Name ___________________________________

Number of Regions – An Introduction to the TI-Nspire CAS Student Worksheet

Created by Melissa Sutherland, State University of New York at Geneseo

Below you will find copies of the notes provided on the various pages of the activity in case you prefer not to scroll the windows on the pages of the activity on the TI-Nspire handheld. Please record your work as you go as asked on this sheet and in the activity in the boxed areas below.

1.1

Number of Regions - An Introduction to TI-Nspire CAS

In this activity you will investigate the problem of counting the maximum number of regions determined by n lines in a plane. In doing so, you will be introduced to the various features of the TI-Nspire. Be sure to read all directions.

1.2

Page 1.3 is a **Graphs & Geometry** page in **Plane Geometry View**. If you have used Geometer's Sketchpad or Cabri before, you should find familiar tools under the **menu** key.

On page 1.3, you will add lines one at a time (see Hints below for how to do this), moving them around to create the maximum number of regions.

As you do, count the maximum number of regions you can make using 0, 1, 2, 3 and 4 lines. Record your counts for the maximum number of regions using 1, 2, 3 and 4 lines in the table provided on page 1.4, which is a **Lists & Spreadsheet** page.

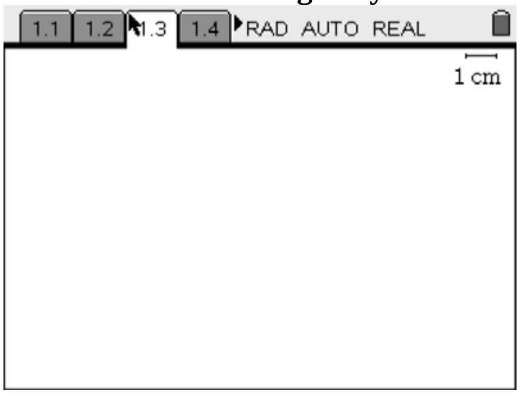
Hints:

1. To add a line, use **menu**, select **Points and Lines**, and choose **Line**. Use the **NavPad**, position the point where you want it, press the **Click button**, and then move the **NavPad** to draw the line. Press the **Click button** to finish the line. The tool will still active and you can add another line. To put the tool away, use **esc**.

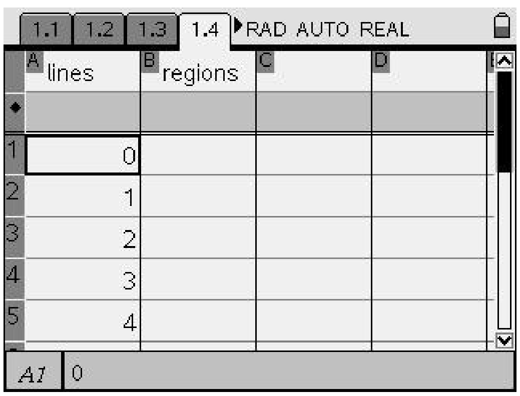
2. To make the lines on the screen longer, grab the "end" of the line by putting the cursor at the "end" of the line and holding down the **Click button** until the hand turns to a fist. Use the **NavPad** to drag the line longer. Use **esc** to release the line.

3. To move a line around the screen, grab (as directed above) the line away from the "ends" and use the **NavPad** to rotate the line or grab the point to move the entire line. Use **esc** to release the line.

1.3
Below record the image of your 4 lines. Number the regions.



1.4
Record the maximum number of regions you determined on page 1.3 for 0, 1, 2, 3, and 4 lines.



	A lines	B regions	C	D
◆				
1	0			
2	1			
3	2			
4	3			
5	4			

A1 0

1.5
We have only collected data for 0, 1, 2, 3, and 4 lines in the plane. While we could probably continue for a few more lines, it quickly becomes clear that we might want a formula that would give us the maximum number of region formed for any number of lines in the plane.
Looking back at your table on page 1.4, is this relationship linear? On the student handout, explain why or why not.

Is this relationship linear? Explain why or why not.

1.6
Let's graph our data points to determine what type of function we have.

Page 1.7 has a split screen containing a **Lists & Spreadsheet** application and a **Data & Statistics** application. You will find that all the data you entered in the table on page 1.4 is now also on page 1.7. To toggle between the parts of the split screen, use the **ctrl** key and the **tab** key to move from the **Lists & Spreadsheet** application on the left of page 1.7 to the **Data & Statistics** application on the right.

To create a scatter plot of your data points in the table on page 1.7, click on the "Click to add variable" on each axes of the graph. Put the variable **lines** on the x-axis and the variable **regions** on the y-axis.

1.7 Split Screen

1.8
You can also plot the data points in a **Graphs & Geometry** application as in page 1.9. To try this, on page 1.9, under the **menu** key, select **Graph Type** and **Scatter Plot**. At the bottom, use **tab** to go from one variable to the other, hitting **enter** and the **NavPad** to select the variable **lines** to store in *x* and the variable **regions** to store in *y*.

