First Law of Thermodynamics - ID: 9115
Time required
By Irina Lyublinskaya
45 minutes
Topic: Heat and Thermodynamics

- Calculate heat, work, and changes in internal energy using the first law of thermodynamics.
- Calculate work done by gases.

Activity Overview
In this activity, students explore the first law of thermodynamics and apply it to various thermodynamics processes in order to calculate heat, work, and changes in internal energy of a very simple system: an ideal gas enclosed in a container fitted with a movable piston.

## Materials

To complete this activity, each student will require the following:

- TI-Nspire ${ }^{\text {TM }}$ technology
- pen or pencil
- blank sheet of paper

TI-Nspire Applications
Graphs \& Geometry, Notes, Calculator

## Teacher Preparation

Before carrying out this activity, you should review with students the concepts of internal energy of an ideal gas, work done by a gas and on the gas, and heat for the processes carried out on an ideal gas. Students should also be comfortable with the ideal gas laws and with the distinction between state and process variables. You may also wish to review with students the equation describing the first law of thermodynamics, $\Delta U=Q-W$.

- The screenshots on pages 2-12 demonstrate expected student results. Refer to the screenshots on pages 13-15 for a preview of the student TI-Nspire document (.tns file).
- To download the .tns file, go to education.ti.com/exchange and enter "9115" in the quick search box.


## Classroom Management

- This activity is designed to be teacher-led with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in this document, so you should make sure to cover all the material necessary for students to comprehend the concepts.
- The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.
- Students may answer the questions posed in the .tns file using the Notes application or on a separate sheet of paper.
- In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.

The following questions will guide student exploration during this activity:

- What are the relationships between heat, work, and internal energy of a gas in various thermodynamic processes?
- How can we calculate heat, work, and changes in internal energy using the first law of thermodynamics?

The purpose of this activity is to provide students with an opportunity to explore the first law of thermodynamics and apply it to calculate heat, work, and internal energy of an ideal gas.
TI-Nspire technology provides students with a dynamic environment for explorations and tools for graphical and numerical analysis for a variety of situations.
This activity consists of four problems. The first three problems explore energy conservation in simple thermodynamic processes. In the last problems, students use TI-Nspire technology to analyze $P-V$ diagrams and apply the first law of thermodynamics to answer specific questions.

## Problem 1 - Isothermal processes and the first law of thermodynamics

Step 1: Students should open the file
PhysWeek02_1stLawThermo.tns, read the first two pages, and then move to page 1.3, which shows an animated piston cylinder and a graph of pressure $(P)$ vs. volume ( $V$ ). In this part of the activity, students will use TI-Nspire features to simulate isothermal expansion and compression of an ideal gas. The equation that is used for the isothermal curve is $P=\frac{2,400}{V}$. The initial state of the gas is preset to $V=$


8 L and $P=300 \mathrm{kPa}$. The work is calculated as a definite integral using the Integral Measurement tool. Thus work is calculated in units of joules.
Step 2: Students should click on the Play ( $ا$ ) button and observe as the piston starts moving up. As the piston moves, the point on the graph describing each state of the ideal gas moves along the curve. When the value of the volume is close to 20 L , students should click on the Pause (II) button. The area under the curve will be shaded, and the value of the integral appears as work done by the gas, $W$. The students should then answer questions 1 and 2 on page 1.3.

Q1. On the $P-V$ diagram, what does the work done
 by the gas correspond to? In other words, how is work represented geometrically on the diagram?
A. Work is the area under the curve on the $P-V$ diagram.

Q2. How much heat was added to the system during this process? Explain your answer.
A. Because this is an isothermal process, $Q=W$. Therefore, the amount of heat added to the system is equal to the work done by the gas. Student values will vary but should be approximately 2,200 J.

Step 3: Next, students should press the Play button again and wait until the piston starts moving down. When the volume of the gas reaches about 3 L , students should press the Pause button to stop the animation. Then, they should answer question 3 on page 1.4.

Q3. Was heat added or removed from the system when the gas was compressed? Justify your answer.
A. As the diagram shows, the work done by the gas in the case of compression is negative, which means work is done on the gas. The amount of heat is negative as well, which means heat is removed from the system per the first law of thermodynamics: $Q=W$.

Step 4: Students should now move the piston up and down by dragging point $M$ with the arrow keys. Students should use the simulation to answer questions 4-9 on pages 1.5-1.11. Because the process is isothermal, the internal energy remains constant.

