The Sprinkler and the Lawn
Teacher Notes

Overview
In this activity students will apply the concepts of

- Angle bisector
- Incenter of a triangle
- Percentage

Supplies/Materials
TI-Nspire or TI-Nspire CAS handheld devices
The Sprinkler and the Lawn Student Worksheet

Pre-requisite Knowledge and Curricular Placement
Students should have a basic understanding of angle bisectors, perpendicular lines, areas of triangles and circles, and percentages. Possible curricular placements include high school geometry or a geometry courses for pre-service or in-service teachers at the college level.

Pedagogical Suggestions
This activity is designed to be student centered and requires minimal student experience with a TI-Nspire handheld device. For students with considerable experience navigating the TI-Nspire menus, you may wish to delete the detailed directions and screen captures from the Student Worksheet. Having students record their findings on the Student Worksheet will encourage engagement, retention, and mathematical communication. Anticipated timing for the activity is one class period of 50 minutes.

Problem Overview
The Parks Department has purchased a new sprinkler whose spray rotates to form a perfect circle. The sprinkler is to water the triangular-shaped lawn that is surrounded by sidewalks. Where should the sprinkler be placed inside the triangular-shaped lawn so as to cover as much of the lawn as possible without spraying the sidewalks?

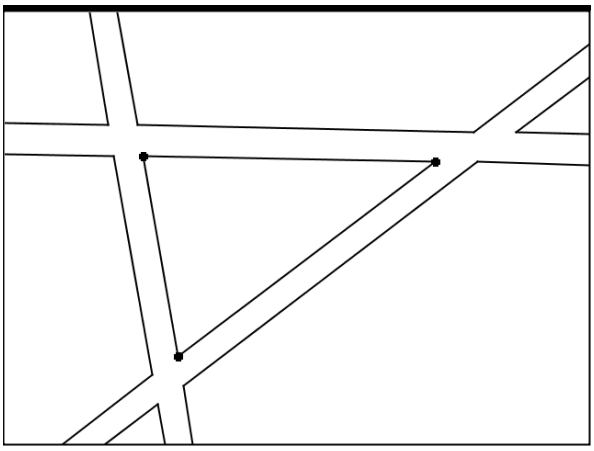
Instructions
Open the file **Sprinkler Incenter_EN.tns** on your TI-Nspire™ handheld device and work through the activity. Use this document as a guide to the activity and to record your answers.

Advance to Page 1.2 by pressing ctrl and the right side of the NavPad.
Examine the triangular-shaped lawn that is surrounded by sidewalks.

1. Explain whether or not you think all of the lawn in the triangle can be watered with a circular sprinkler.

Answers will vary but the preferred response would be no since the water would be confined to the inside of the circle and therefore couldn't reach outside the circle if the sidewalk is to remain dry.

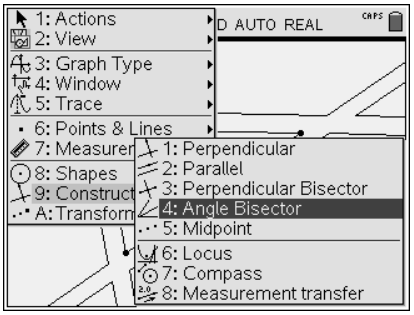
2. Predict where you think the sprinkler should be placed and label it on the diagram.
   Placement will vary but should be inside the Triangle.



3. Draw a sketch on the diagram of the circle that the sprinkler's spray would make.
   A circle should be drawn inside the circle and tangent to each side of the triangle.

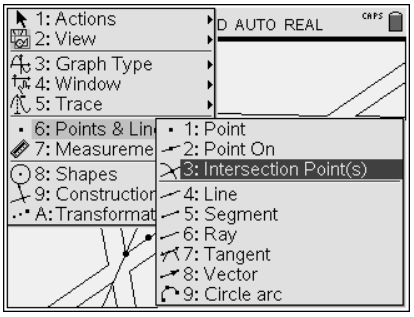
Construct the angle bisectors of two angles of the triangle.

- Select (menu), choose 9: Construction, choose 4: Angle Bisector, and press (enter).
- Click once on one side of the desired angle, click once on the vertex, and click once on the second side of the angle.



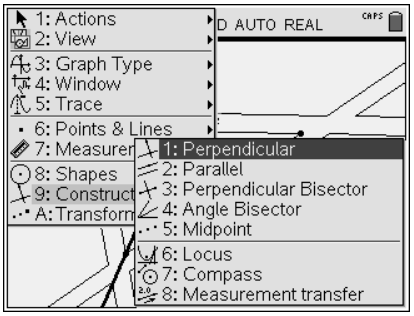
Construct a point at the intersection of any two angle bisectors. This intersection point is called the **incenter** of the triangle.

- Select (menu), choose 6: Points & Lines, choose 3: Intersection Point(s), and press (enter).
- Select each of the two angle bisectors.



Construct a perpendicular line from the **incenter** to one side of the triangle.

- Select (menu), choose 9: Construction, choose 1: Perpendicular, and press (enter).
- Select the incenter and then one side of the triangle.

