

Swinging Ellipses: Plotting an Ellipse

Any ellipse centered at the origin can be expressed in the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

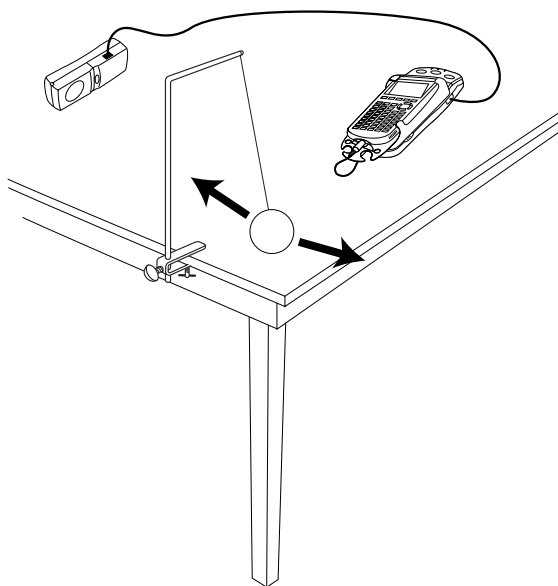
where $\pm a$ and $\pm b$ represent the x - and y -intercepts of the ellipse.

To graph an ellipse on a calculator, the expression above must first be solved for y to obtain

$$y = \pm b \sqrt{1 - \frac{x^2}{a^2}}$$

This equation is entered into the calculator in two parts, one expression for the positive part (upper half of the ellipse) and one for the negative part (lower half of the ellipse).

In this activity you will use the Motion Detector to record the position and velocity of a swinging pendulum. You will find that the plot of velocity versus position is elliptical, and that you can model it with the standard equation of an ellipse.



OBJECTIVES

- Record position and velocity versus time data for a swinging pendulum.
- Plot data as a velocity versus position phase plot.
- Determine an ellipse that fits the phase plot.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator	pendulum bob
EasyData application	string
CBR 2 or Go! Motion and direct calculator cable or Motion Detector and data-collection interface	meter stick

PROCEDURE

1. Set up the Motion Detector and calculator.
 - a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
 - b. Turn on the calculator and connect it to the Motion Detector. (This may require the use of a data-collection interface.)
2. Connect the pendulum bob to an 80 cm long string and suspend it from a support. Arrange the Motion Detector so that the bob swings back and forth in front of it and comes no closer than 50 cm.
3. Set up EasyData for data collection.
 - a. Start the EasyData application, if it is not already running.
 - b. Select **File** from the Main screen, and then select **New** to reset the application.
4. So that the distance data is measured from the pendulum equilibrium position, you need to zero the Motion Detector. Do this by allowing the bob to hang straight downward, with no motion.
 - a. Select **Setup** from the Main screen, and then select **Zero...**
 - b. Confirm that the bob is hanging still, move your hands away from the bob, and select **Zero**.
5. Practice swinging the bob by pulling it back about 15 cm, and then releasing it so that the bob swings in a line directly away from the Motion Detector.
6. With the bob swinging properly, select **Start** to begin data collection. Data collection will run for five seconds.
7. When data collection is complete, a graph of distance versus time will be displayed. Examine the distance versus time graph. The distance versus time graph should be sinusoidal and centered on zero. Check with your teacher if you are not sure whether you need to repeat the data collection.

To repeat data collection, select **Main** and repeat Step 6.
8. To see a graph of velocity versus distance, you need to leave the EasyData application.
 - a. From the distance graph, select **Main**.
 - b. Exit EasyData by selecting **Quit**, and then selecting **OK**.



ANALYSIS

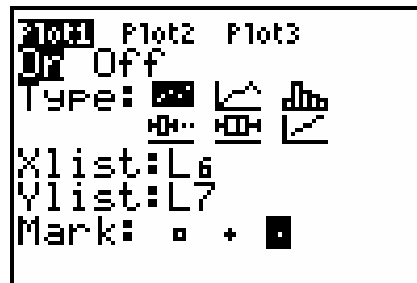
- The graph of velocity versus distance is known as a *phase plot*. The distance data is stored in L6, while the velocity data is in L7. Display the phase plot.

- Press 2nd [STAT PLOT] and press ENTER to select Plot 1.

- Change the Plot1 settings to match the screen shown here. Press ENTER to select any of the settings you change.

- Press ZOOM and then select ZoomStat (use cursor keys to scroll to ZoomStat) to draw a graph with the x and y ranges set to fill the screen with data.

- Press TRACE to determine the coordinates of a point on the graph using the cursor keys.



- The phase plot has multiple x - and y -intercepts. To determine the parameters for the ellipse model, you need to determine some average values of these intercepts. One way to do this is by choosing a typical intercepts among those available.

Move the cursor along the velocity versus distance plot and record a typical positive x -intercept. Then determine a typical negative x -intercept. Do the same for positive and negative y -intercepts. Round these values to 0.001 and record them in the Data Table on the *Data Collection and Analysis* sheet.

- Find the average of the absolute values of the x -intercepts. Record this value as a in the Data Table.

With the result of the average calculation on screen, press STO A ENTER to store the value in the variable A.

- Find the average of the absolute values of the y -intercepts. Record this value as b in the Data Table.

With the result of the average calculation on screen, press STO B ENTER to store the value in the variable B.

- Enter the two halves of the ellipse equation for graphing.

- Press Y= .

- Press CLEAR to clear the Y_1 equation.

- Enter the positive half of the ellipse as $B*\sqrt{(1-X^2/A^2)}$ in the Y_1 line.

- Press 2nd until the icon to the left of Y_1 is blinking. Press ENTER until a bold diagonal line is shown to display the model with a thick line.

- Press 2nd 2nd and press ENTER to move to the next line.

- Press CLEAR to clear the Y_2 equation.

- Enter the negative half of the ellipse as $-B*\sqrt{(1-X^2/A^2)}$ in the Y_2 line.

- Press 2nd until the icon to the left of Y_2 is blinking. Press ENTER until a bold diagonal line is shown to display your model with a thick line.

- Press GRAPH .

\Rightarrow Answer Questions 1-3 on the *Data Collection and Analysis* sheet.

EXTENSION

Show how the algebraic expression $y = \pm b\sqrt{1 - \frac{x^2}{a^2}}$ used in this activity can be derived from the ellipse equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLE

First x -intercept	Second x -intercept	a

First y -intercept	Second y -intercept	b

QUESTIONS

- How well do the curves of the model ellipses and the data points agree?

- At what point in the pendulum's swing is its velocity the largest? Where is the velocity zero? How are these values related to the x - and y -intercepts?

- How would the data change if the amplitude (the largest distance from the middle position) of the pendulum's swing were increased? Would this change affect the values of the constants a and b ? If so, how?