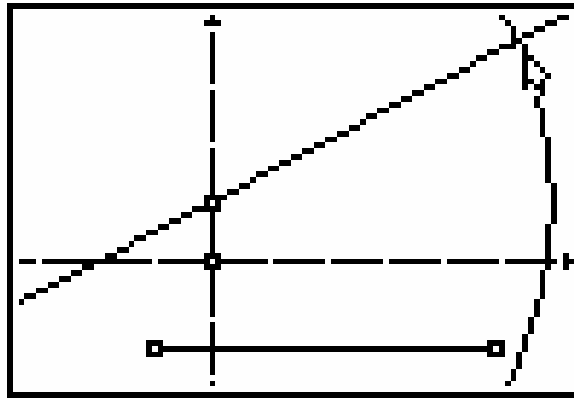


Lines in the Plane

Cabri® Jr. Activity

Exploration

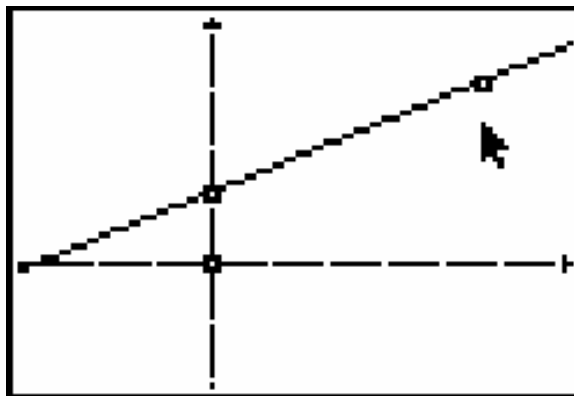
- Open a new *Cabri® Jr.* file and select **Hide/Show (F5)**, select **Axes** from the submenu and press **ENTER** to show the coordinate axes, if necessary.
- Draw a horizontal **Segment (F2)** in the lower portion of the screen.
- Construct a **Point on (F2)** the y-axis.
- Use the **Compass tool (F3)** to create a circle centered at the point on the y-axis with radius of the segment drawn.
- Enlarge the circle so that the point in the first quadrant is in the upper right corner of the screen by dragging the right end of the segment.
- Construct a **Point on (F5)** the circle in the first quadrant.
- Draw a **Line (F2)** using the center of the circle and the point on the circle in the first quadrant.



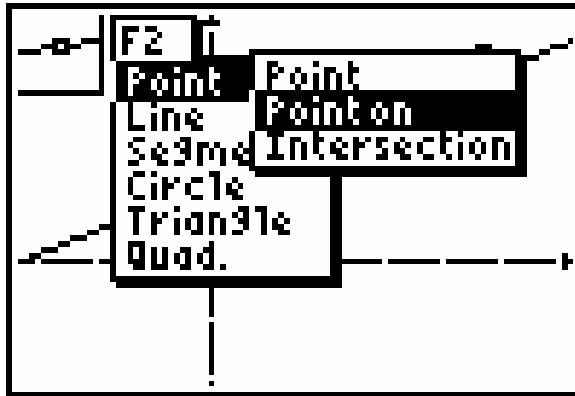
- **Hide (F5)** the circle and the segment.

Note: The construction of the line using the circle's center and a point on the circle will make the slope triangle, which will be created later, have a constant hypotenuse. The slope triangle will better display the change in the legs as the first-quadrant point that defines the line is moved.

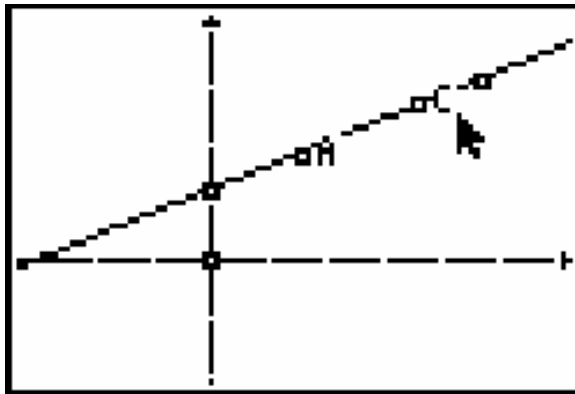
- Draw a **Line (F2)** using some point on the y-axis and another point in the upper right corner of quadrant I as two defining points.



