EXPLORATIONS

Activity 3

Watching Your Weight

Objectives

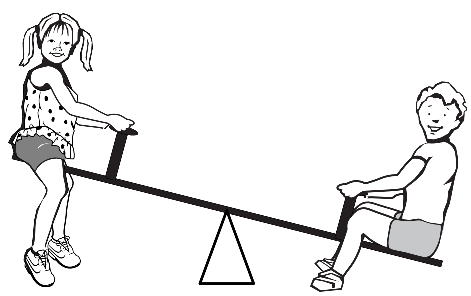
- To find the *y* value of a function, given the *x* value
- To use technology to find a best fit line
- To use technology to plot a set of ordered pairs

Materials

- TI-73 graphing device
- Bathroom scale, kitchen scale, or small scale
- Block(s) equal in height to the height of the scale
- Wooden plank 2 – 4 cm thick by 25 – 30 cm wide and 120 – 140 cm long (or meter stick or ruler), one per group
- Textbooks that weigh at least 10 – 14 kilograms or 25 – 30 pounds (or a bathroom size paper cup and at least 50 pennies), one set of weights per group
- Meter stick or tape measure, one per group

Introduction

There is a toy that children used to play on called a seesaw. It is shown in the illustration to the right. Many playgrounds have removed them for safety reasons. The seesaw was a board that was hinged on a bar. When one child pushed off the ground and went up, the child on the opposite end went down. Children loved to go up and down on the seesaw. If the children were of unequal weight, it became a problem, since the heavier child would weigh down the lighter child. This problem could be solved if one of the two children moved closer to the center of the board.

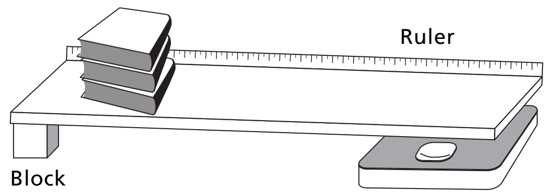


Problem

How does moving a weight along a board affect the downward force on the board? How might a heavier child at one end, or a lighter child at the other end, balance a seesaw when their weights are different?

Collecting the data

To solve this problem, set up a board on a scale and add some books as shown in the illustration to the right. The board represents a portion of the seesaw. The block represents the triangular fulcrum in the diagram of the seesaw shown previously. The books are similar to the child who is sliding back and forth, trying to balance the lighter child.



The block is used as the triangular fulcrum so that you can measure the apparent weight at the end of the board as the books slide up and down the seesaw.

1. Each group of students should obtain a scale, a wooden plank, and blocks from your teacher. Position the scale and the block(s) far enough apart so that the wooden plank is supported on one end by the scale and the other end by the block(s). Starting at the edge of the scale, place marks at 10-centimeter increments on the wooden plank. See the diagram above.

   ***Note**: If you have a small scale, you will use this setup with a meter stick. If you have a kitchen scale, you will use this setup with a ruler.*

2. If you are using a wooden plank, obtain textbooks that weigh at least 10 – 14 kilograms (or 25 – 30 pounds) to use for the weight. (If you are using a small scale, use a paper cup with pennies or marbles for the weight.)

3. Place the books on the wooden plank (or cup of pennies on the meter stick or ruler) at the edge of the scale. Record the weight shown on the scale for 0 centimeters in the table on the **Data Collection and Analysis** page.

4. Move the books a distance of 10 centimeters away from the scale. Record the weight shown on the scale in the table on the **Data Collection and Analysis** page.

5. Move the books a distance of 20 centimeters from the scale. Record the weight shown on the scale in the table on the **Data Collection and Analysis** page.

