Guppies and Frogs

About the Lesson

In this activity, students will create equations with two or more variables to represent relationships between quantities. They will also graph equations on coordinate axes with labels and scales. As a result, students will:

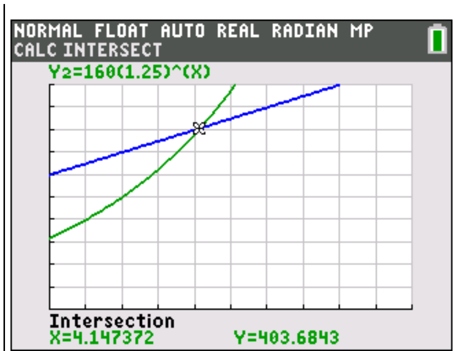
- Write functions that describe a linear and exponential relationship between two quantities.
- Interpret and compare the average rate of change of a function with the average percent change of a function.
- Model real-world scenarios with equations and line graphs on a coordinate plane.
- Analyze the equations and graphs of linear and exponential models to determine properties of the models.

Vocabulary

- population
- linear functions
- exponential growth
- intersection

Teacher Preparation and Notes

- Students should be familiar with how to graph the formula of a function and display a table of its values.
- Students should be able to use the Last Answer feature to build a table of values recursively on the home screen.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-84 Plus family with the latest TI-84 Plus operating system (2.55MP) featuring MathPrint™ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- GuppiesandFrogs _Student.pdf
- GuppiesandFrogs _Student.doc

A large pond contains 300 guppies and 160 frogs. Suppose the guppy population increases by 25 guppies per year. Suppose the frog population increases by 25% per year.

1. What does the future hold for this pond? Make a prediction.

   Answer: Answers will vary. Students should make a specific prediction about how the guppy and frog populations will change over time.

> **Teacher Tip:** Students might predict that the pond will be very crowded. If they predict the frogs will overtake the guppies, ask them to commit to a guess on when this will happen. Some may think that it will take longer than it really does.

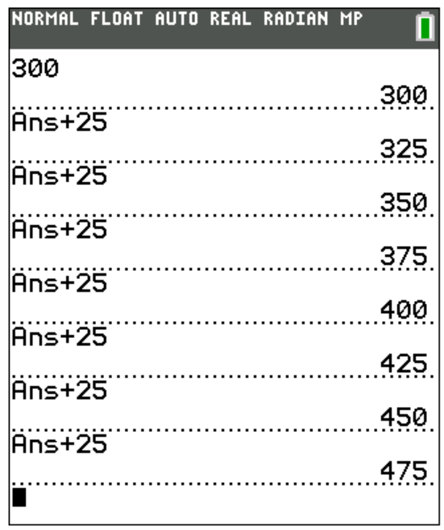
> **Tech Tip:** Students can press + 25 [ENTER], then repeatedly press [ENTER] to use the Last Answer feature to build a table of values recursively on the home screen.

2. What factors might affect the growth of the guppy and frog populations?

   Answer: The food supply and limited size of the pond will affect the guppy and frog populations. Additional factors include predators in the pond, or an increased chance of disease as the population grows and overcrowds the pond.

3. Use the words NOW and NEXT to write a rule showing how the guppy population changes each year. On the home screen of your calculator, simulate 6 years of guppy growth.

   Answer: NEXT = NOW + 25



4. Let the function *G* represent the guppy population at year *t*. Complete the table.

Year, *t*	Guppies, *G*
0	300
1	325
2	350
3	375
4	400
5	425
6	450

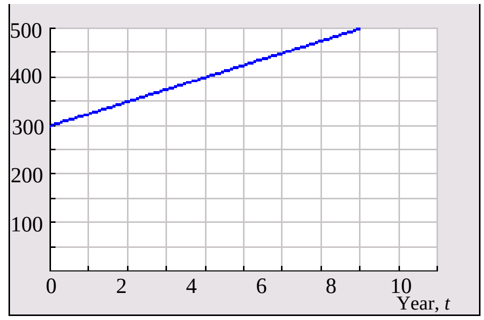
5. How does the guppy population change every 2 years?

   Answer: Every 2 years the population increases by 50 guppies.

> **Teacher Tip:** Have students consider whether the increase in the guppy population is an example of linear or exponential growth. This is a good opportunity to discuss and interpret average rate of change.

6. Graph the data ((t, G)) on the set of axes provided. Then, connect the data points.

   Answer: Students might use the grid to climb by 50 guppies every 2 yrs.



7. Write a formula to model the population growth.

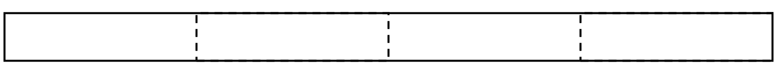
   Answer: $G = 25t + 300$.

Teacher Tip: Once it is established that the model is a linear function, you might ask students to interpret the slope and *y*-intercept in the context of the scenario. (The slope represents how much the population increases each year. Specifically, the population increases by 25 guppies each year. The *y*-intercept is the population of guppies in the pond when $t = 0$. That is, there are initially 300 guppies in the pond.)

8. Use your formula to predict the population at year $t = 10$.

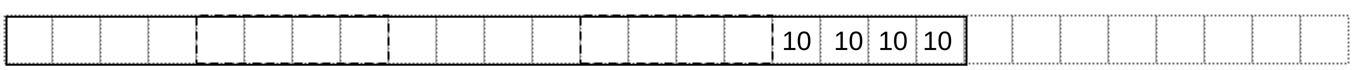
   Answer: At year $t = 10$ there are 550 guppies. $G = (25)(10) + 300 = 550$.