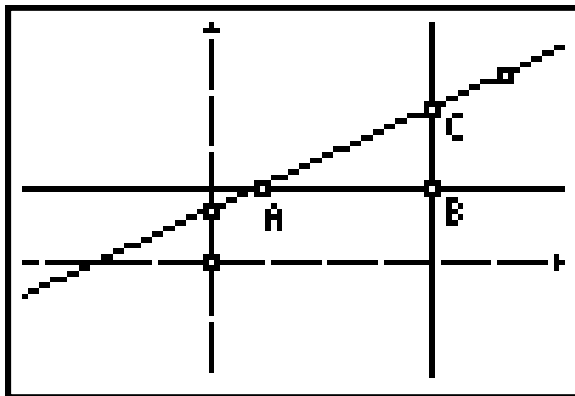
- Construct two points on the line using the **Point:Point on tool (F2)**.



- Label the two point **A** and **C**, as shown in the figure below. Do not use the points defining the line as either point **A** or **C**.

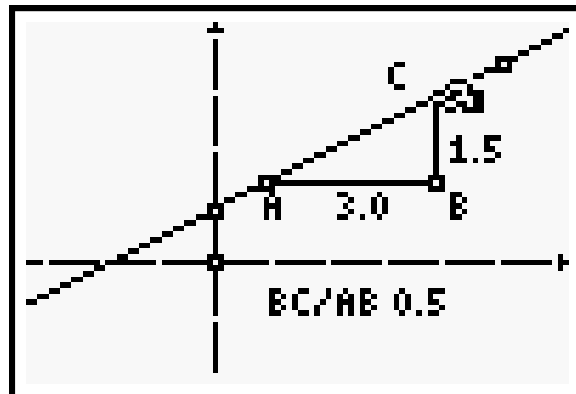


- Construct a line **Perpendicular (F3)** to the x -axis through **C** by selecting the x -axis first and then the point.
- Construct a second line **Perpendicular (F3)** to the y -axis through **A**.
- Construct the point of intersection of the two perpendicular lines just drawn by using **Point:Intersection** tool (**F2**).
- Label the point of intersection of the two perpendicular lines **B**.



- Draw **Segments (F2)** **AB** and **BC** and hide the perpendicular lines.
- Use the **Measure:D. & Length** tool (**F5**) to find the length of segments **AB** and **BC**.

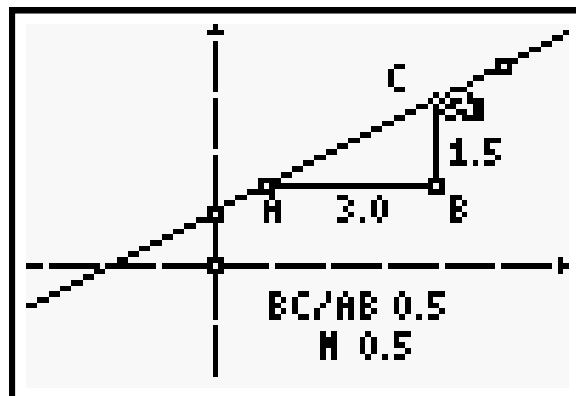
- Use the **Calculate** tool (F5) to compute the value of the ratio $\frac{BC}{AB}$.
- Drag point C (or A) along the line to change the lengths of the legs of the right triangle $\triangle ABC$.



Is there any variation in the value of the ratio? Explain.

- Drag the second point defining the line (upper right portion of the screen) around the screen.

Explain what happens to the ratio $\frac{BC}{AB}$ as you drag the line so it rotates around the point on the y-axis (the y-intercept).



The ratio $\frac{BC}{AB}$ is the ratio of $\frac{\text{Change in } y \text{ coordinates}}{\text{Change in } x \text{ coordinates}}$ and is called “the slope of the line.”

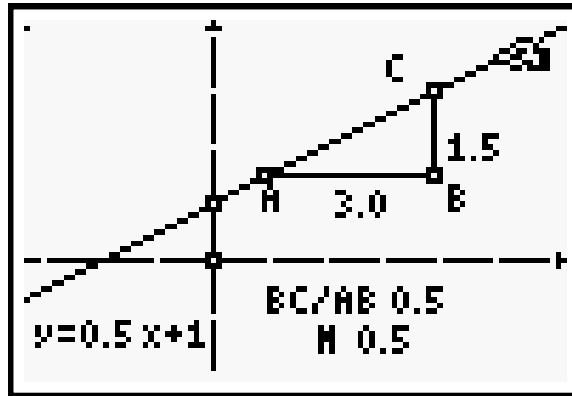
- Use the **Measure:Slope** (F5) to display the slope of the line.

