Driver Assisted Parking

Teacher Notes & Answers

7 8 9 10 **11** **12**

TI-Nspire™ Investigation Student 120 min

Introduction

Five years ago autonomous driving was possible, it now appears inevitable. Companies like Texas Instruments, Tesla and Google are all contributing to this landscape that is expected to contribute more than $10 trillion to the global economy. The horseless carriage is becoming the driverless car.

Teacher Notes:

> Show students the video referenced here as an introduction. Students may also choose to scan the barcode to view the video. Other videos that provide more detailed information include:
>
> Fusion combines Radar & Camera: https://www.youtube.com/watch?v=sPQGtIXaXqA
>
> Challenges facing vehicle automation: https://www.youtube.com/watch?v=KhnlOD4Euyg

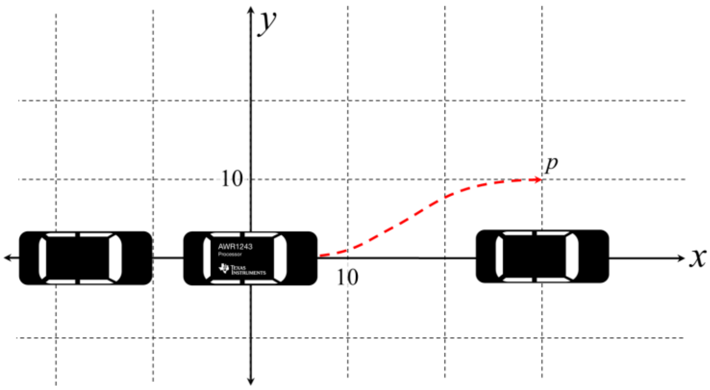
Autonomous vehicles contain an enormous array of technology. Check out some of the features that are already possible: http://bit.ly/DrivingFutureTI

The apparent intelligence of these vehicles all comes down to numbers, and lots of them. Data is collected by a series of sensors, the data is interpreted and a multitude of mathematical algorithms are applied so decisions can be made. In this investigation you will create some basic equations to enable a vehicle (Rover) to safely enter and exit a parking space using information collected from just two sensors.

Simulation – Exiting a parking space

Before a solution is tested on an actual vehicle, it is simulated many times over. The simulations generally start with a simplified version of the final problem; this is generally referred to as 'proof of concept'.

To begin we start by imagining the car as a point located at the origin. The path of the car is indicated by the dotted red line that terminates at point P (30, 10).



A number of curves could be used to generate a path similar to the one shown.

- Trigonometric (sine)
- Polynomial (cubic)
- Piecewise (Quadratic/Quadratic)

Author: P. Fox

Question: 1

Suppose the curve is modelled by the function: $f(x) = a\sin(b(x-h)) + k$

a) Determine appropriate values for a, b, h and k.

The parameters can be determined from the conditions that include the origin: (0, 0) and P: (30, 100).

$\therefore$ a = 5, k = 5, $b = \dfrac{\pi}{30}$ and h = 15. Variations for h of course include: $15 \pm 60n \quad n \in J$.

Overall equation: $f(x) = 5\sin\left(\dfrac{\pi}{30}(x-15)\right) + 5$

b) State the domain restrictions for the function so that it only moves along the path shown.

Domain: $[0, 30]$

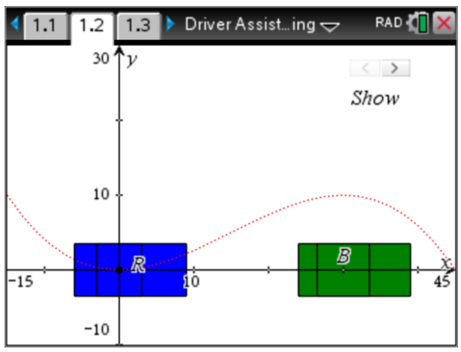
Students can apply these domain restrictions either on the original function definition or in the Graph application which means the restrictions apply only to the graphical representation.