1.9 Graphs & Geometry page

1.10
What type of function does it appear might best fit our data points (at least for positive integer values of *x*)?

Doesn't it seem to be quadratic?

To investigate this, we'll graph $f(x) = x^2$ on the same axes as our scatter plot on page 1.9 and drag the graph to hand fit it to our data points. To do this, on page 1.9, under the menu key, select **Graph Type** and **Function**. This should keep the plot of the data points and also allow you to now enter x^2 in $f1(x) =$

We will now drag the graph of $f1(x) = x^2$ to hand fit it to the data points. First, you may need to hit **esc** to get the cursor to move up to the graph.

Next, grab the graph by moving the cursor to the graph until the graph is blinking and holding down the **Click button** until the hand closes. If you grab on the curve away from the vertex, you can stretch and shrink the graph. If you grab on the curve near the vertex, you can drag the curve horizontally and vertically.

Now drag the graph with the **NavPad** until the graph best fits the data points.
Use **esc** to release your grab.

Note that the quadratic expression changes as you drag the curve.

Do this now and write the quadratic function that you get to best fit the data points on the student worksheet.

Hand fit quadratic function
$f1(x) =$

1.11
Now we'll see how to do a quadratic regression using our data points and then we'll compare the regression to our hand fit quadratic. We'll do this in two different applications so you can see how to do regression in both.

First, we'll use the **Lists & Spreadsheet** application. You'll find a copy of your table from page 1.4 on page 1.12. To find the quadratic regression in this application put the cursor in column C, use the **menu** key and select **Statistics**, **Stat Calculations**, and **Quadratic Regression**. Choose **lines** for the X List and **regions** for the Y List and *f2* for where to **Save RegEqn to** and c[] for **1st Result Column**. Use **tab** to go from input to input.

Information from the regression will be put into columns C and D.

The quadratic from the regression is in $f2(x)$. To see its graph, go back to page 1.9's entry line and the **NavPad** to select f2(x) and hit enter to plot $f2(x)$.

Record the quadratic you got from this regression.
$f2(x) =$

1.12 Lists & Spreadsheet page where the regression is done.

1.13
Next we'll show how to add a page to a problem, in this case a **Data & Statistics** page, so we can explore an alternate way to get a quadratic regression of our data points.

To do this, hit the **ctrl** key and then the **Home** key, to get the **Tool** menu. Select **Insert** and **Page**. Choose **Data & Statistics**.

Click on the axes to select **lines** for the x-axis and **regions** for the y-axis.

If your data points do not show up, hit **menu**, **Window/Zoom**, and **Zoom-Data**.

To get the quadratic regression, hit **menu**, **Analyze**, **Regression**, and **Show Quadratic**.

1.14 (It will become 1.15 after you insert the page as directed by page 1.13.)
We want to compare the quadratic you found by hand fitting the graph of x^2 to the data points and the quadratic found using regression. Note that to compare them, we would like to expand the hand fit quadratic.

To do this, use **ctrl** and **tab** to toggle to the **Calculator** application on the right side of this page. Then hit the **catalog** key under the **clear** key, **2**, find **Algebra** and select **expand**. Type $\text{expand}(f1(x), x)$ and hit **enter**. Write this expanded version of $f1(x)$ on the student worksheet.

Expanded form of $f1(x) =$

How well does your $f1(x)$ compare to the regression quadratic, $f2(x)$?

In the screen on the right, type $f2(x)$ in the input line and hit enter to compare.

$f2(x) =$

Describe how well your $f\,1(x)$ compares to the $f\,2(x)$.

1.15 (Will become page 1.16)
Let's now use algebra to find the quadratic function passing through our data points.
Let *x* be the number of lines in the plane.
Let *y* be the maximum number of regions determined by the *x* lines.

We want to find coefficients A, B, and C such that $Ax^2 + Bx + C = y$.

Use three of your (x,y) coordinates to set up a system of three equations and three unknowns.

Record your system on the student worksheet.

System of three equations and three unknowns:

On the next page, enter your augmented matrix. Use **tab** or the **NavPad** to go from entry to entry, entering your values. Hit enter to find the reduced row echelon form of your matrix.

You can now read off the values for A, B, and C. Record the values of A, B, and C on the student worksheet.

How does this quadratic compare to the hand fit quadratic $f1(x)$ and the regression quadratic $f2(x)$ that you found earlier?

1.16 (Will become page 1.17)

Record the matrix you entered.

$$rref\left(\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}\right)$$

Record the reduced row echelon form of your matrix.

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}$$

Record the coefficient solutions to your system below.

***A* =**

***B* =**

***C* =**

Record the quadratic $Ax^2 + Bx + C = y$ obtained by substituting these values in:

How does this quadratic compare to the hand fit quadratic $f1(x)$ and the regression quadratic $f2(x)$ that you found earlier?

1.17 (Will become page 1.18)
You have now had some experience with what the TI-Nspire CAS is capable of and its ability to allow students to look at problems through multiple linked representations.