Explain how the slope of the line compares to the ratio $\frac{BC}{AB}$ as the line rotates around the y-intercept. Are these two values always equal? Explain.

- Drag points A and C along the line.

Explain how the slope is affected and why.

- Use the **Coord. & Eq.** tool (F5) to display the equation of line AC.



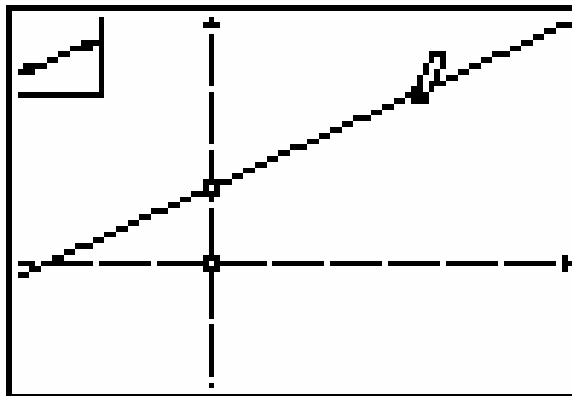
Explain how this compares to the ratio $\frac{BC}{AB}$ and the slope as you rotate the line around its y-intercept.

- Display the coordinates of the y-intercept of the line using the **Alph-Num** tool (F5).
- Drag the y-intercept up and down the y-axis.

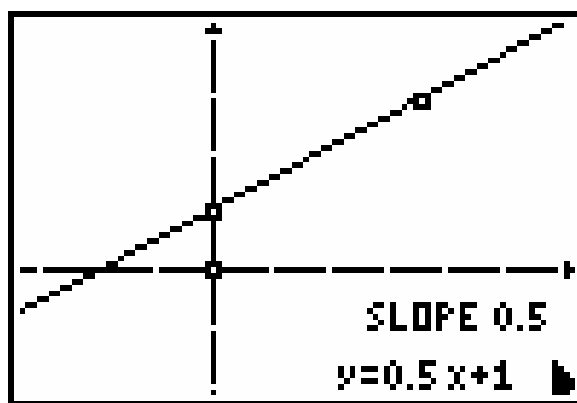
How does dragging the y-intercept change the equation? Explain the relationship between the equation of the line, the coordinates of the y-intercept, and the slope of the line as the line is rotated and translated.

Changing the Slope

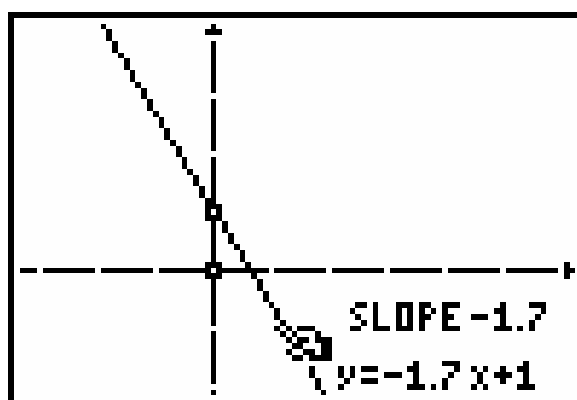
- On a **New** (F1) screen with the coordinate axes showing, construct a line through the origin extending through the first quadrant using the **Line** tool (F2).



- Display the slope and the equation of this line using **Measure:Slope** (F5) and **Coord. & Eq.** (F5).



- Rotate the line by dragging the point in quadrant I and observe the value of the slope in various positions.



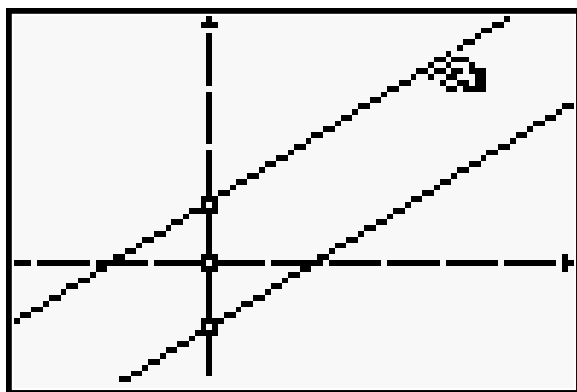
Sometimes the value of the slope is positive and sometimes it is negative. Sometimes the slope has a large absolute value and sometimes it has a small absolute value. Sometimes the slope is zero, sometimes it is one, and sometimes it is infinitely large.