6. Continue to move the books away from the scale in 10-centimeter increments and record the weights.

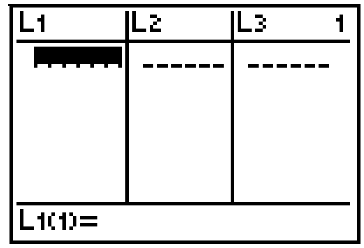
   ***Note**: If you are using a meterstick, place the cup 3 cm from the scale and continue to move the cup in 3 cm increments. If you are using a ruler, place the cup 1 cm from the scale and continue to move the cup in 1 cm increments.*

Setting up the TI-73

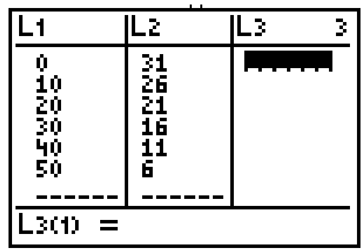
Before starting your data collection, make sure that the TI-73 has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

Entering the data in the TI-73

1. Press [LIST].

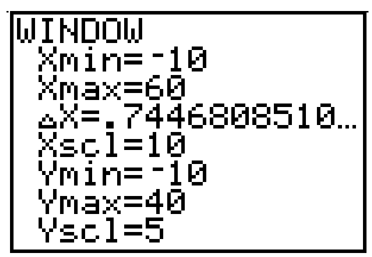


2. Enter the distance from the scale in **L1**.
3. Enter the weight shown on the scale in **L2**.



Setting up the window

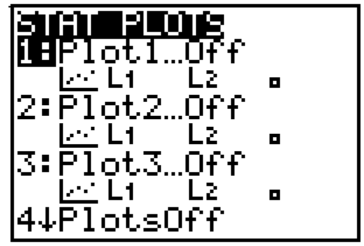
1. Press [WINDOW] to set up the proper scale for the axes.
2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.



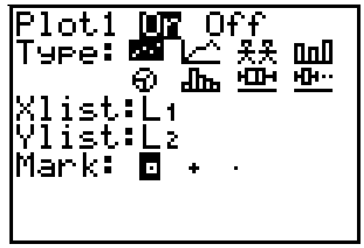
3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. **Do Not Change the ΔX Value.** Set the **Xscl** to **10**. (Set **Xscl** to **3** if you are using a small scale or **1** if you are using a kitchen scale.)
4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.
5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **5**. (Set **Yscl** to **0.25** if you are using a small scale or a kitchen scale.)

Graphing the data: Setting up a scatter plot

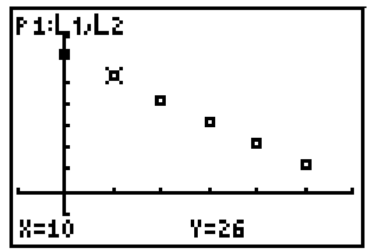
1. Press [2nd] [PLOT]. Select **1:Plot1** by pressing **1** or [ENTER].



2. Set up the plot as shown by pressing [ENTER] [▼] [ENTER] [▼] [2nd] [STAT] **1:L1** [▼] [2nd] [STAT] **2:L2** [▼] [ENTER].

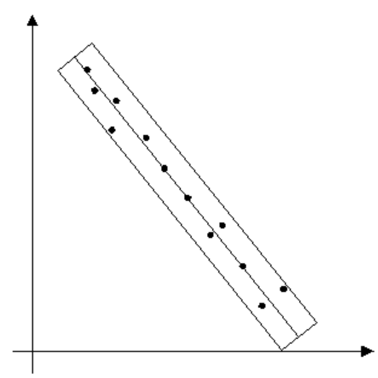


3. Press [TRACE] to see the plot. Press [◄] and [►] to move between the points.



You can analyze and make predictions using the data that you collected. In order to make predictions, you must describe the data using a mathematical model. Data analysis is not an exact science, and several different methods may be used to find mathematical models. Your data may not fit any model exactly; however, the challenge is to search for a model that best fits the data. The data that you have collected should appear linear; therefore, you will find a *line of best fit* or a *trend line*. You will use two different methods to find a line of best fit or a trend line. The first method is visual and the second method uses the *linear regression* feature of the TI-73.

