Powers and Roots of Complex Numbers

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Let's explore the graphical and numerical representations of the powers of the complex number $(-1 + i)^t$. First, numerically, a few powers of the number to see if there is a pattern:

$(-1 + \boldsymbol{i})^4$

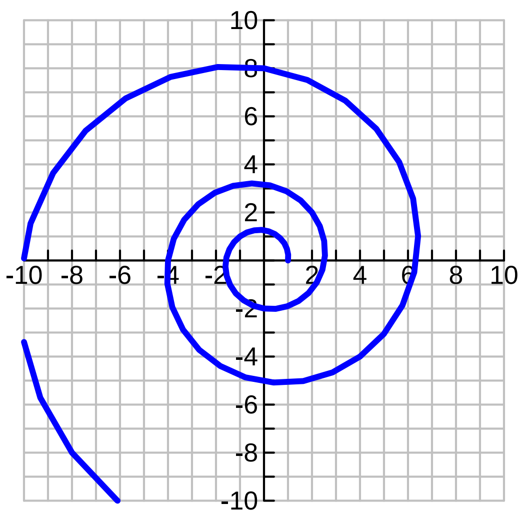
-4

Complex numbers can be represented graphically using parametric equations. Let the x definition represent the real part of the number and the y definition represent the imaginary part of the number. First, store a complex number in Z and then use that in the parametric definitions

x = real(Z^t)

y = imag(Z^t)

$(-1 + 1 \cdot \boldsymbol{i}) \to Z$



The Roots of Complex Numbers

The roots of complex numbers, including the roots of unity, can be represented graphically using De Moivre's theorem and parametric equations.

x = R^(1/N)*cos((A+360*t)/N)
y = R^(1/N)*sin((A+360*t)/N)

In these equations, R = absolute value of the complex number, N = the number of roots, A = the angle of the complex number in the Argand plane, and t = the index that moves through the N roots.

Example 1: Investigate the numerical values and the graph the 6 roots of $Z^6 = -64$.

Numerically, the computation below gives only one of the 6 roots, called the principal root. What about the other five roots?

$\sqrt[6]{-64}$

$\sqrt{3} + i$

$64 \rightarrow R$

$180° \rightarrow A$

$6 \rightarrow N$