Open the TI-Nspire file "Driver Assisted Parking". This document contains several interactive diagrams that will help explore the problems presented in this investigation in addition to a series of programs that will drive the TI-Innovator Rover. Rover will be used as a test vehicle to help evaluate the success of your calculations including the use of Rover's park assist sensors.

Page 1.1 of this document consists of a Notes application where the function $f(x)$ has been defined. Change this definition then navigate to page 1.2 to see your curve (vehicle path) plotted in the scenario provided.

Here you will notice that the car is no longer a point source. The diagram allows us to visualise some of the problems associated with the real situation.

Grab point R to move it along the vehicle path (function). The exiting vehicle (R) should clear the parked car (B)?



The exiting vehicle now passes through the point (30, 8).

c) Determine the new values for a, b, h and k.

Changes to point P, y coordinate only effect only a and k.

$\therefore$ a = 4, k = 4, $b = \dfrac{\pi}{30}$ and h = 15. Variations for h of course include: $15 \pm 60n \quad n \in J$.

Overall equation: $f(x) = 5\sin\left(\dfrac{\pi}{30}(x-15)\right) + 5$

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d) The closest point on the parked car (B) to the exiting vehicle (R) has the coordinates: Q: (24, 3.5). Define a function $d(x)$ that represents the distance between the centre of car R and the point Q.

$$d(x)=\sqrt{(x-24)^2+(f(x)-3.5)^2}$$

This expression demonstrates sufficient understanding of how the general distance is computed and effectively creates a function in terms of a single variable, x. Students may however elect to express the function as follows:

$$d(x)=\sqrt{(x-24)^2+\left(5\sin\left(\frac{\pi}{30}(x-15)\right)+1.5\right)^2}$$

Whilst a small simplification is included, the result is an equation that is slightly more visually complicated; expanding produces a significantly more visually complicated expression that does not provide any cognitive benefits.

e) Determine the closest point R comes to point Q and therefore the maximum possible width of vehicle R.

Students can use the definition (above) for the distance and their calculator to solve: $\frac{d\left(d(x)\right)}{dx}=0$.

Result: x=22.11806

The closest distance from the centre of R is therefore: 5.2409 units. As this measurement is from the centre of the vehicle, the maximum vehicle width would be: 2 × 5.2409 = 10.4818 units.

Question: 2

The trigonometric path for vehicle R is replaced with a polynomial: $f(x)=ax^3+bx^2+cx+d$. The vehicle starts at the origin and passes through the original coordinates of point P: (30, 10).

a) Explain why *d* must equal 0.

As the function passes through the origin: $f(0)=0 \therefore d=0$

b) Assume vehicle R is parallel to the curb at the start and finish of its journey as it passes through P: (30, 10). Determine an appropriate function for: $f'(x)$ and the subsequent polynomial: $f(x)$.

$f'(x)=a_1x(x-30)$ therefore $f(x)=\frac{a_1x^2(x-45)}{3}$ since $f(30)=10$ then $a_1=\frac{-1}{450}$

$$f(x)=\frac{x^2(45-x)}{1350}$$

Redefine $f(x)$ in the Notes application on Page 1.1 of the TI-Nspire document and check to see if vehicle R successfully exits the parking space by dragging vehicle R along the path of $f(x)$.

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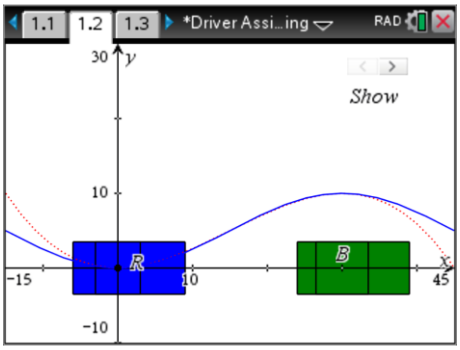
c) Graph the original trigonometric solution and the polynomial on the same set of axis and comment on the result.