Determine a way to organize the different slope values so you can quickly estimate the slope of any line.

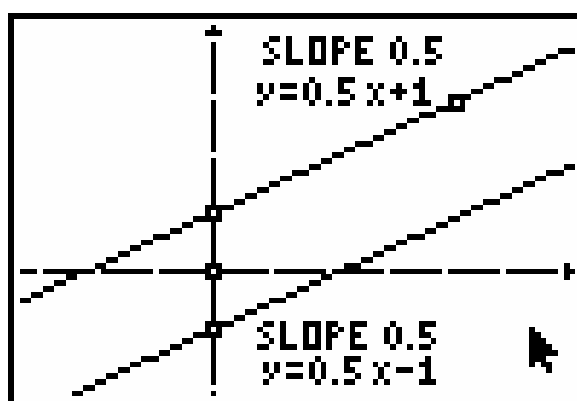
Parallel and Perpendicular Lines

Explore the slopes of parallel and perpendicular lines.

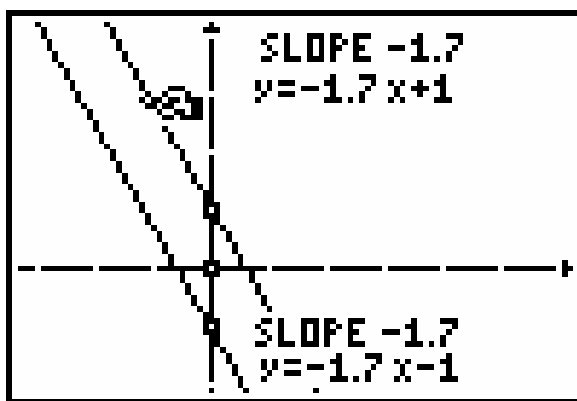
- Open a **New (F1)** screen and show the coordinate axes.
- Draw a line with a y-intercept extending through the first quadrant.
- Use the **Parallel tool (F3)** to construct another line parallel to the first line with its defining point on the y-axis. [Note: when using the **Parallel** tool, select the line first and then place the defining point on the y-axis.]



- Use the **Measure:Slope** tool (F5) to compute slopes and the **Coord. & Eq.** tool (F5) to compute the equations of both lines.



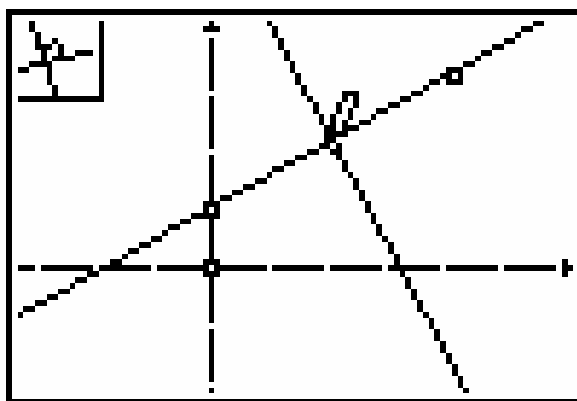
- Drag the original line by selecting it but not selecting either of the line's defining points.



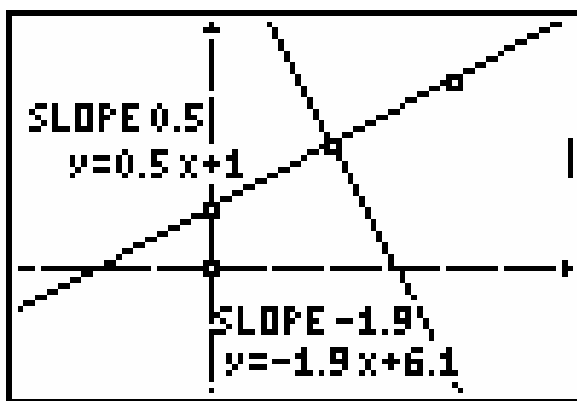
Make a conjecture about the slopes of parallel lines.

