Circle Product Theorems

About the Lesson

Students will use dynamic models to find patterns. As a result, students will:

- Summarize the relationships proven by the Chord-Chord, Secant-Secant, and Secant-Tangent Product Theorems.

Vocabulary

- chord
- secant
- tangent

Teacher Preparation and Notes

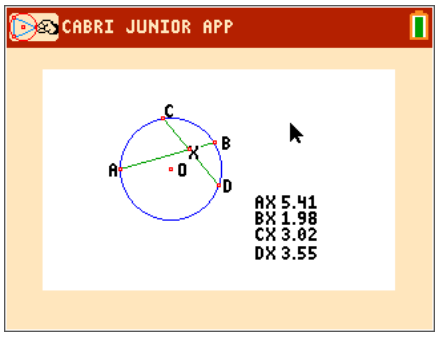
- This activity was written to be explored with the Cabri Jr. application on the TI-84 Plus CE.
- Before beginning this activity, make sure that all students have the Cabri Jr. application, and the Cabri Jr. files PRODUC1.8xv, PRODUC2.8xv, and PRODUC3.8xv loaded on their TI-84 Plus CE calculators.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

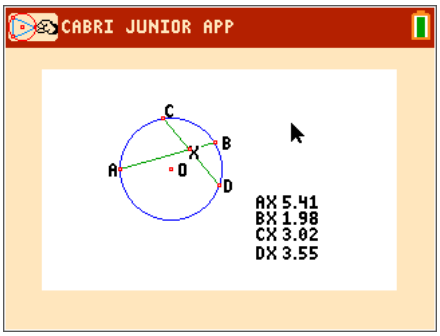
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Circle_Product_Theorems_Student.pdf
- Circle_Product_Theorems_Student.doc
- PRODUC1.8xv
- PRODUC2.8xv
- PRODUC3.8xv

Tech Tip: Before beginning the activity, the Cabri Jr. files PRODUC1.8xv, PRODUC2.8xv, and PRODUC3.8xv need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ CE software.

Problem 1 – Chord-Chord Product Theorem



Students will begin this activity by investigating the intersection of two chords and the product of the length of the segments of one chord and the product of the length of the segments of the other chord.

Students will be asked to collect data by moving point *A*. They are also asked to calculate the products either by hand on their worksheet or using the calculator as described below.

To calculate $AX \cdot BX$, they need to

- Press [graph],
- Select the tool **Calculate**,
- Move the cursor to the measurement of *AX* until it is underlined and then press [alpha] to select it,
- Press [×],
- Move the cursor to the measurement of *BX* and press [alpha] to select it,
- Move the resulting calculation to a clear area of the screen and
- Press [enter].

Students are asked several questions about the relationship among the products.

As an extension, prove the Chord-Chord Product Theorem using similar triangles.

1. Move point *A* to four different points and collect the data in the table below and calculate the products $AX \cdot BX$ and $CX \cdot DX$.

Sample Answer:

Position	*AX*	*BX*	*CX*	*DX*	$AX \cdot BX$	$CX \cdot DX$
1	2.70	0.99	1.51	1.78	2.67	2.68
2	2.51	1.05	1.38	1.90	2.63	2.62
3	2.93	0.91	1.80	1.49	2.67	2.68
4	1.80	1.08	2.51	0.77	1.94	1.93

2. What do you notice about the products $AX \cdot BX$ and $CX \cdot DX$?

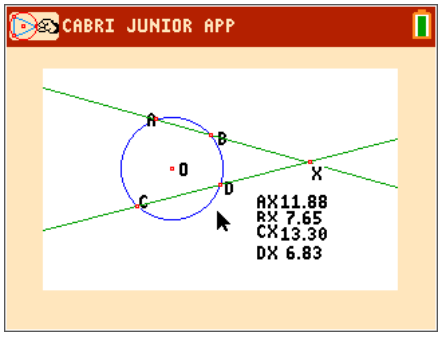
Answer: They are equal.

3. Summarize the relationship between the lengths of the segments of two chords if the two chords intersect in the interior of a circle.

Answer: If two chords intersect in the interior of a circle, then the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

Problem 2 – Secant-Secant Product Theorem

Students will investigate the intersection of two secants and the product of the lengths of one secant segment and its external segment and the product of the lengths of the other secant segment and its external segment.



Students will be asked to collect data by moving point *A*. They are to calculate the products by hand on their worksheet or using the calculator as described in Problem 1. Students are asked several questions about the relationship among the products.

