Stacking Bricks

About the Lesson

This activity presents a real-world situation—stacking bricks in a pile—that can be modeled by a polynomial function. Students create a small table to show how the number of bricks relates to the number of rows, then calculate the first, second, and third differences of the data in order to determine what degree of polynomial model to use. Next, they use the graphing calculator's statistical calculation functions to perform the correct regression. Finally, they evaluate the model using a variety of methods: by graphing the model and the data together, by examining the value of R^2, and by discussing the model's applicability to the real-world situation. As a result, students will:

- Use finite differences to find the degree of a polynomial that will fit data.
- Use a polynomial function to model data.

Vocabulary

- degree
- regression
- coefficient of determination

Teacher Preparation and Notes

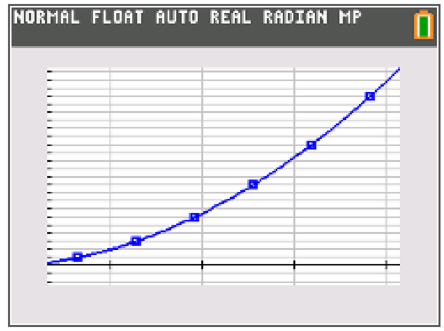
- This activity is designed to be used in an Algebra 2 or Pre-calculus classroom. This activity is intended to be mainly student-centered, and could be performed in pairs or small groups with some periods of teacher-led discussion.
- Prior to beginning this activity, students should have an introduction to basic polynomial (linear, quadratic, cubic, and quartic) functions, their graphs, and the concept of degree.
- Before beginning, clear out any data from the calculator's lists with the command ClrAllLists (found in the catalog).

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

*** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

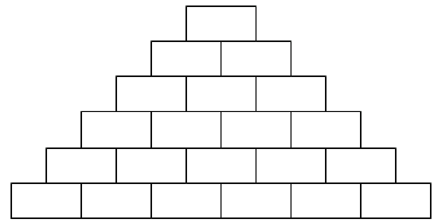
- Stacking_Bricks_Student.pdf
- Stacking_Bricks_Student.doc

Stacking Bricks

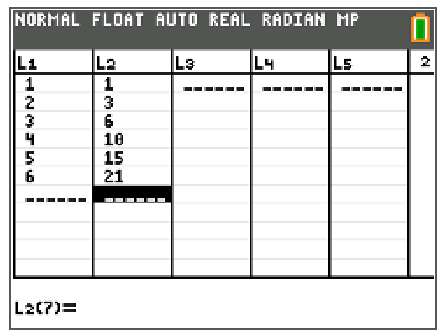
Tech Tip: If your students are using the TI-84 Plus CE have them turn on the GridLine by pressing [2nd] [zoom] to change the [format] settings. If your students are using TI-84 Plus, they could use GridDot.

Problem 1 – A Flat Triangular Stack

In Problem 1, students will find an appropriate model so that they can calculate the number of bricks in a stack given the number of rows in the stack.



Students start by determining the number of bricks in a stack with 1, 2, 3, 4, 5, and 6 rows. Then they will calculate the successive differences and record the values in the table on the worksheet. Students also need to enter the data for **L1** and **L2** in the calculator.



If students need help calculating the number of bricks in a stack, suggest that on the back of the worksheet they draw a picture a stack with 5 rows and then add another row for 6 rows. (A stack with 6 rows is shown above.)

Explain to students how to calculate successive differences if they don't already know. The differences between the values of the function (in this case the number of bricks) are called first differences. The differences between the first differences are called second differences, and so on.

Students should see that the second differences are the first time that the successive differences appear to be constant, so a second degree or quadratic model is a good choice for a model.

L1	L2	L3	L4	L5
rows	**bricks**	**1stDiff**	**2ndDiff**	**3rdDiff**
1	1	2	1	0
2	3	3	1	0
3	6	4	1	0
4	10	5	1	
5	15	6		
6	21			

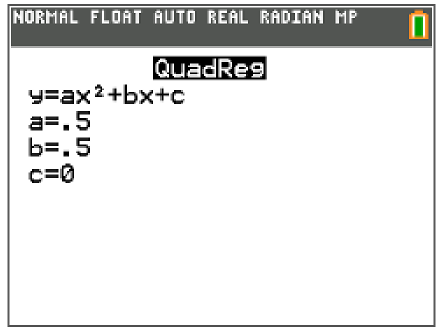
1. Which set of differences is constant?

Answer: second differences

2. What degree polynomial is the best fit for this data?

Answer: a second degree or quadratic polynomial

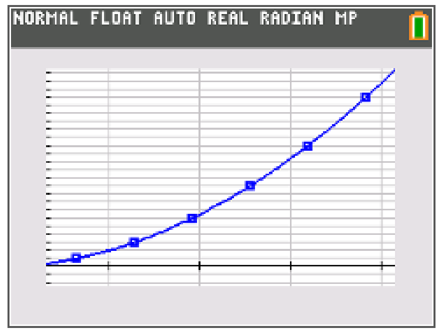
Now students should use the **QuadReg** command from the **Stat > Calc** menu. Explain to students that when they enter **L1** for **Xlist**, **L2** for **Ylist**, **Y1** for **Store RegEQ**, and select **Calculate**, this tells the calculator that the *x*-values they want to model are in **L1**, the *y*-values are in **L2**, and the equation for the model should be stored in **Y1**.