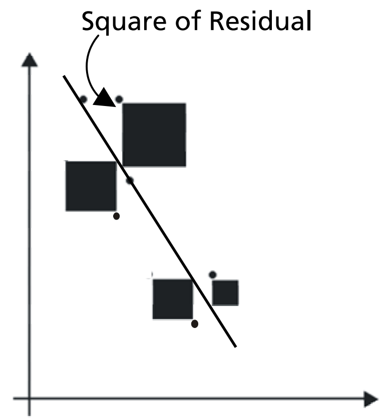
In the first method, use a crude but natural method for finding a line of best fit or a trend line: visually estimate the trend line. Follow the guidelines below to find the trend line.



1. Find the smallest rectangle that contains all the points and shows the direction of the points.
2. Find a line that contains as many of the points as possible.
3. Find a line that divides the points equally above and below the line.

The points above or below the line should not be concentrated at one end.

In the second method, use the TI-73 to find a line of best fit. The method of finding the line of best fit employed by the TI-73 uses the formula that minimizes the sum of the squares of the residuals. A *residual* is the vertical distance between the data point and the point on the line. Look at the diagram to the right that shows the squares of the residuals. The formula used by the TI-73 finds the equation where the sum of the squares of the residuals is as small as possible. (This is referred to as the method of least-squares.)



Analyzing the data

Finding a trend line (method 1)

The data appears to be linear; therefore, you can determine an equation for a trend line for the data. The equation for a line is $Y = MX + B$, where M is the *slope* and B is the y-intercept.

The *slope* of the line is defined as the change in y (Δy) divided by the change in x (Δx).

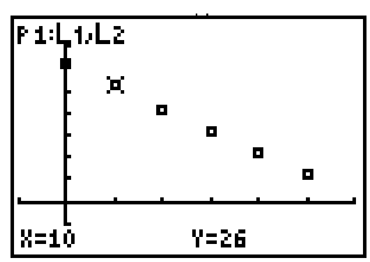
Slope = M

$$M = \Delta y/\Delta x$$

$$= \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

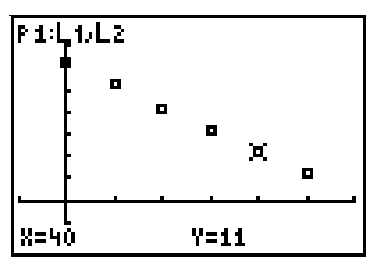
1. Find the *slope* of the line. Press [TRACE] [▸] to move the cursor to a point on the plot. Record the x and y values shown at the bottom of the screen.

X1 = __________ **Y1 =** ___________



2. Press [▸] as many times as you need to find a second point on the plot. Record the x and y values shown at the bottom of the screen.

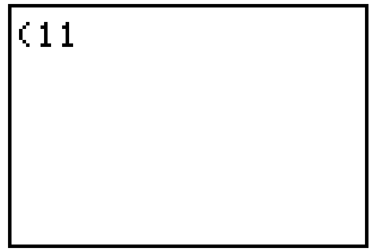
X2 = __________ **Y2 =** ___________



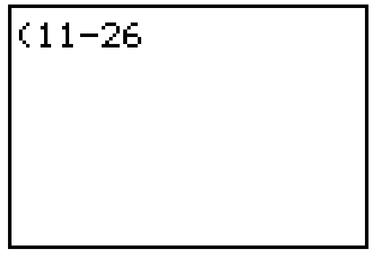
Use the following steps to calculate the *slope* of the line and to store the *slope* to **M** in the TI-73.

3. Press [2nd] [QUIT] to return to the Home screen. Press [CLEAR] to clear the Home screen.

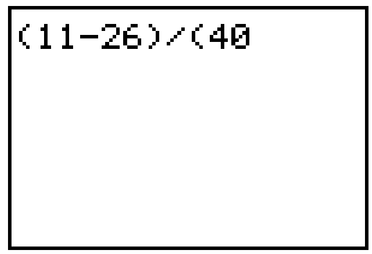
   a. Press [(] and enter the value for **Y2**.



