also changes sign when x is between 3.1 and 3.2 $(\approx \pi)$. This is similar to the behavior of $^{-}\sin(x)$.
8. 
9. The agreement is generally about six decimal places.
10. $\frac{1}{x}$
11. The symmetric difference quotient appears to always be nearly the reciprocal of the X column.

12. 

13. The agreement is generally about six decimal places.

14. e^x

15. The **Y1** column and the **Y0** column appear to be about equal. In other words, the values of *f* closely match the values of the symmetric difference quotient.

16. Yes; the graphs of *f* and the symmetric difference quotient are practically indistinguishable.

17. The agreement is generally about five decimal places.

18. Answers will vary. Students should mention that they used the symmetric difference quotient at many points to develop formulas for the derivatives of sine, cosine, natural log, and natural exponential functions. They should state these formulas.

Extension Solution

The command **nDeriv** appears to compute the symmetric difference quotient with $h = 0.001$.