Q4. An ideal gas expands isothermally, performing $1.128 \times 10^{3} \mathrm{~J}$ of work in the process. Calculate the change in internal energy of the gas and the heat absorbed during this expansion.

A. Because the process is isothermal, $\Delta U=0$. The heat absorbed is equal to the work done by the gas: $Q=W=1.128 \times 10^{3} \mathrm{~J}$.

Q5. The volume of 1 mole of an ideal gas is slowly increasing from 8 L to 20.2 L . How much heat is being added to the system, how much work is being done by the gas, and what is the change in internal energy of the gas?
A. Because the process is slow, it is an isothermal process, so $\Delta U=0$ and $Q=W=2,223 \mathrm{~J}$.

Q6. How much work must be done on this gas to compress it from 8 L to 4 L isothermally?
A. $W=-1,663.6 \mathrm{~J}$

Q7. What is the change in internal energy in an isothermal process? What is the relationship between the work done by the system and the heat added to the system in an isothermal process?
A. Because the internal energy of an ideal gas is directly proportional to the temperature, and temperature change is zero, the internal energy of the system remains constant. Thus, the work done by the system is equal to the heat added to the system, $Q=W$.

Q8. Is it possible for the temperature of a system to remain constant even though heat flows into or out of it? If so, give one or two examples.
A. Yes, in an isothermal expansion or compression, the heat is added or removed from the system, but the temperature stays constant. This occurs because any added heat goes into work done by the system (expansion), and any work done on the system (compression) releases heat.

Q9. In an isothermal process, 370 J of work is done by an ideal gas. Is this enough information to tell how much heat has been added to the system? If so, how much?
A. Yes, it is enough information. The amount of heat added to the system is equal to the work done by the gas, so $Q=370 \mathrm{~J}$.

Problem 2 - Isobaric and isovolumetric processes and the first law of thermodynamics

Step 1: Next, students should move to page 2.1 and read the text there. Students should then answer question 10 on page 2.2 before proceeding with the rest of the activity.
Q10. Derive an equation to calculate the work done by the gas during an isobaric process in terms of its volume, $V$, and pressure, $P$.
A. Since $W=\Delta(P V)$ and pressure is constant, $W=P \Delta V$ or $W=P\left(V_{2}-V_{1}\right)$.

Step 2: Next, students should move to page 2.3, which shows the same system as page 1.2; however, in this case, the gas compression and expansion are carried out at constant pressure. The corresponding $P-V$ diagram is shown on the right. As before, the pressure is given in units of kilopascals and volume is in liters. The amount of work done by the gas, $W$, is displayed in joules. Students should use this simulation to confirm the equation they derived in question 10. They should enter the equation into the left-hand side of the screen, using the Text tool (Menu > Tools > Text). They should move the cursor to an empty place on the screen and press S.inter. The text box will appear. They should then enter the equation derived above, $W=P \cdot\left(V_{2}-V_{1}\right)$. Note: Students must enter a multiplication sign between $P$ and the parentheses, or the calculation will not work correctly.
Step 3: Next, students should start the animation. They should pause it when the final volume is between 15 L and 20 L .

The two other simple thermodynamic processes are isobaric and isovolumetric, or isochoric, processes. An isobaric process is one in which the pressure is kept constant ( $P=$ const). An isovolumetric process is one in which the volume does not change ( $V=$ const).


Step 4: Students should now use the Calculate tool (Menu > Tools > Calculate) to assign values to the variables in the equation they entered in step 2 . After selecting the Calculate tool, they should move the cursor over the equation and press ยूiner. They will be prompted to select the value of $P$. Students should move the cursor over the $y$-coordinate of a point representing the state of the ideal gas and press Sefier (Because pressure is constant, students may select either of the two $y$-coordinates on the plot.) Students will then be similarly prompted to select $V_{1}$. They should repeat the same procedure, moving the cursor over the $x$-coordinate of the initial point. Finally, they are prompted to select $V_{2}$. They should choose the $x$ coordinate of the final point for $V_{2}$. When the calculated value of the work appears on the screen, students can drag it to a position close to their equation and then press Sinure. They should then press (esc) to quit the Calculate tool.
Step 5: Students should now play the animation and compare the values of work done by the gas as calculated according to their equation with the values of work shown on the graph. They should then answer questions 11 and 12 on pages 2.4 and 2.5.
Q11. Does your equation for the work done by the gas produce the same value as the value displayed on the graph?
A. If the students entered the correct equation, the values should be the same.