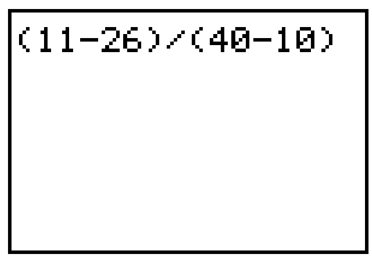
   b. Press [-] and enter the value for **Y1**.



   c. Press [)] [÷] [(] and enter the value for **X2**.

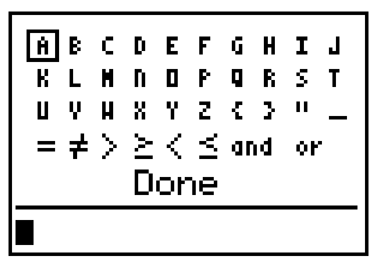


   d. Press [-] and enter the value for **X1**. Press [)].

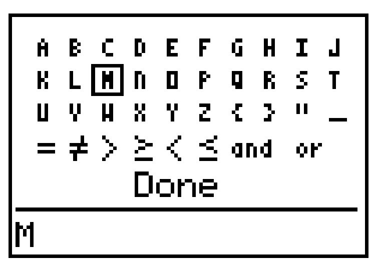


4. To store the *slope* to M:

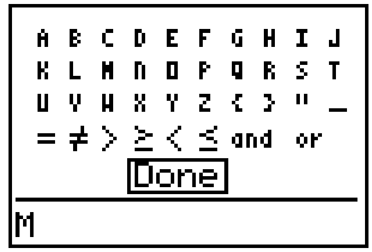
   a. Press [STO▶] [2nd] [TEXT].



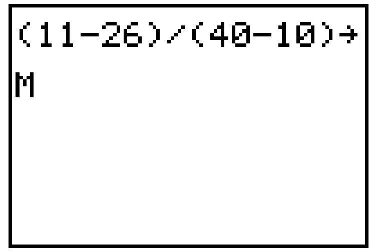
   b. Press [▼] [▶] [▶] [ENTER] to select M.



c. Press [▲] [▲] to highlight **Done**.

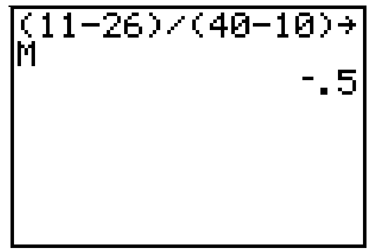


d. Press [ENTER] to exit the Text editor.



5. Press [ENTER] to calculate the *slope* and store the *slope* to **M** in the TI-73.

M= ___________.

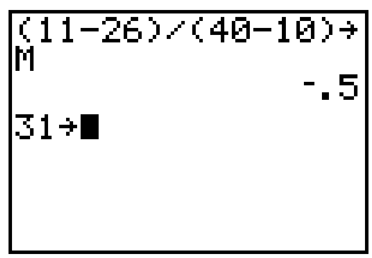


6. The *y*-intercept of a line is the point at which the line crosses the *y*-axis*.* The *y*-intercept of the trend line is the first value in **L2**.

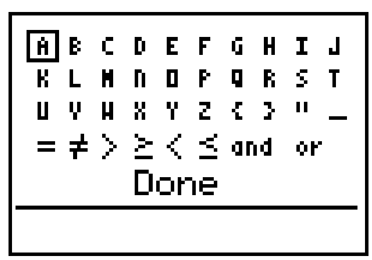
B = ____________

7. Store the *y*-intercept to **B** in the TI-73.

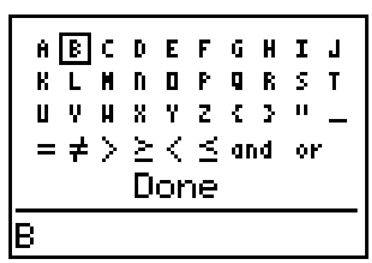
a. Enter your *y*-intercept value, then press [STO▶].