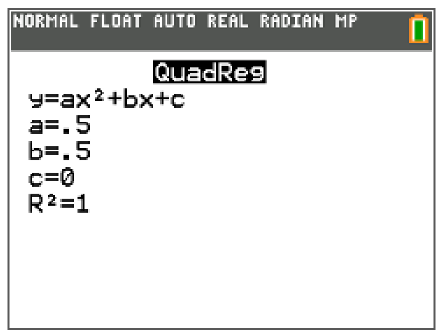
3. Record the equation of the model here:

<u>Answer:</u> $f(x) = 0.5x^2 + 0.5x$

Students will now compare the model to the data by creating a scatter plot. The graph screen should appear as shown at the right with the line passing through all of the points.

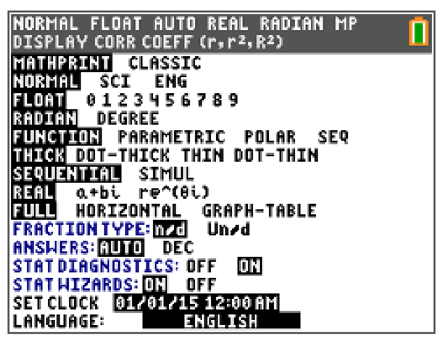


To do a second type of check, students look at the coefficient of determination, R^2. They first turn on the stat diagnostics from the catalog. Then they run the same regression again.



The Diagnostic feature can be turned on from the Mode screen. Students can press [mode] and press enter on **ON** next to **STATDIAGNOSTICS**.

Then press [2nd] [quit] to return to the Home screen.



4. What is the R^2 value for your model? What does that mean?

<u>Answers:</u> 1; The model fits the data perfectly.

5. If your model is correct, use it to calculate the number of bricks in a stack 50 rows high. (Remember that **Y_1(X)** is the number of bricks and **X** is the number of rows.)

Answer: 1275 bricks

6. Discuss the shortcomings of the model for this situation. For what numbers of rows is it valid? For what numbers of rows does it not make sense?

Answer: The model does not make sense for negative numbers of rows.

7. Write a domain for this model.

Answer: $\{x \mid x \geq 0\}$

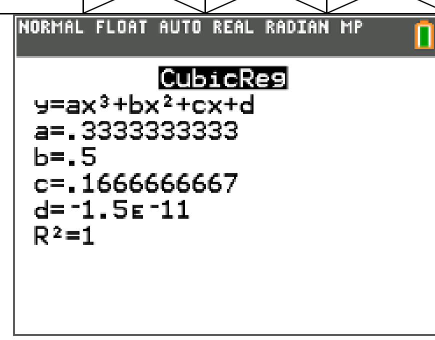
Problem 2 – A Pyramidal Stack

In Problem 2, students repeat the process of Problem 1 to find the number of bricks in a pyramidal stack given a certain number of layers in the stack.

Students should see that the third differences are the first time that the successive differences appear to be constant, so a third degree or cubic model is a good choice for a model.

L_1	L_2	L_3	L_4	L_5
rows	**bricks**	**1stDiff**	**2ndDiff**	**3rdDiff**
1	1	4	5	2
2	5	9	7	2
3	14	16	9	2
4	30	25	11	
5	55	36		
6	91			

Students are to perform the regression, choosing **CubicReg** from the stat > CALC menu. They can store the equation in **Y_1**.



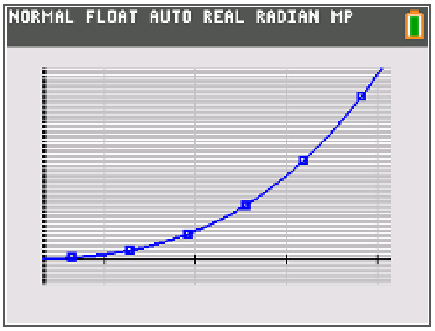
8. Choose and perform and polynomial regression. Record it here.

Answer: $\frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}x$

9. Look at the R^2 value for the regression. What is it? What does this mean?

Answers: 1; The model fits the data perfectly.

Then students compare the model to the data by creating a scatter plot. The graph screen should appear as shown at the right with the line passing through all of the points.



Note: Students should press zoom and select **ZoomStat** to view the plot in an appropriate window.

10. Check your model. Graph total bricks vs. number of layers as a scatter plot together with your model. Does the model go through all the points?

Answer: yes

11. If your model is correct, use it to calculate the number of bricks in a pyramid 50 layers high.

Answer: 42,925 bricks

12. Discuss the shortcomings of your model for this situation. For what numbers of layers is it valid? For what numbers of layers does it not make sense?

Answer: The model does not make sense for negative numbers of layers.

13. Write a domain for this model.

Answer: $\{x \mid x \geq 0\}$