Q12. Show that your derived equation for work agrees with the definition of work, $W=F \cdot d$, where $F$ is the force of the gas on the piston and $d$ is the distance the piston moves.
A. Force, F , is the product of pressure and crosssectional area of the piston: $F=P \cdot A$. Combining this equation with the equation for work above yields $W=P \cdot A \cdot d$. The product of the cross-sectional area and the distance the piston moves is equal to the change in the gas volume: $\Delta V=A \cdot d$. Therefore, the equation for work becomes $W=P \Delta V$.


Step 6: Next, students should move to page 2.6 , which contains a blank Graphs \& Geometry application. They should adjust the window to show the first quadrant only (Menu > Window > Zoom Quadrant 1). They should then change the $x$-axis label to $V$ and $y$-axis label to $P$ by moving the cursor over each label in turn, pressing Senier twice, pressing and pressing 气iner . They should then show the positive arrows on the axes by moving the cursor over an axis, choosing Attributes from the content-sensitive menu ( © trrl) menu > Attributes), selecting the ( $1 / 3$ ) Positive Arrows option, and pressing enimer . Students should then answer questions 13-16.

Q13. What is the representation of an isovolumetric process on the $P$ - $V$ diagram? On page 2.6, plot a curve that represents an isovolumetric process. Explain your answer.
A. In an isovolumentric process, volume does not change. Therefore, an isovolumetric process is
 represented by the vertical line $V=$ constant. Students can use the Line tool (Menu > Points and Lines > Line) or the Perpendicular tool (Menu > Constructions > Perpendicular) to plot the vertical line.

Q14. What is the work done by the gas in an isovolumetric process? Use both equations and graphical reasoning to explain your answer.
A. If there is no change in volume, $\Delta \mathrm{V}$ is zero, and work is also zero because $W=P \Delta V$. From the geometric perspective, a graph of an isovolumetric process is a vertical line. The work is the area under this line, which is zero.

Q15. The pressure on an ideal gas in a container with rigid walls is slowly decreased by half. In the process, 265 kJ of heat leaves the gas. What is the change in internal energy of the gas during this process? Explain your answer.
A. Since work is zero, $\Delta U=Q$. Therefore, the change in internal energy is 265 kJ .

Q16. Heat is allowed to flow out of one mole of an ideal gas at constant volume so that its pressure drops from 2.2 atm to 1.6 atm . Then, the gas expands at constant pressure from 6.8 L to 9.3 L . The temperature of the gas at the end of the process is equal to the temperature at the beginning. Calculate the total work done by the gas in the process, the change in internal energy of the gas in the process, and the total heat flow into or out of the gas. Use the

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|  | 1/99 | Calculator application on page 2.12 to help you solve this problem.

A. To calculate work in joules, students will first need to convert pressures from atmospheres to kilopascals using the conversion factor $1 \mathrm{~atm}=101 \mathrm{kPa}$. The total work done by the gas is equal to the work done by the gas during isobaric processes only. Therefore, the work done by the gas is equal to $P_{f}\left(V_{f}-V_{i}\right)=405 \mathrm{~J}$. Because the initial and final temperatures are equal, the total change in internal energy is zero, and by the first law of thermodynamics, $Q=W=405 \mathrm{~J}$.

## Problem 3 - Adiabatic processes and the first law of thermodynamics

Step 1: In this part of the activity, students explore adiabatic processes. Students should read page 3.1 before moving on to the simulation. Page 3.2 shows the simulation for the ideal gas in an adiabatic process. The equation for this particular adiabatic process is $P=\frac{2,400}{V^{3 / 2}}$. Students should study the animation and then answer question 17 on page 3.3.


Q17. Compare the work done by an ideal gas in an adiabatic expansion with that in an isothermal expansion from the same initial state. Explain your observations using the first law of thermodynamics.
A. The work done by the gas in an isothermal process is greater than the work done in an adiabatic expansion. This is because the pressure drops faster in an adiabatic process than in an isothermal process. As a result, the adiabatic curve is below the isothermal curve, so the area under the curve is smaller.