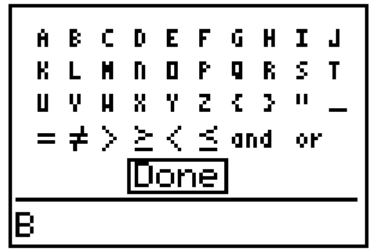
b. Press [2nd] [TEXT].



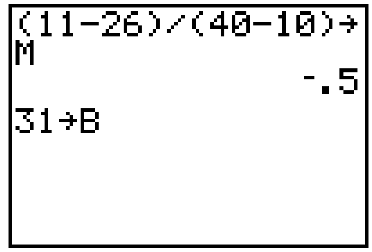
c. Press [▶] to select **B**, then press [ENTER].



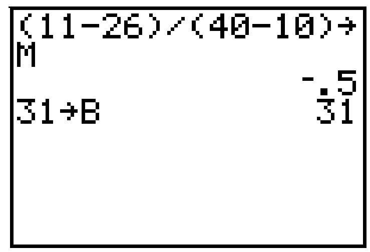
d. Press [▲] to highlight **Done**.



e. Press [ENTER] to exit the Text editor.

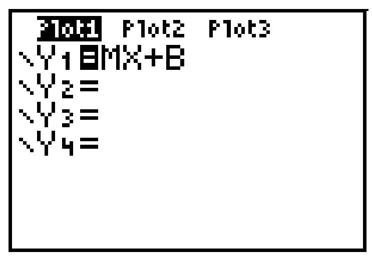


f. Press [ENTER] to store the value to **B.**



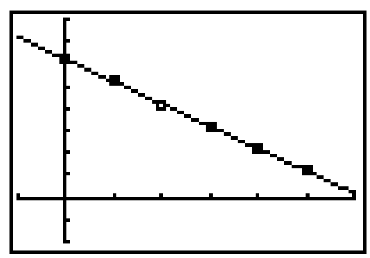
8. Enter the *slope-intercept* form of a linear equation in **Y1**.

Press [Y=] [2nd] [TEXT] [▼] [▶] [▶] [ENTER] [▼] [▶] [ENTER] [+] [▲] [▲] [◀] [◀] [ENTER] [▲] [ENTER] to place the equation *Y* = *MX* + *B* in the Y= menu.



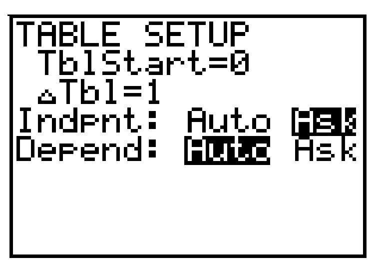
9. Press [GRAPH] to see the graph of the trend line. Record the equation on the line below.

Equation: ______________________________

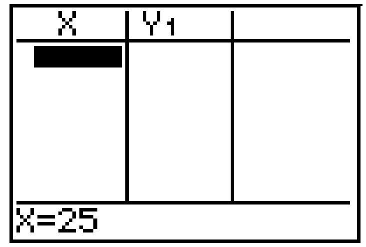


Finding the weight at different distances

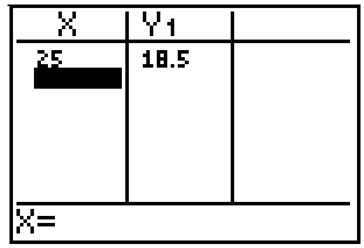
1. Find the weight of the objects at a distance of 25 centimeters from the scale. Press [2nd] [TBLSET]. Press [▼] [▼] [▶] [ENTER] to set the Independent variable to **Ask**.