The two graphs are remarkably similar over the required domain.



Teacher Notes:

Students can graph $f_2(x) - f_1(x)$ and locate the maximum value, largest difference. This occurs when $x \approx 8.357$ with $f_2(x) - f_1(x) \approx 0.100$. Students should therefore conclude that both of these functions provide equally applicable solutions for the path of the vehicle.

The similarity between these two curves can be re-visited after students have completed this investigation. The idea that polynomials of a higher degree can provide an acceptable model for trigonometric functions leads to the Taylor Polynomial series.

Rover's first test drive

Question: 3

The polynomial solution needs to be generalised in preparation for vehicle testing. Let point P = (m, n).

a) Determine the general equation for the polynomial where the vehicle's path starts at the origin (parallel to the curb) and completes the exit at point P (parallel to the parked vehicle).

Applying the same logic as the specific case yields:

$f'(x) = a_1 x(x-m)$ therefore $f(x) = \dfrac{a_1 x^2(2x-3m)}{6}$ since $f(m) = n$ then $a_1 = \dfrac{-6n}{m^3}$

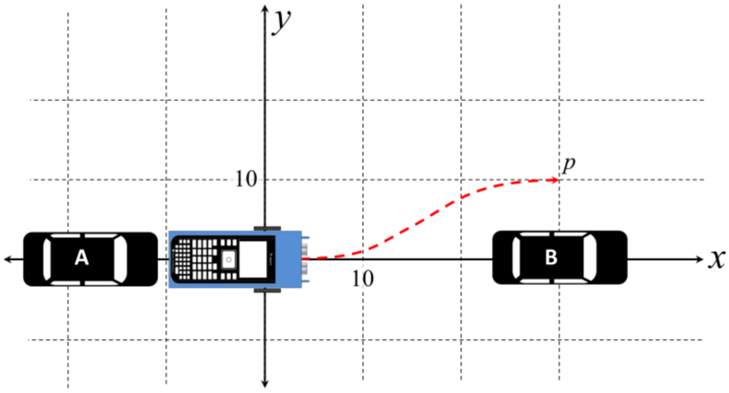
$$f(x) = \frac{-nx^2(2x-3m)}{m^3}$$

A program has been included in the Driver Assisted Parking file that will drive the TI-Innovator Rover along a path according to the defined function: $f(x)$. There a several things you need to do to ensure the first test drive is successful.

Warning

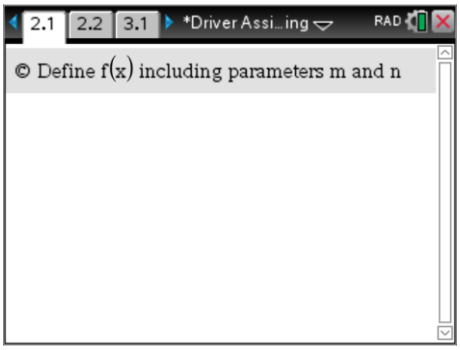
For this section of the investigation the ultrasonic motion sensor of the vehicle is not turned off.

Be ready to move the obstacle to help Rover in the event of an impending collision!



- ❖ Rover needs a smooth[1], flat level surface upon which to drive.
- ❖ If you are using paper to record Rover's path, make sure the paper is taped down.
- ❖ The origin should be located half way between Rover's wheels; this is the point upon which Rover pivots.
- ❖ Place vehicle B approximately 30cm in front of rover, the exact location is not essential but must be measured accurately.
- ❖ Determine a reasonable location for your point P, record the coordinates of this point as the program will prompt you for these values.
- ❖ All measurements are in centimetres.