Step 2: Next, students verify their answers to question 17 by setting up a process with a specific final volume. First, they should graph the curve for an isothermal process with the same starting conditions as the displayed adiabatic process. The function that should be entered for the isothermal process is
$P V=106.1 \cdot 8=848.8$, so $P=\frac{848.8}{V}$. To display the function entry line, students should press ©ctr) (G), enter the new function in $f 2(x)$, and press (atr) again to hide the entry line. (Make sure students use $x$ instead of $V$ as the variable in the equation.) They should then hide the equation so that the screen does not become too cluttered. For a better view, the $y$-axis maximum value can be adjusted to a smaller value.

Step 3: Next, students should calculate the area under the curve for the isothermal process. They should use the Integral Measurement tool (Menu > Measurements > Integral) to measure the area under the curve. After selecting this tool, students should click on the curve, then the initial point, and then the final point. Students can label this measurement as $W_{T}$. To change the precision of the calculation, they should move the cursor over the calculated value, press 䠌 once (the value will be shaded), press ©trl) (ment), select Attributes, and use the arrow keys to change the precision. They should press ) fine to finish this process. Students should then compare the values of work displayed for each process. Then, they should answer questions 18-20.


Q18. One mole of an ideal gas has a temperature of 500 K and a volume of 10 L . It is compressed adiabatically to half its volume. Set up a $P-V$ diagram for this process, and answer the following questions: How much heat flows into or out of the gas? What is the change in internal energy of the gas? Does its temperature rise or fall? Use the Graphs \& Geometry application on page 3.5 to help you solve this problem.
A. Because the process is adiabatic, $Q=0$. The equation relating pressure and volume is $P=\frac{n R T}{V^{3 / 2}}=\frac{4,155}{V^{3 / 2}}$. The change in internal energy is equal to the work done on the gas, $\Delta U=-W$, which is represented by the area under the curve. Since $W<0, \Delta U>0$ and $\Delta T>0$, so the gas temperature rises. To construct the diagram, students should use the Graphs \& Geometry application on page 3.3. They should enter the function $f 3(x)=4155 / x^{3 / 2}$ in the function entry line and adjust the window settings (Menu > Window $>$ Window Settings) so that the y-axis runs from approximately -50 to 500 and the x-axis runs from approximately -5 to 15 . They should then use the Integral Measurement tool to measure the area under the curve between $V=5$ and $V=$ 10. To use the Integral Measurement tool, students should click once on the curve and then move the cursor along the x-axis until a point appears at $x=5$ (at these window settings, each tick mark represents one unit). They should click on $x=5$ and then again on $x=10$. The device should then display the value of the integral (the work done), which should equal 1,088.5 J.


Q19. Use conservation of energy to explain how and why the temperature of gas changes when it is quickly compressed or expanded.
A. When a gas is quickly compressed or expanded, the heat cannot flow out of or into the system, because heat transfer is a slow process. As a result, all work done by the gas or on the gas is converted into the change in its internal energy, which is directly proportional to the change in temperature of the gas.

Q20. One and one-half moles of an ideal monatomic gas expand adiabatically, performing 7,500 J of work in the process. What is the change in temperature of the gas during this expansion?
A. By the first law of thermodynamics, $\Delta U=-W$. By
definition, $\Delta U=\frac{3}{2} n R \Delta T$, so
$\Delta T=-\frac{7,500}{\frac{3}{2}(1.5)(8.31)}=-401 \mathrm{~K}$. The temperature drops by 401 K .

## Problem 4 - Applications of the first law of thermodynamics to problem solving

Step 1: Next, students should use the application on page 4.2 to answer questions 21 and 22.

Q21. A gas within a closed chamber undergoes the cycle shown in the $P-V$ diagram on page 4.2. Calculate the net energy added to the system as heat during one complete cycle. On the diagram, the units of pressure are $\mathrm{N} / \mathrm{m}^{2}$ and the units of volume are $\mathrm{m}^{3}$.
A. Since it is a closed cycle, the total change of internal energy is zero. Thus, by first law of thermodynamics, $Q=W$. The work done by the gas is equal to the area enclosed by the cycle. The direction of the process means the work is negative. Therefore, the work is equal to the area of the triangle: $W=-\frac{1}{2}(30-10) 2=-20 \mathrm{~J}$.
Students should confirm their calculations using the diagram on page 4.2. They should draw triangle ABC on page 4.2 using the Triangle tool
 (Menu > Shapes > Triangle). They should then use the Area Measurement tool (Menu > Measurement > Area) to find the area of the triangle. If you wish, you may also allow students to confirm that the total work is equal to the area of the triangle by having them calculate the work done during the expansion, the work done during the compression, and the isovolumetric work; add them together; and compare this value to the area of the triangle.