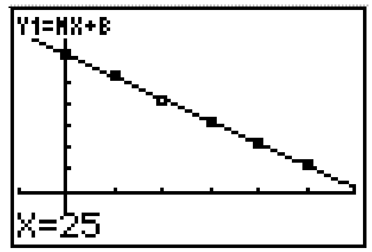
2. Press [2nd] [TABLE]. Enter **25.**



3. Press [ENTER] to see the desired weight. (The *y* value is the desired weight.)



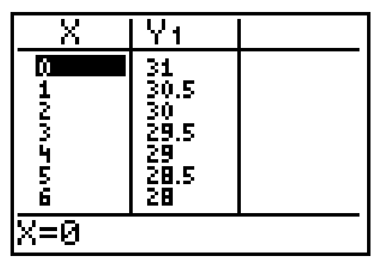
4. The [TRACE] key can be used to find the value of *y* given the value of *x.* Press [TRACE] [▼] to place the cursor on the graph of **Y1**. Enter the value of *x*, **25**.



Fining the distance given the weight

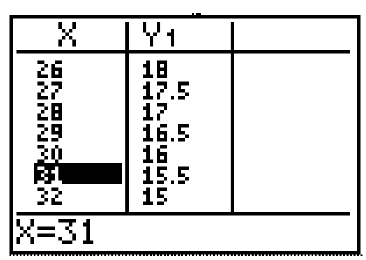
1. Press [2nd] [TBLSET]. Type **0** then press [ENTER] to set the **TblStart** value. Press [▼] **1** to set the ΔTbl value. Press [▼] [ENTER] to set the independent variable back to **Auto**.

2. Press [2nd] [TABLE]. Use [▼] and [▲] to scroll the table.



3. Find the point at which **Y1** is equal to 15.5. The *x* value is the solution to the problem.

Note*: Your* ***Y1*** *value may not exactly equal 15.5. If this occurs, choose the value closest to 15.5.*

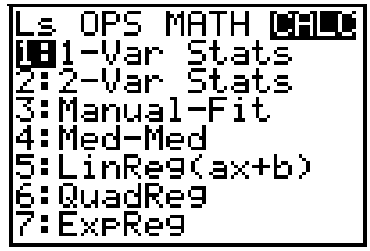


Use your equation to answer questions 1 through 6 on the **Data Collection and Analysis** page.

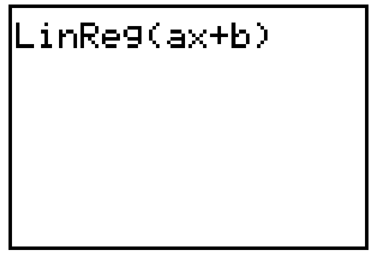
Finding a best fit line (method 2)

You can use the TI-73 to find the statistical line of best fit for the data. Clear the Home screen before you begin.

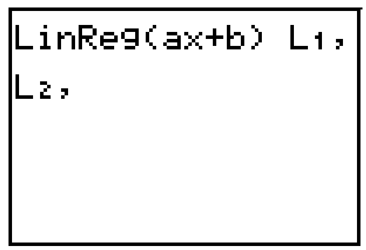
1. Find a linear regression equation for the data. Press [2nd] [STAT] [◄] to move the cursor to the **CALC** menu.



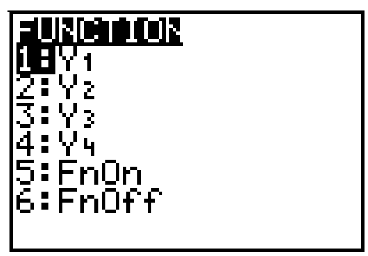
2. Select **5:LinReg(ax + b)** by pressing **5**.



