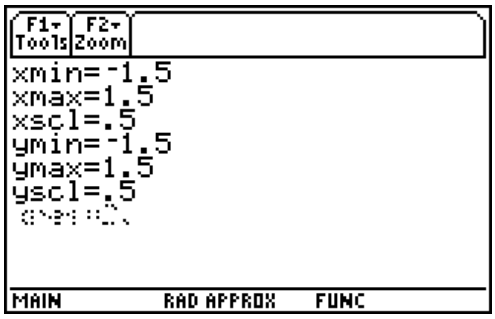
The Mean Value Theorem

The Mean Value Theorem (MVT) is stated as follows:

Let f be differentiable on (a, b) and continuous on [a, b]. Then there is at least one point, c, in (a, b) where $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Your goal for this activity is to find a value of c such that the slope of the tangent line at $f(c)$ is the same as the slope of the secant line that goes through the points (a, $f(a)$) and (b, $f(b)$) for each given function.

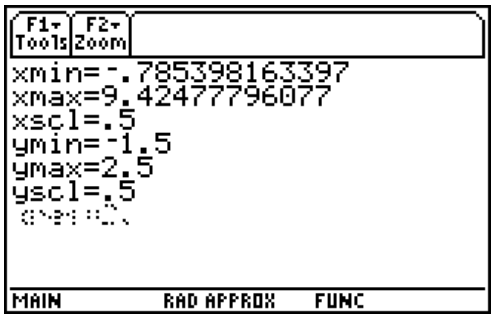
First, let's examine the function $y = x^2$ on the interval (0, 1). Enter the function **y1**(x) = x^2 in your calculator and change the window settings to match those on the right. Start the program *MVT.89p*.



To run the program, enter **mvt()** on the HOME screen. When prompted for a, enter 0 and press ENTER twice. When prompted for b, enter 1 and press ENTER twice. When prompted for c, make a guess for the value of x such that the slope of the tangent at $f(x)$ is the same as the slope of the secant line. Observe the lines, then press ENTER. If the slope of the tangent line does not match the slope of the secant line, select **Yes** when prompted to try another value of c. If the slopes match, then select **No** to quit the program.

1. Are the hypotheses of the MVT met? If so, find the value of c guaranteed to exist by this theorem using the HOME screen. If the hypotheses are not met, then state why.

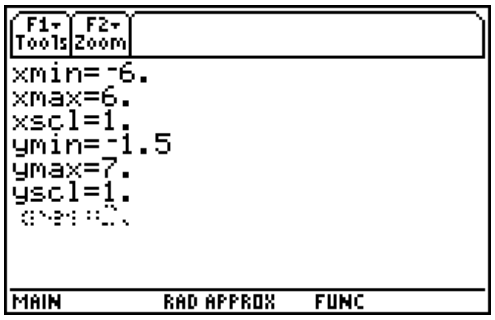
Next, examine the function **y1**(x) = sin x on the interval (0, 2π). Set your window settings to match those on the right. Run the program *MVT* again to try to find a location where the slope of the tangent line is equal to the slope of the secant line.



2. How many values of c are there? Does this violate the MVT?

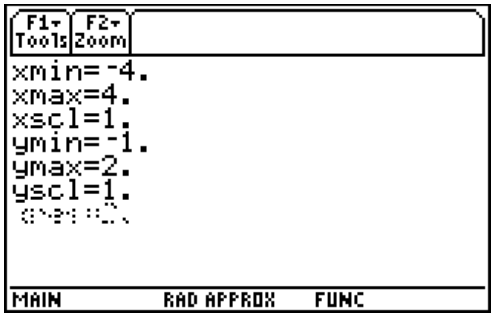
3. If the hypotheses of the MVT are met, then find the value(s) of c guaranteed to exist by this theorem using the HOME screen. If the hypotheses are not met, state why.

Examine the graph of **y1**(x) = $x^{\frac{2}{3}}$ on the interval (–2, 2). Set your window settings to match those on the right. Run the program *MVT* again to try to find a location where the slope of the tangent line is equal to the slope of the secant line.



4. Is there a value of *c* that satisfies the MVT? If the hypotheses of the MVT are met, then find the value(s) of *c* guaranteed to exist by this theorem. If the hypotheses are not met, state why.

Examine the graph of **y1**(x) = $\frac{1}{x^2}$ on the interval (–2, 2). Set your window settings to match those on the right. Run the program *MVT* again to try to find a location where the slope of the tangent line is equal to the slope of the secant line.



5. Is there a value of *c* that satisfies the MVT? If the hypotheses of the MVT are met, then find the value(s) of *c* guaranteed to exist by this theorem. If the hypotheses are not met, state why.

Extension – Application

The Mean Value Theorem can be applied to velocity. For example, if a car averages 60 mph on a road trip, then there must be at least one time during the trip where the instantaneous velocity (the measurement on the speedometer) is 60 mph.

6. Two race horses finish a race as a tie. Show that the two horses had the same velocity at least once during the race.