As an extension, prove the Secant-Secant Product Theorem using similar triangles.

4. Move point *A* to four different points and collect the data in the table below and calculate the products $AX \cdot BX$ and $CX \cdot DX$.

Sample Answer:

Position	*AX*	*BX*	*CX*	*DX*	*AX · BX*	*CX · DX*
1	5.94	3.82	6.65	3.42	22.69	22.74
2	4.85	3.16	5.85	2.62	15.33	15.33
3	4.03	2.75	5.32	2.09	11.08	11.12
4	7.47	4.96	7.92	4.68	37.05	37.07

5. What do you notice about the products $AX \cdot BX$ and $CX \cdot DX$?

Answer: They are equal.

6. Summarize the relationship between the lengths of the secant segments and their external segments if the two secant segments share the same endpoint outside of a circle.

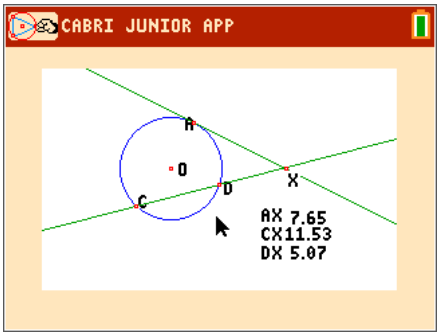
Answer: If two secant segments share the same endpoint outside of a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

Problem 3 – Secant-Tangent Product Theorem

Students will investigate the intersection of the product of the lengths of one secant segment and its external segment and the square of the tangent segment.

Students will be asked to collect data by moving point *A*. Students are asked to calculate the products by hand on their worksheet or using the calculator. Students are asked several questions about the relationship among the products.

As an extension, prove the Secant-Tangent Product Theorem using similar triangles.



7. Move point *A* to four different points and collect the data in the table below and calculate AX^2 and $CX \cdot DX$.

Sample Answer:

Position	*AX*	*CX*	*DX*	AX^2	$CX \cdot DX$
1	4.22	6.13	2.90	17.81	17.77
2	3.42	5.40	2.16	11.70	11.66
3	2.45	4.56	1.32	6.00	6.02
4	9.46	11.21	7.98	89.49	89.46

8. What do you notice about the products AX^2 and $CX \cdot DX$?

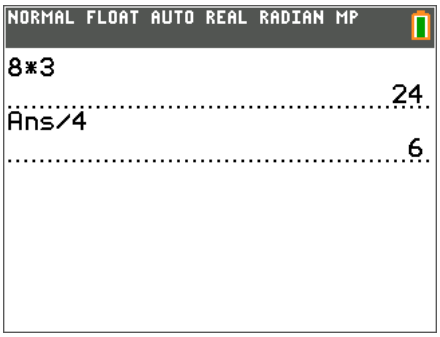
Answer: They are equal.

9. Summarize the relationship between the lengths of the secant segment, its external segment, and the tangent segment if the secant and tangent segments share the same endpoint outside of a circle.

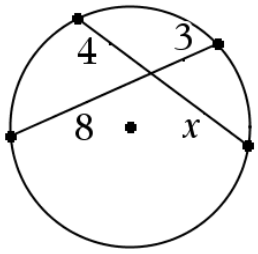
Answer: If a secant segment and a tangent segment share an endpoint outside of a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

Problem 4 – Application of the Product Theorems

Students will be asked to apply what they learned in Problems 1–3 to solve a few problems.

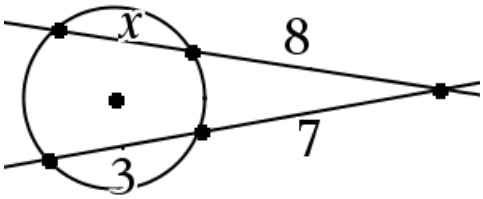


10. Find the value of *x*.



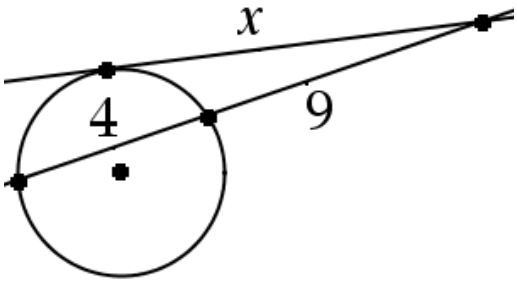
Answer: 6

11. Find the value of *x*.



Answer: $\frac{3}{4}$

12. Find the value of *x*.



Answer: $3\sqrt{13}$ or 10.817