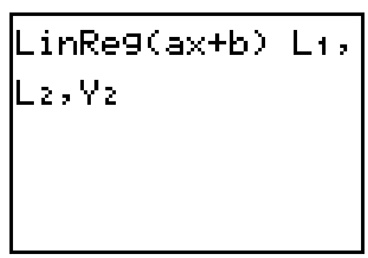
3. Enter **L_1**, **L_2**, and **Y_2**. Press [2nd] [STAT] **1:L_1** [ENTER] [,] [2nd] [STAT] **2:L_2** [,].



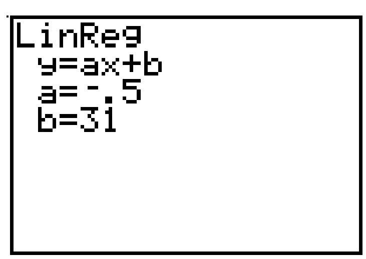
4. Press [2nd] [VARS]. Select **2:Y-Vars** by pressing **2**.



5. Select **2:Y_2** by pressing **2**.

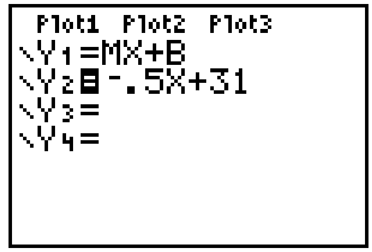


6. Press [ENTER] to calculate the equation for the best fit line. The function is pasted in **Y_2**.

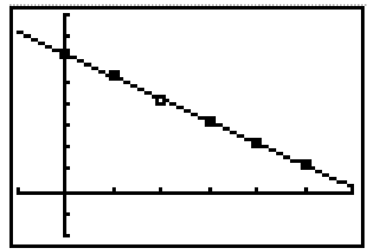


7. Press [Y=] to see the function.

Note: *Turn OFF the equation in* **Y1***. Press* [Y=] [◄] [ENTER].



8. Press [GRAPH] to see the graph of the best fit line.



9. Repeat the **Finding the weight at different distances** and **Finding the distance given the weight** sections using the linear regression model (**Y2**).

Answer questions 7 through 10 on the **Data Collection and Analysis** page.

Data Collection and Analysis

Name____________________________

Date ____________________________

Activity 3: Watching Your Weight

Collecting the data

Record your data in the appropriate column below.

Distance from scale (cm)	Weight shown on scale (__)

Analyzing the data

Use your equation from Step 9 in the **Analyzing the data: Finding a trend line (method 1)** section to answer questions 1 through 6.

1. The *slope* of the line is ____ .

2. Explain what the *slope* represents.

____

____

3. The *y*-intercept of the line is ____________________________________ .

4. Explain what the *y*-intercept represents.

____

____

5. Find the weight if the objects that you are using are placed at a distance of 25 centimeters from the scale. (If you used a small scale, find the weight of the objects at a distance of 13 centimeters from the scale.)

____

6. At what distance are the objects if the weight shown on the scale is one-half the original weight of the objects? ______________________

Use the regression equation that you found in the **Analyzing the data: Finding a best fit line (method 2)** section to answer questions 7 through 10.

7. The *slope* of the regression line is ______________________ .

8. The *y*-intercept of the regression line is ______________________ .

9. Find the weight if the objects that you are using are placed at a distance of 25 centimeters from the scale. (If you used a small scale, find the weight of the objects at a distance of 13 centimeters from the scale.)

____

10. At what distance are the objects if the weight shown on the scale is one-half the original weight of the objects?

____

11. How do the values that you found in questions 1, 3, 5, and 6, using your model, compare with the values that you found in questions 7 through 10, using the regression model?