Perpendicular Lines & Slopes

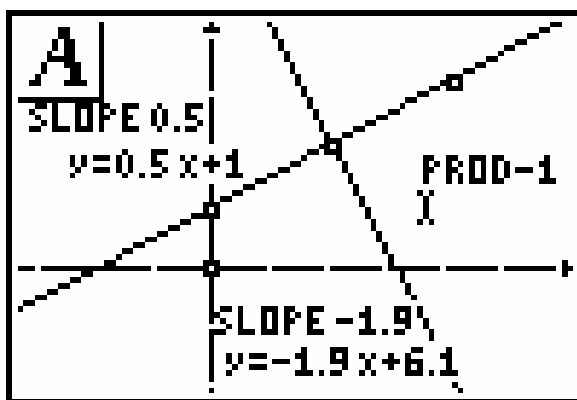
- Open a **New (F1)** screen with the coordinate axes showing.
- Draw a **Line (F2)** with a y-intercept extending through the first quadrant.
- Use the **Perp. tool (F3)** to construct a perpendicular line at any point on the original line.



- Use the **Measure:Slope** tool (F5) to compute the slope and the **Coord. & Eq.** tool to compute the equation of both lines.



- Use the **Calculate** tool (F5) to compute the product of the slopes of the two lines.

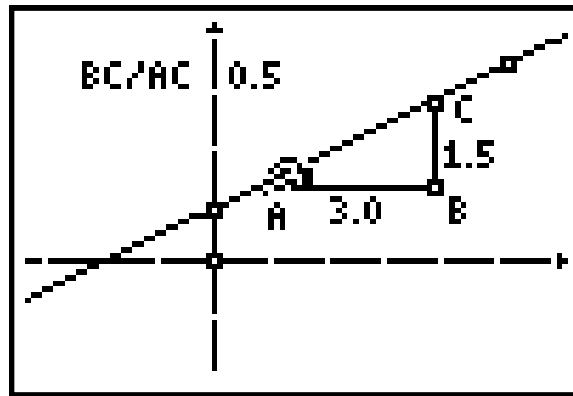


- Drag the original line around the screen by using the point in quadrant I.

Make a conjecture about the slopes of perpendicular lines.

Teaching Notes

1. This construction creates a "slope triangle" attached to a line showing the relationship $\frac{\Delta y}{\Delta x}$, the definition of slope of a line. As points **A** and **C** are dragged along the line, the ratio $\frac{BC}{AB}$ remains unchanged.



Measuring angles of $\triangle ABC$ may help convince students that no matter where points **A** and **C** are located, all versions of the triangle are similar to each other. Because similar triangles have proportional sides, the ratio $\frac{BC}{AB}$ will stay constant as long as the line is not rotated.

As the line is rotated around the y -intercept, the angles of the triangle change producing non-similar triangles. The ratio $\frac{BC}{AB}$ changes, becoming larger in absolute value as the line approaches a vertical position and smaller in absolute value as it approaches a horizontal position.

The absolute value of the ratio $\frac{BC}{AB}$ and slope are equal. The ratio $\frac{BC}{AB}$ gives only positive values for slope because both quantities represent distances and distance is always positive. The **Slope** tool computes slope with the correct sign for all lines.

Like the ratio $\frac{BC}{AB}$, the value of slope remains unchanged as **A** and **C** are dragged along the line. The slope is constant at all points on a fixed line.

2. One function version of the equation of a line is $y = mx + b$, where m equals the slope of the line and b is the y -intercept, $(0, b)$. As the line is rotated around y -intercept, the slope of the line is equal to the coefficient of x in the equation. When the line is neither horizontal nor vertical, the ratio $\frac{BC}{AB}$ is the same as the absolute value of the coefficient of x .

The ratio and the coefficient are equal when the line passes through the first and third quadrants with a positive slope. The ratio and the slope differ by their sign when the line passes through the second and fourth quadrants with a negative slope.

Dragging the y -intercept changes the value of b in the equation.

This exploration forms the basis for understanding the slope-intercept form of a linear function, $y = mx + b$, and helps build the notion for the future study of the vertical shifting of the linear function from $y = mx$ to $y = mx + b$.

3. Construct the lines $y = x$ and $y = -x$ to organize different zones for estimating slope.

An effective way to construct these two lines is to use the **Angle Bis.** tool (F3) to bisect the first and second quadrants.] These lines create four distinct zones:

- a) When the line is between $y = x$ and the y -axis, $m > 1$ (Figure 1).
 b) When the line is between $y = -x$ and the y -axis, $m < -1$ (Figure 2).

