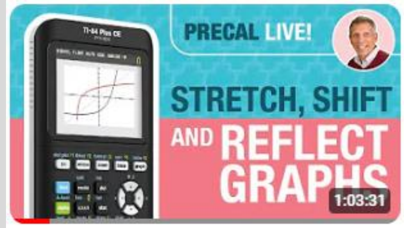
Thursday Night Precalculus
October 26, 2023

In this AP Precalculus Live session, we will explore inverses, transformations, and compositions.



New Functions from Old: Inverses, Transformations, and Compositions

About the Lesson

- This Teacher Notes guide is designed to be used in conjunction with the AP Precalculus Live session and Student Problems document that can be found on-demand: https://www.youtube.com/watch?v=jZ94r3wv6mM&list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI&index=3&t=83s&pp=iAQB
  - *Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the "Live Session" but can be enriched by that resource.*
- This session involves functions and their inverses, transformations of functions, and function composition.
- The transformations include:
  - Additive transformations: horizontal and vertical translations,
  - Multiplicative transformations: horizontal and vertical dilations.
- Students should be able to use the TI-Nspire CX II to explore inverses, compositions, and transformations.
- **Class Discussion**: Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

AP Precalculus Learning Objectives

- 1.12.A: Construct a function that is an additive or multiplicative transformation of another function.
- 2.7.A: Evaluate the composition of two or more functions for given values.
- 2.7.B: Construct a representation of the composition of two or more functions.
- 2.8.B: Determine the inverse of a function on an invertible domain.

Source: AP Precalculus Course and Exam Description, The College Board

Materials:

TI-Nspire document
- New_Functions_from_Old.tns

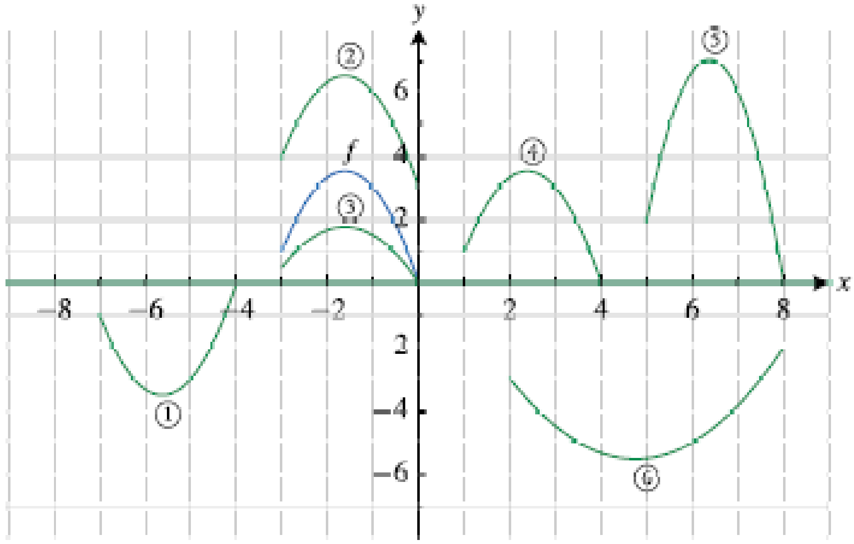
Student document
- Problems_10_26_23V3

Solutions
- Problems_solutions_10_26_23V3

YouTube
- https://www.youtube.com/watch?v=jZ94r3wv6mM&list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI&index=3&t=83s&pp=iAQB
- ***Documents and materials can be downloaded from this site.***

Problem 1. (a) – (f)

The graph of f is given in the figure. Match each equation with its graph and give a reason for each choice.



(a) $y = f(x-4)$ (b) $y = f(x)+3$ (c) $y = 2f(x-8)$

(d) $y = \frac{1}{2}f(x)$ (e) $y = -f(x+4)$ (f) $y = -f\left(\frac{1}{2}(x-8)\right)-2$

Teacher Tip: The graph of $y = f(x)$ is blue in the figure above. The transformations are green.

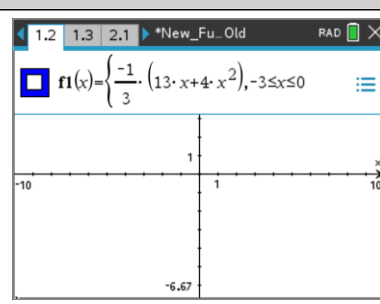
Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Teacher Tip: AP Precalculus refers to these transformations as translations, dilations, and reflections.

Using the TI-Nspire with Problem 1. (a) – (f)

In the video, the demonstration uses the TI-84. The "Using the TI-Nspire" notes in this document provide directions and screen shots that are similar to the use of the TI-84.
All parts of this problem are not discussed in the video. Screen shots of (a) – (f) from the TI-Nspire are shown on the next page.