____

____

Teacher Notes

Activity 3

Watching Your Weight

Objectives

- To find the *y* value of a function, given the *x* value
- To use technology to find a best fit line
- To use technology to plot a set of ordered pairs

Materials

- TI-73 graphing device
- Bathroom scale, kitchen scale, or small scale
- Block(s) equal in height to the height of the scale
- Wooden plank 2 – 4 cm thick by 25 – 30 cm wide and 120 – 140 cm long (or meter stick or ruler), one per group
- Textbooks that weigh at least 10 – 14 kilograms or 25 – 30 pounds (or a bathroom size paper cup and at least 50 pennies), one set of weights per group
- Meter stick or tape measure, one per group

Preparation

- The wooden plank can be obtained from a lumberyard or a home improvement store. You can also use a bookshelf.
- A bathroom scale works well. You can also get a small scale from the science department in your school or use a kitchen scale.
- Make sure that students place the wooden plank at the middle of the scale.
- If you are using a bathroom scale with textbooks, make sure that the books weigh at least 10 – 14 kilograms or 25 - 30 pounds.
- If you are using small scales, use a bathroom cup with at least 50 pennies. You can also use marbles, metal washers, or any other small object with weight.

Answers to Data Collection and Analysis questions

Collecting the data

- Sample data for a bathroom scale with textbooks:

Distance from scale (cm)	Weight shown on scale (kg)
0	11.8
10	11.3
20	10.9
30	10.4
40	10.0
50	9.5
60	9.1
70	8.6
80	8.2

- Sample data for a small scale with pennies:

Distance from scale (cm)	Weight shown on scale (g)
0	4.25
3	4.00
6	3.50
9	3.00
12	2.75
15	2.25

Analyzing the data

Use your equation from number 9 in the **Analyzing the data: Finding a trend line** section to answer questions 1 through 6.

1. The *slope* of the line is ____ .

   Answers may vary.
   For the sample data in Table 1 the slope is approximately -0.2625.
   For the sample data in Table 2 the slope is approximately -1.3333.

2. Explain what the *slope* represents.

   The slope represents the decrease in the number of kilograms per centimeter increase in distance from the scale.

3. The *y*-intercept of the line is ____ .

   The y-intercept for the sample data in Table 1 is approximately 26.
   The y-intercept for the sample data in Table 2 is approximately 4.25.

4. Explain what the *y*-intercept represents.

   The y-intercept represents the weight of the books at a distance of zero centimeters from the scale.

5. Find the weight if the objects that you are using are placed at a distance of 25 centimeters from the scale. (If you used a small scale, find the weight of the objects at a distance of 13 centimeters from the scale.)

   The weight for the sample data in Table 1 is 19.4375 kilograms.
   The weight for the sample data in Table 2 is 2.5166 grams.

6. At what distance are the objects if the weight shown on the scale is one-half the original weight of the objects?

   The distance for the sample data in Table 1 is 49.52 centimeters.
   The distance for the sample data in Table 2 is 15.94 centimeters.

Use the regression equation that you found in number 4 in the **Analyzing the data: Finding a best fit line** section to answer questions 7 through 10.

7. The *slope* of the regression line is ____________________________________ .

   For the sample data in Table 1 the slope is approximately -0.2683.
   For the sample data in Table 2 the slope is approximately –0.1357.

8. The *y*-intercept of the regression line is _______________________________ .

   The y-intercept for the sample data in Table 1 is approximately 25.6222.
   The y-intercept for the sample data in Table 2 is approximately 4.3095.

9. Find the weight if the objects that you are using are placed at a distance of 25 centimeters from the scale. (If you used a small scale, find the weight of the objects at a distance of 13 centimeters from the scale.)

The weight for the sample data in Table 1 is 18.9138 kilograms.
The weight for the sample data in Table 2 is 2.5452 grams.

10. At what distance are the objects if the weight shown on the scale is one-half the original weight of the objects?

The distance for the sample data in Table 1 is 47.04 centimeters.
The distance for the sample data in Table 2 is 16.10 centimeters.

11. How do the values that you found in questions 1, 3, 5, and 6, using your model, compare with the values that you found in questions 7 through 10, using the regression model?

The values should be close.