Navigate to Page 2.1

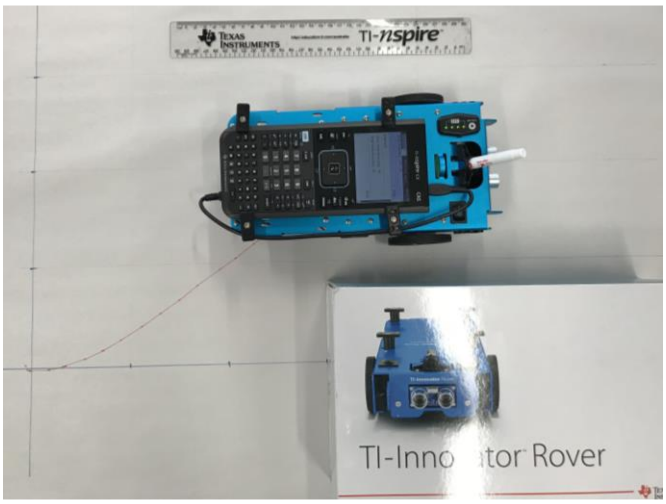


Define your general equation as: $f(x)$. The parking program uses this function to define the path of Rover. Make sure your definition includes the parameters m and n.

Make sure Rover is in position!

Run the "TestDrive" program from the VAR menu and enter the coordinates of point P when prompted.

b) Record the path that Rover travels as it exits the parking space. Discuss any additional considerations that need to be accounted for to ensure Rover safely exits the parking space.



Shown opposite is a sample graph produced by Rover exiting the parking space using the defined function and the "testdrive" program.

Vehicle B (Rover box) was set up 30cm from the origin. Rover was set up so the midpoint of the two wheels was located at the origin.

The point P selected for this test drive was located at P (40, 20).

The path followed by Rover suggests a reasonable clearance, however it was very close! Rover is not a single point; self-awareness for any vehicle must include consideration of all points on the vehicle's surface.

[1] Smooth – Polished concrete, linoleum, timber floor boards or melamine work best.

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c) Notice that Rover exits on the 'wrong' side for a right-hand drive vehicle. Determine the equation to the function so that Rover exits the parking space on the most common side for a right-hand drive vehicle.

There are two simple ways to make Rover exit the parking space on the 'opposite' side, the side applicable to a right-hand drive vehicle:

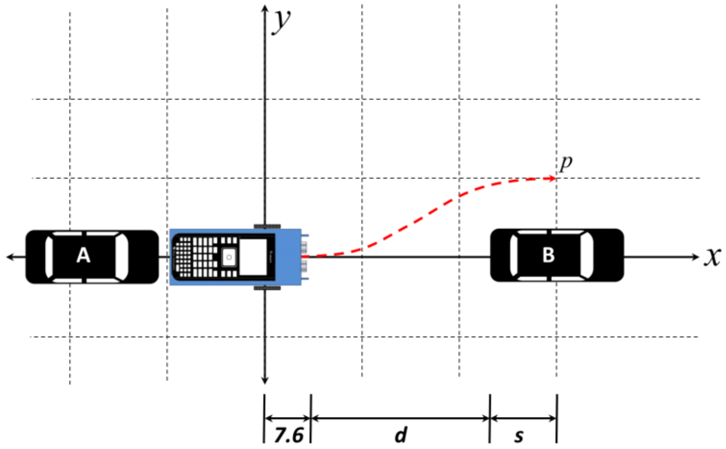
i) Set the coordinates of point P (m, n) such that m < 0

ii) Using the original coordinates in quadrant 1, change $f(x)$ to $-f(x)$

Rover becomes Autonomous

In this phase of the investigation, a vehicle (B) will be placed an unknown distance in front of Rover. The ultrasonic sensor on the front of Rover will measure the distance to this vehicle and store the result in *d*.

Your new equation for Rover's path must include *d* as one of its parameters. You will **not** be prompted for the location of point P so you must think about how much clearance you would like to leave between Rover and vehicle B and how far along this vehicle Rover will be when it comes to a stop.

