Relating Rates

ID: 11642

Time Required
45 minutes

Activity Overview

In this activity, students are introduced to the topic or related rates. The process is explained step-by-step to help students build understanding and put what they learned into practice with homework or extension questions.

Topic: Related Rates

- *Applications of implicit differentiation*
- *Formula relating to real world phenomena*

Teacher Preparation and Notes

- *This activity can be completed with or without the use of CAS functionality. If CAS functionality is not used, then students should complete the implicit differentiation and unit conversion by hand.*
- *The activity is designed to be a student-centered discovery and instruction of related rates. At the conclusion of the activity, reinforce the method for solving these problems. After completing the activity, students should be more successful with AP* questions like 2002 formB AB6, 1999 AB6, 96BC5&6, 95AB5 BC3, 94AB5 BC2, 92AB6, 92 BC5, and multiple -choice questions 1998AB90, AB&BC78*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "11642" in the quick search box.***

Associated Materials

- *CalcWeek18_RelatingRates_Worksheet_TI89.doc*

Suggested Related Activities

- *The Falling Ladder (TI-Nspire CAS technology) — 8318*
- *Related Rates (Learning Check) — 7080*

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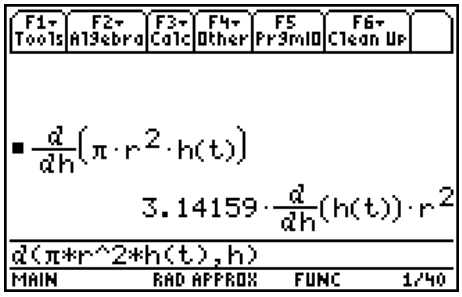
Problem 1 – Example & Explanation

Students are given a situation of water draining out of a cylindrical tank. This problem serves as the basis for explaining the process of solving related rates questions.

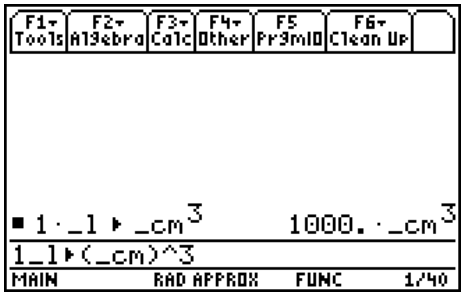
Students have difficulty realizing that if the volume is decreasing, the rate is negative.

Another difficulty students have is remembering an appropriate formula that relates what they know and what they want to find. Make sure that students are using the correct equation.



Students often either want to treat *r* and *h* as variables when one of them is not changing, or they treat one as a constant when it is really changing with respect to time. The question associated with Step 3 brings that out.

Another issue students need to pay attention to is units. The unit conversion capabilities of the CAS functionality can be utilized as shown to the right.



To use units with the TI-89 Titanium, the underscore must be entered. To do this, press ◆ + [_] (◆ + MODE). The convert symbol can be found by pressing 2nd + [▸] (2nd + MODE).

Student Solutions

Step 1: Variables: height *h*, radius *r*, volume *V*, time *t*

Given information: $\frac{dV}{dt}$ = –4 L/s, *r* = 2

Unknown(s): $\frac{dh}{dt}$

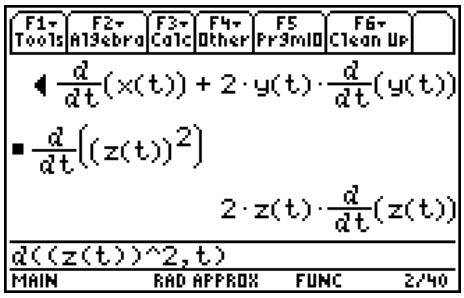
Step 2: $V = \pi r^2 h$

Step 3: No, *r* is constant; $\frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$

Step 4: $\frac{-1000}{\pi}$ cm/sec

Problem 2 – Additional Example & Explanation

Implicit differentiation and the steps for related rates questions are exemplified again. The example of two cars leaving a single point is a typical Pythagorean problem that needs to be carefully differentiated implicitly so as not to forget the Chain Rule.



To add more depth to this question, ask the students if they would get the same answer if the car going to the east actually started from 60 units away and traveled west at 15 units/hour. Ask them to explain their reason.

Student Solutions

Step 1: Variables: x is the distance east, y is the distance north, z is the distance between the cars, and t is the number of hours passed

Given information: $\frac{dy}{dt} = 8$, $\frac{dx}{dt} = 15$

Goal: $\frac{dz}{dt}$ when $x = 30$

Step 2: $x^2 + y^2 = z^2$

Step 3: $2x \cdot x' + 2y \cdot y' = 2z \cdot z'$

Step 4: $2 \cdot 30 \cdot 15 + 2 \cdot 16 \cdot 8 = 2 \cdot 34 \cdot \frac{dz}{dt}$

17 units

Problem 3 – Homework/Extension

Students apply what was learned to various real-world and graphical problems. Students can use the **HOME** screen if needed to help them with their calculations.

Student Solutions

1. $\frac{1}{4\pi}$ mm/sec
2. decreasing 0.5 units/sec
3. –36
4. -160π cm^2/hr
5. approximately 28.83 mph
6. 70.686 mL/s