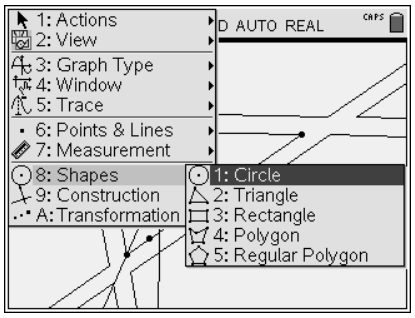


Construct a point at the intersection of the perpendicular line and the side of the triangle.

- Select (menu), choose 6: Points & Lines, choose 3: Intersection Point(s), and press (enter).
- Select the perpendicular line and the triangle side.

Construct a circle centered at the incenter with radius equal in length to the distance between the incenter and the intersection point of the perpendicular line and the triangle side.

- Select (menu), choose 8: Shapes, choose 1: Circle, and press (enter).
- Select the incenter first and then the intersection point of the perpendicular line and the triangle side.

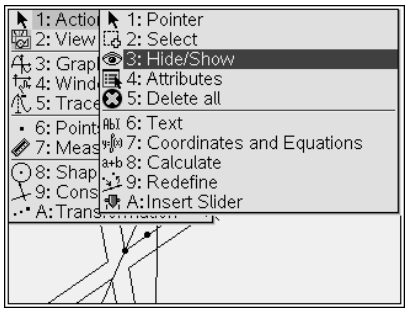


Construct the other two points where the circle and the triangle intersect.

- Select (menu), choose 6: Points & Lines, choose 3: Intersection Point(s), and press (enter).
- Select the circle and the second triangle side.
- Repeat for the circle and the third triangle side.

Hide all lines and any points that are not triangle vertices, incenter, or circle/triangle points of tangency

- Select each line or point and press (enter).
- Press (esc) to exit Hide/Show.



4. Describe the relationship between the triangle and the circle.
   *The circle is inscribed inside the triangle – thus the name **incenter**.*

Move the vertices of the triangle to change its shape and size.

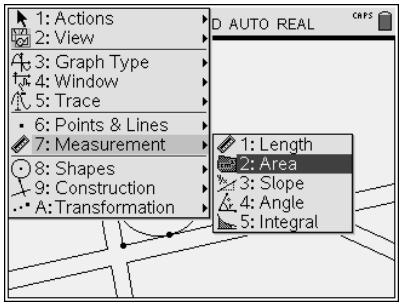
- Cursor to a vertex and press (ctrl) (touchpad) when ✎ shows to activate ✎.
- Move a vertex using the arrow keys.

5. Explain whether or not the relationship between the triangle and the circle is affected.
   The circle remains inscribed inside the triangle no matter where the vertices are moved.

6. If the bisector of the third angle of the triangle were constructed, through which two points would it pass?
   Incenter and the third vertex

Find the area of the circle and the area of the triangular-shaped lawn.

- Select (menu) and chose 7: Measurement, 2: Area.
- When the circle blinks, press (touchpad) or (enter). Repeat for the triangle.
- Press (esc) to exit the Measurement menu.



Store the circle and triangle areas as variables.

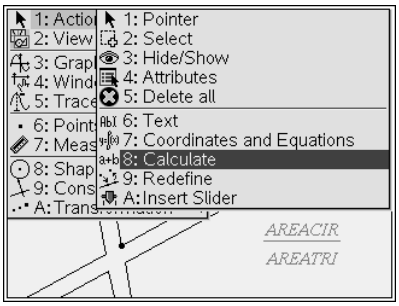
- Point at the circle's area value, select (sto→ var), 1:Store Var, (·) or (enter), and type ***areacir***. Note that variables are not case sensitive.
- Repeat this process to store the triangle area as ***areatri***.

Insert a Text box to represent the ratio of the circle area to the triangle area.

- Select (menu), 1: Actions, 6: Text, and (enter).
- Press (·) or (enter), type **areacir/areatri**, and press (enter).
- Press (esc) to exit the Text menu.

Use the Calculate tool to compute the ratio of the circle area to the triangle area.

- Select (menu), 1: Actions, 8: Calculate, and (enter).
- Select in order the expression for the ratio, the value of the circle area, and the value of the triangle.
- Move the result to a convenient location and double click to display the result.



7. What percent of the triangular-shaped lawn is covered by the circular sprinkler's spray?
   Answers vary but an approximate answer of 55% is likely.

Advance to Page 1.3. Observe how the Automated Data Capture tool dynamically collects measurement data for the inscribed circle and the triangle. Observe that your values for the circle area and triangle area are already listed in cells A1 and B1. The ratio of the areas of the circle and the triangle is listed in C1.

To collect additional data, return to Page 1.2 and drag a triangle vertex around the screen.

Return to your spreadsheet on Page 1.3.

8. Record several of the collected areas in the table below.

Circle Area	Triangle Area	Circle Area/Triangle Area
18.219	32.173	0.566
16.204	29.693	0.546
19.925	36.984	0.538
16.715	30.230	0.553
17.274	33.053	0.522

9. For which type of triangular-shaped lawn would the circular sprinkler spray the largest percentage of area? Justify your answer.
   Equilateral