Q22. The first law of thermodynamics is sometimes whimsically stated as "You can't get something for nothing." Explain how this statement could be equivalent to the formal statement.
A. This is an energy conservation statement that says that energy cannot be created from nothing and cannot disappear; it can only change form. The first law of thermodynamics states that the change in internal energy, $\Delta U$, of a system is equal to the heat added to the system, Q, minus the work done by the system, W. Essentially, this equation, $\Delta U=Q-W$, states that a system can gain energy only if its surroundings lose an equivalent amount of energy. As a corollary, an isolated system (which can exchange neither matter nor energy with its surroundings) can never gain or lose energy (that is, $\Delta U=0)$.

## First Law of Thermodynamics - ID: 9115

(Student)TI-Nspire File: PhysWeek02_1stLawThermo.tns


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3. Was heat added or removed from the system when the gas was compressed? Justify your answer.

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4. An ideal gas expands isothermally, performing $1.128 \times 10^{3} \mathrm{~J}$ of work in the process. Calculate the change in internal energy of the gas and the heat absorbed during this expansion.

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5. The volume of 1 mole of an ideal gas is slowly increasing from 8 L to 20.2 L . How much heat is being added to the system, how much work is being done by the gas, and what is the change in internal energy of the gas

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5. How much work must be done on this gas to compress it from 8 L to 4 L isothermally?

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7. What is the change in internal energy in an isothermal process? What is the relationship between the work done by the system and the heat added to the system in an isothermal process?

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9. In an isothermal process, 370 J of work is done by an ideal gas. Is this enough information to tell how much heat has been added to the system? If so, how much?

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The two other simple thermodynamic processes are isobaric and isovolumetric, or isochoric, processes. An isobaric process is one in which the pressure is kept constant ( $P=$ const). An isovolumetric process is one in which the volume does not change ( $V=$ const).


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11. Does your equation for the work done by the gas produce the same value as the value displayed on the graph?


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13. What is the representation of an isovolumetric process on the $P-V$ diagram? On page 2.6 , plot a curve that represents isovolumetric process. Explain your answer.

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15. The pressure on an ideal gas in a container with rigid walls is slowly decreased by half. In the process, 265 kJ of heat leaves the gas. What is the change in internal energy of the gas during this process? Explain your answer.

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16. Heat is allowed to flow out of one mole of an ideal gas at constant volume so that its pressure drops from 2.2 atm to 1.6 atm . Then, the gas expands at constant pressure from 6.8 L to 9.3 L . The temperature of the gas at the end of the process is equal to the temperature at the beginning. (continued on next page)

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16. (continued) Calculate the total work done by the gas in the process, the change in internal energy of the gas in the process, and the total heat flow into or out of the gas. Use the Calculator application on page 2.12 to help you solve this problem.

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An adiabatic process is one in which no heat is allowed to flow into or out of the system: $Q=0$. This situation can occur if the gas in the cylinder is well insulated, or if the process happens so quickly that heat - which flows slowly - has no time to flow in or out.



17. Compare the work done by an ideal gas in an adiabatic expansion with that in an isothermal expansion from the same initial state. Explain your observations using the first law of thermodynamics.

18. One mole of an ideal gas has a temperature of 500 K and a volume of 10 L . It is compressed adiabatically to half its volume. Set up a $P-V$ diagram for this process and answer the following questions: How much heat flows into or out of the gas? What is the change in internal energy of the gas? Does its temperature rise or fall? Use the Graphs \& Geometry application on page


| 3.3 | 3.4 | 3.5 | 3.6 | RAD AUTO REAL |
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19. Use conservation of energy to explain how and why the temperature of gas changes when it is quickly compressed or expanded.

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20. One and one-half moles of an ideal monatomic gas expand adiabatically, performing 7,500 J of work in the process. What is the change in temperature of the gas during this expansion?

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21. A gas within a closed chamber undergoes the cycle shown in the $P-V$ diagram on page 4.2. Calculate the net energy added to the system as heat during one complete cycle. On the diagram, the units of pressure are $\mathrm{N} / \mathrm{m}^{2}$ and the units of volume are $\mathrm{m}^{3}$.


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22. The first law of thermodynamics is sometimes whimsically stated as "You can't get something for nothing." Explain how this statement could be equivalent to the formal statement.
