

## Growth - A Recursive View

Contributed by Allan Bellman

Name

Date

The approximate population of the city of Denver, Colorado in 1994 was 494000. If the city's population grows at a 1.3% annual rate, what should the city's population have been in 1998? For the 2000 census? In 2010?

The population in 1995 would equal the population in 1994 + 1.3% of the population in 1994.

Notationally we could write this expression:

$$\text{pop}_{1995} = \text{pop}_{1994} + .013\text{pop}_{1994}$$

Using a MathBox, the population in 1995 should have been approximately:

$$494000 + .013 \cdot 494000 = 500422.$$

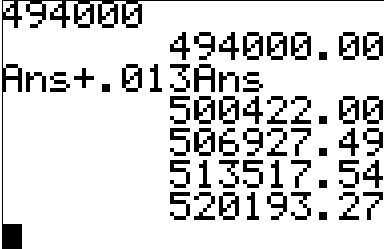
Using the same method the population in 1996 would be:

$$500422 + .013 \cdot 500422 = 506927.$$

You could continue in this manner, but there must be an easier way! It sure would be nice to be able to recall the result of the previous operation without retyping it every time.

On the calculator you can recall the last result by entering [2nd][ANS]. These keystrokes produce a variable ANS that contains the result of the previous operation. This method allows the use of a recursive formula on the home screen of the calculator.

The picture shows the population growth up to 1998 using this "home screen recursion" method.



A calculator screen showing a sequence of recursive calculations for population growth. The initial value is 494000. The first calculation is 494000 + .013 \* 494000, resulting in 500422.00. The second calculation is 500422 + .013 \* 500422, resulting in 506927.49. The third calculation is 506927.49 + .013 \* 506927.49, resulting in 513517.54. The fourth calculation is 513517.54 + .013 \* 513517.54, resulting in 520193.27. The screen shows the use of the ANS key to recall previous results.

You can do the same type of operation with TI-Interactive, but it is not quite as simple. You would need to cut and paste the formula in each new MathBox. With Interactive the problem could be solved as follows.

$$494000 \quad 494000$$

$$\text{ans} + .013 \cdot \text{ans} \quad 500422.$$

$$\text{ans} + .013 \cdot \text{ans} \quad 506927.$$

ans + .013-ans      513518.  
ans + .013-ans      520193.

The formula was entered the first time, highlighted and copied. Each time enter was pressed to perform the operation, a new MathBox appears, and the formula is "pasted" in? We get the same answer for 1998.

If you want to predict the population for another city with a population of 750000 in 1994 that also grew at a 1.3% rate, you could click on the first line and change the 494000 to 750000. All calculators below would be updated.

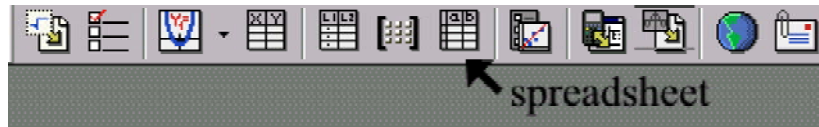
Try it yourself. Did you get 789767 as your answer?

You still do not have the answer for 2000 or 2010. This method would work but there is a better way.

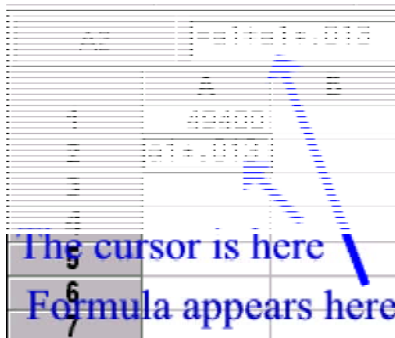
### The Spreadsheet

The steps listed below were used to create a spreadsheet to solve these problems. The spreadsheet is on the next page. Look at the steps and then use the already created spreadsheet to answer some questions.

Activate a spreadsheet by clicking the spreadsheet icon on the tool bar.



Go to the top row and enter 494000. When you press enter, the cursor will go down one row. Enter the formula =A1+A1\*.013.



To copy the formula into the cells below these two, click on cell A2. Click on the copy point and while holding the left mouse button down, drag the formula down. The formula will be copied into any cell that is highlighted.

	A
1	49400
2	50042.2
3	
4	
5	
6	
7	
8	

Click and drag from here.

Now add the year to column B. Go to B1 and enter 1994. In B2 enter =B1+1. Copy this formula down.

494000	1994
500422	1995
506927	1996
513518	1997
520193	1998
526956	1999
533806	2000
540746	2001
547775	2002
554896	2003
562110	2004
569418	2005
576820	2006
584319	2007
591915	2008
599610	2009
607405	2010

You can now use this spreadsheet to answer the three questions from above. The population in 1998 would have been 520193.

For the 2000 census the population should have been approximately 533806.

We would estimate the population in 2010 to be 607405.

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## Use the Spreadsheet

1. If a suburban city had a population of 800000 in 1994 and grew at a 1.3% annual rate, what would you estimate it's population to be in 2000? You can use the same spreadsheet.
2. When would the city in #1 first reach a population of 1000000?

## Create Your Own

3. Create a spreadsheet that shows the yearly value if \$1000 was invested at a 5% annual rate. Let your table show 10 years.
4. After how many years will the \$1000 double?
5. Assume a car depreciates at a 15% yearly rate. Create a spreadsheet to show the value of a \$20000 car every year until it's value is \$10000?

### A New Spreadsheet Idea

Original Car Value =	20000	Year	Value
		2000	20000.00
Depreciation Rate =	0.12	2001	17600.00
		2002	15488.00
Year the car was new =	2000	2003	13629.44
		2004	11993.91
		2005	10554.64
		2006	9288.08
		2007	8173.51
		2008	7192.69
		2009	6329.57
		2010	5570.02
		2011	4901.62
		2012	4313.42

6. Double click on the spreadsheet above. Once the spreadsheet is opened, click on G3 and record the formula entered in that cell. Scroll down the G column and record one or two more cell formulas. What is the purpose of the term  $\$D\$3$ ?