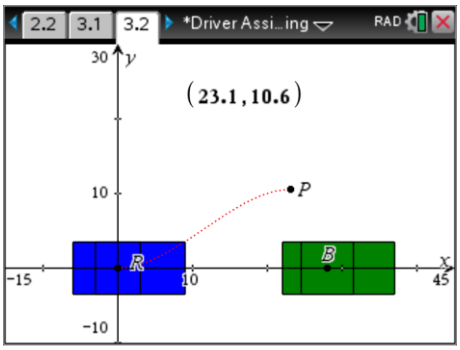


Navigate to page 3.2.

A function has been defined that passes through a general point. The function is updated every time point P is moved.

Vehicle (B) can also be dragged along the x axis.

Note: A slight delay will be experienced when point P is moved as the equation is recalculated.



Explore various locations for Point P and vehicle B. Drag Rover along the curve to help get a sense of the best target location for point P with regards to clearance and distance along the side of vehicle B.

Calculator Tip

The Geometry > Construction menu in the Graph application includes an option to construct a locus. Selecting the 'near side' of vehicle R followed by point R will produce a series of lines that represent the progressive location of the side of vehicle R as it moves along the curve. The locus is also dynamic and therefore update whenever the point P is moved.

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Question: 4

Use your explorations to determine an appropriate value for S (refer previous page). Explain your considerations with regards to the overall position of point P and Rover's ability to exit the parking space safely based on the selected function.

If the point P is aligned with the rear of vehicle B the probability of a collision between Rover and vehicle B is virtually eliminated. The problem with this location is that it reduces the likelihood of a real autonomous vehicle being able to exit the parking space due to a lack of turning space. Giving vehicle B a wider berth (increasing the value of n) also works, but in a practical situation Rover would likely be on the opposite side of the road.

Student answers will vary for the value of s. Values greater than 10 are likely to lead to a collision if vehicle B is parked relatively close to Rover.

Rover is about to enter its autonomous phase! When the program runs it will use the function you store in $f(x)$, measure the distance to the vehicle in front using its ultrasonic ranger and apply this distance (d) to the function. When you run the DriveSense program you will not be prompted for anything, Rover will drive itself!

Question: 5

Determine a general function $f(x)$ that includes the parameter *d*. This equation must be defined on Page 3.1 of the Driver Assisted Parking document.

The simplest derivation for this new function is to substitute d + 7.6 + s for m.

$$f(x)=\frac{-nx^2(2x-3m)}{m^3} \quad \text{General form:} \quad f(x)=\frac{-nx^2(2x-3(7.6+d+s)}{(7.6+d+s)^3}$$

Example: s = 7.4cm and n = 20

$$f(x)=\frac{-20x^2(2x-3(d+15)}{(d+15)^3}$$

Ask your teacher to put vehicle B in place. Draw a diagram of the set up. The distance between Rover and vehicle B will be recorded and accessible after the program has finished and Rover has exited the parking space.

Teacher Notes:

Place vehicle B (box) at least 15cm from the front of Rover and not more than 30cm. If Vehicle B is placed closer than 15cm then Rover will most likely clip the box as it passes by, depending on the value of n selected by the student(s). Further than 30cm can increase the error margins associated with Rover's movement. The aim is for students to achieve a successful car park exit providing they have correctly completed their calculations. When the distance is between 15 and 20cm away with $n \approx 20$, Rover will pass very close to vehicle B.
Students may record Rover's path using a video (phone) to provide evidence that their exit was successful.

Question: 6

Record the distance *d* measured by Rover and the updated function definition. Discuss how successful Rover was at exiting the parking space.
Answers will vary depending on the actual distance to the box, similarly with the function definition.
Students should comment on how close Rover gets to the vehicle in front (depending on their selection of n). A sharper series of turns at the start of the exit would improve Rover's ability to exit the parking space.