9. The initial population of 160 frogs grows by 25% each year. Suppose the rectangle below represents a population of 160 frogs.



160

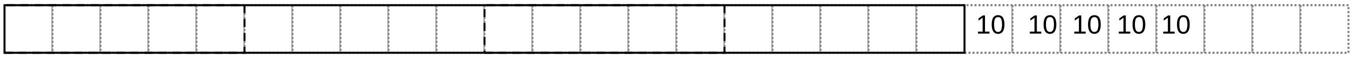
On the grid below, the total rectangle represents the population after the first year of 25% growth.



What is the population of frogs at year $t = 1$?

Answer: At year $t = 1$, there are 160 + 40 = 200 frogs. Each box represents 10 units.

10. On the grid below, the model represents the population after 1 year.
Use the grid to represent the population after 2 years.

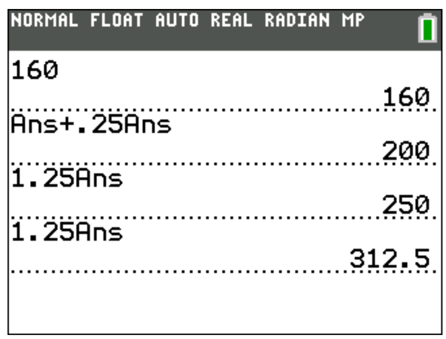


What is the population of frogs at year $t = 2$?
Answer: At year $t = 2$ there are 200 + 50 = 250 frogs.

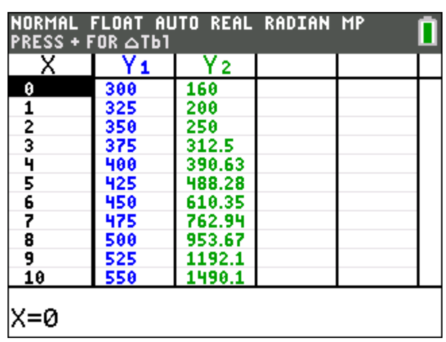
Does the population increase by the same number of frogs each year? Explain your answer.
Answer: No, the population increases by a different number of frogs each year because the total number of frogs changes each year. Since the population increases by 25% each year, it will increase by 40 frogs the first year, 50 frogs the second year, and so forth.

> **Teacher Tip:** The geometric model might help students distinguish that growth by a constant rate of 25 is different than growth by a constant rate of 25 percent, since the length of the total rectangles are not the same. This might help students who could be tempted to model the population of frogs as $F = 160 + 40t$.

11. Use the words NOW and NEXT to write a rule showing how the frog population changes each year. On the home screen of your calculator, simulate 6 years of frog growth.
Answer: NEXT = NOW + 0.25NOW or NEXT = 1.25NOW



12. Interpret the meaning of the population at year $t = 3$ in terms of the context of the situation.
Answer: The model indicates 312.5 frogs, but in the real world, since half of a frog does not make for a living frog, this could be interpreted as 313 frogs.



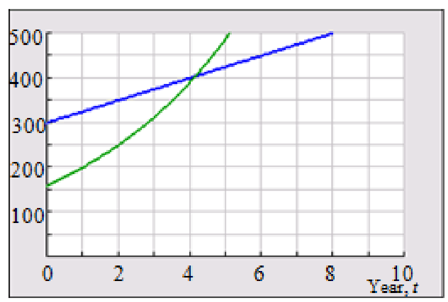
X	Y1	Y2
0	300	160
1	325	200
2	350	250
3	375	312.5
4	400	390.63
5	425	488.28
6	450	610.35
7	475	762.94
8	500	953.67
9	525	1192.1
10	550	1490.1

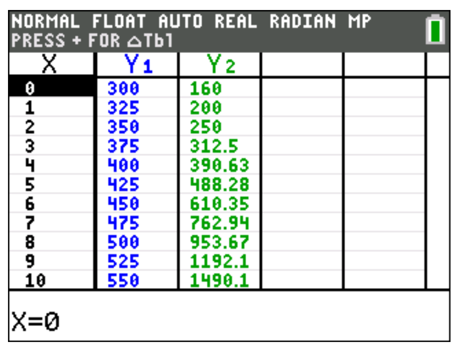
13. Let the function *F* represent the frog population at year *t*. Write a formula that models the population growth of frogs.
Answer: $F = 160(1.25)^t$.

Teacher Tip: Once it is established that the model is an exponential function, you might ask students to interpret the growth factor and *y*-intercept in terms of the scenario. When *t* = 0, *F* = 160, which is the initial amount of frogs. The growth factor of 1.25 means that the total population increases by a factor of 25% each year. Have students consider how this model is different from the linear model

Tech Tip: The table is accessed by pressing [2nd] [GRAPH] for [TABLE].

14. Use the table feature of your calculator to validate your formula. Draw the graph of the frog population on the set of axes on the previous page.
Answer: See the screen shots to the right.

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X=0

Tech Tip: Turn on the GridLine by pressing [2nd][ZOOM] to change the [FORMAT] settings. The GridLine feature is unique to the TI-84 Plus C.

15. In what year does the population of frogs overtake the population of guppies?
Answer: This could be solved graphically or with a table. It is worth discussing that it is not possible to solve the equation $300+25t = 160(1.25)^t$ without technology. Students could report that it occurs at the beginning of year 4. Depending on the level, you could ask students to report the month, which would be about 12 x 0.147 = 1.764 months after the start of the fourth year.

16. List any limitations of these models for population growth.
Answer: In the real world situation, fish and frogs die. The populations cannot grow without bound since the pond, though large, is not of infinite size.