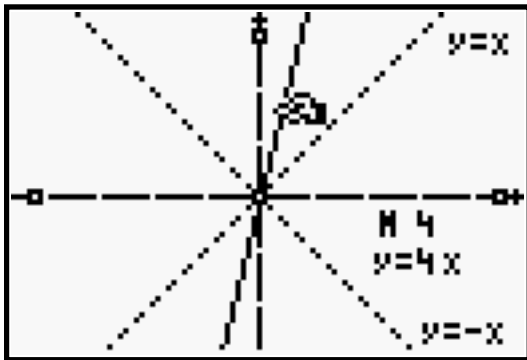


Figure 1

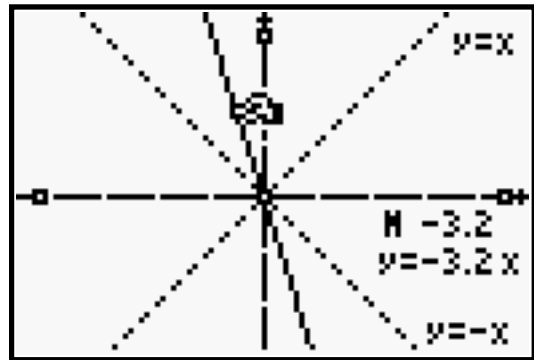


Figure 2

- c) When the line is between $y = x$ and the x -axis, $0 < m < 1$ (Figure 3).
 d) When the line is between $y = -x$ and the x -axis, $-1 < m < 0$ (Figure 4).

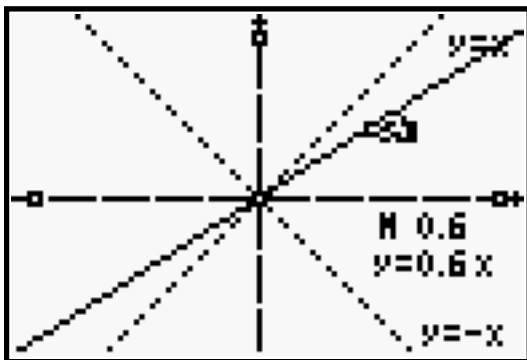


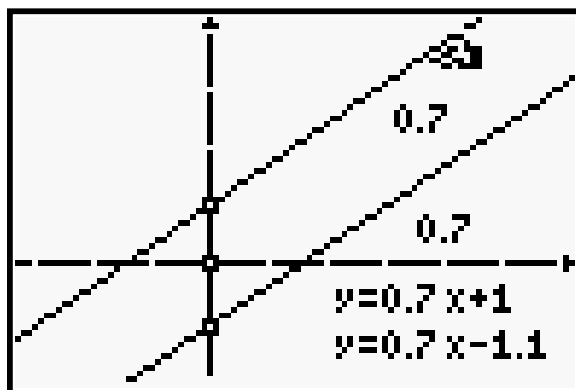
Figure 3



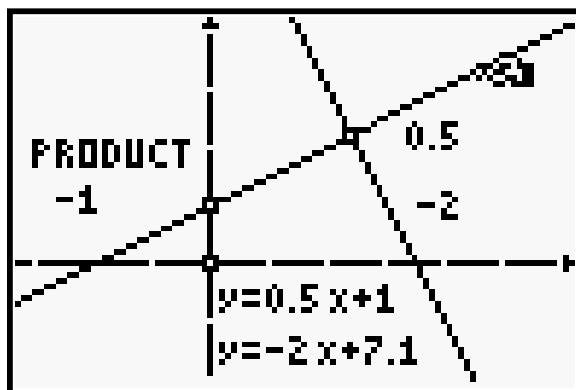
Figure 4

The line $y = x$ has slope of 1 and the line $y = -x$ has slope of -1 . When the line is on the x -axis, the slope is 0. When the line lies on the y -axis, the line has infinite or undefined slope.

4. By dragging the original lines, the students should easily discover that parallel lines have the same slope.



The relationship of the slope of perpendicular lines is more difficult to discover without a hint. Because perpendicular lines have negative reciprocal slopes, suggest that the students compute the product of the two slopes and then drag the original line on the screen .



Negative reciprocal slopes can be deduced from the fact that the product of the slopes always equals negative one.