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Improving the Model

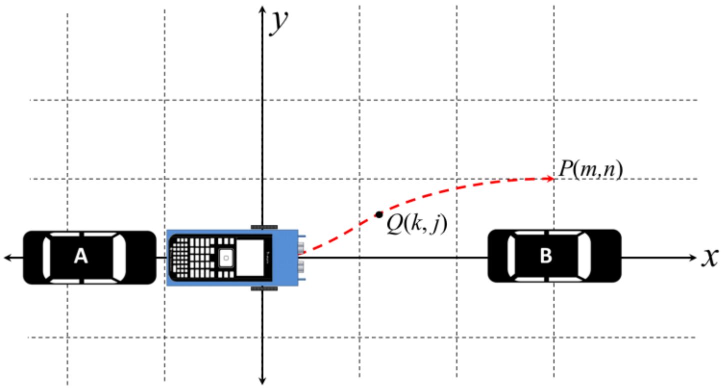
Both the trigonometric function and the cubic function have rotational symmetry about the point of inflexion which means the drive path is somewhat restricted. The new drive path $f(x)$ will be defined as a piecewise function consisting of two quadratic functions: $g(x)$ and $h(x)$. The new path must be continuous and differentiable to ensure a smooth ride.

$$f(x)=\begin{cases} g(x) & x<k \\ h(x) & x\geq k \end{cases}$$

The point $Q(k, j)$ is located along the path from the origin to point $P(m, n)$, $g(x)=ax^2$ and $h(x)=b(x-m)^2+n$.

Question: 7

Use the following questions to help determine the general equation for $f(x)$



a) Determine an expression for j in terms of a and k.

$j=ak^2$ - Using point Q and g(x)

b) Determine an expression for a in terms of b, k, m and n.

$a=\dfrac{b(k-m)^2+n}{k^2}$ - Using point Q and $h(x)=ak^2$

c) Determine an expression for the gradient at point Q using both $g(x)$ and $h(x)$ and hence determine an expression for b in terms of k, m and n.

Gradient at Q needs to be the same for g(x) and h(x) so that the path is 'smooth' (differentiable).

$$2ak=2b(k-m)$$
$$a=\frac{b(k-m)}{k}$$

Equating this expression with the previous one:

$$\frac{b(k-m)}{k}=\frac{b(k-m)^2+n}{k^2}$$
$$bk(k-m)=b(k-m)^2+n$$
$$bk(k-m)-b(k-m)^2=n$$
$$b(k-m)(m)=n$$
$$b=\frac{n}{(k-m)m}$$

Calculator Tip

Define your piecewise function on page 4.1. To test your function as a simulation, copy the function definition into page 5.1. Page 5.2 contains a graph of your function including interactive values for k and point P.

The reason why it is necessary to copy and paste the function definition is that k, m and n are defined in problem 5 and will therefore appear as specific values rather than parameters.

Question: 8

The aim of the new function is to enable Rover to exit tighter parking spaces, explain how this piecewise function might achieve this outcome.

The curvature of the first quadratic can be made sharper and the second less sharp allowing the front of the vehicle to enter the turn in less distance whilst still producing a 'smooth' (differentiable) curve. The curve is no longer symmetrical about the 'point of inflexion'.

Students may use a graph of the derivative function to illustrate the sharper initial curve.

Question: 9

Experiment with a range of values for *k* before putting Rover back on the test track. When you are ready, set up Rover with car B located just 12cm in front[2]. Determine corresponding values for k, m and n, make sure $f(x)$ is defined on Page 4.1, then run the TightSqueeze program. You will be prompted for values of k, m and n. (All measurements in cm)

Answers will vary depending on student's values for m, n and k. Students can either use the pen feature of Rover to record the path or a video. Students should note however that for particularly sharp curves (g(x)) that Rover's rear end goes well into the 'fourth quadrant' or from a practical perspective, the vehicle would hit the gutter if such a tight turn were possible. From a practical perspective however it does raise the issue with regards to 'four wheel steering' that has essentially been made possible thanks to technology.

Include a copy of the path that Rover followed; your final function and the corresponding values for k, m and n. Comment on the practical situation for an autonomous vehicle.

[2] Distance measured from front of Rover to object, note that this is approximately 20cm to the centre of Rover's wheels (Origin).

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