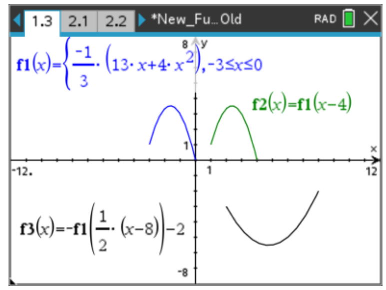
Use the graphing application to graph $y = f(x)$, where

$f(x) = -\frac{1}{3}(13x + 4x^2) | -3 \le x \le 0.$

Technology Tip: To access the vertical bar to restrict the domain, select ctrl =. As an alternative, use the piecewise template with 1 piece from the math template key.

To check the matches of the functions to the graphs, type the new function in f2 using f1 with the various transformations. For instance, for 1 (a), enter $f2(x) = f1(x-4)$ and for 1 (f), enter

$$f3(x) = -f1\left(\frac{1}{2}(x-8)\right) - 2.$$

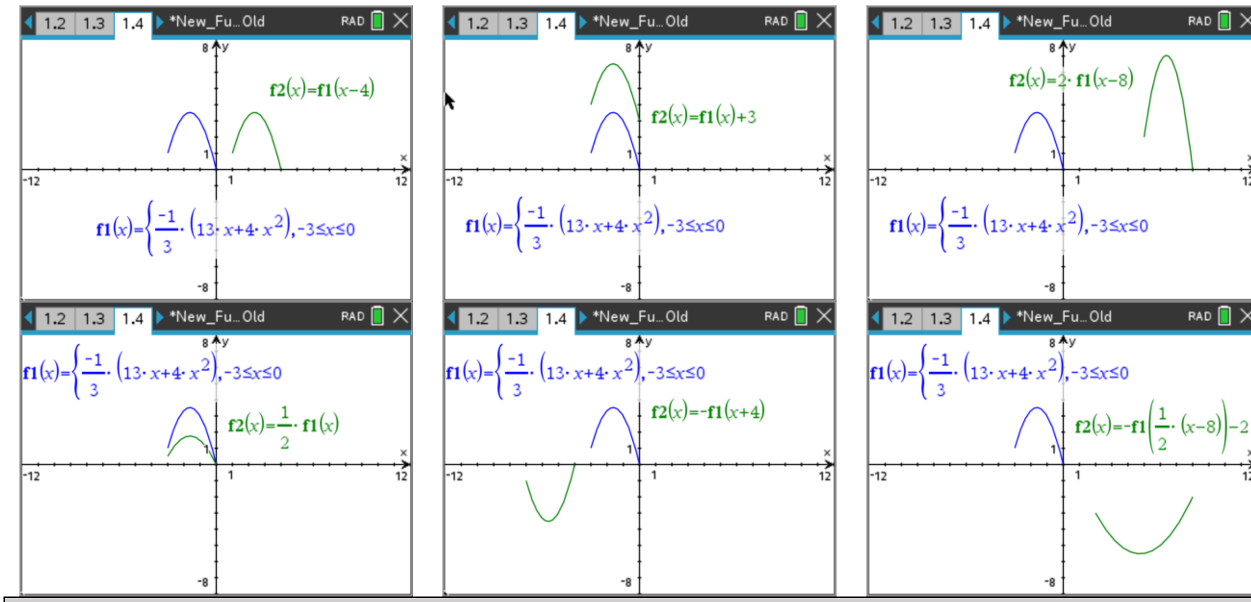


Technology Tip: You may need to adjust the window settings to check some of the graphs given in Problem #1.

Teacher Note: The graph of 2. (f) in the video did not have the –2 vertical translation.

Showing grid lines may also help students visualize the transformations.

Menu – View – Grid – Lined Grid



Problem 2. (a) - (d) (i)

Teacher Note: Before studying precalculus, all students should develop proficiency skills in topics typically found in the Algebra 1-Geometry-Algebra 2 (AGA) content sequence. The sum, difference, product, and quotient of functions are proficiency skills.

Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g, and state their domains.

$f(x)=2^x, \quad g(x)=3^x$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Class Discussion:

What do we notice about the graphs of $(f+g)(x), (f-g)(x)$, and $(f\cdot g)(x)$ as x decreases without bound?

Possible Answers: The graphs of $(f+g)(x)$, $(f-g)(x)$, and $(f\cdot g)(x)$ of $(f\cdot g)(x)$ approach zero as x decreases without bound. We could write these as a limits:
$\lim_{x\to-\infty}((f+g)(x))=0$, $\lim_{x\to-\infty}((f-g)(x))=0$, $\lim_{x\to-\infty}(f\cdot g)(x)=0$.

Class Discussion:

What do we notice about $\left(\frac{f}{g}\right)(x)$ as x increases without bound? Use limits to express your answer.

Possible Answers: The graph of $\left(\frac{f}{g}\right)(x)$ approaches zero as x increases without bound. We would write this as $\lim_{x\to-\infty}\left(\frac{f}{g}\right)(x)=0$.

Teacher Note: Revisit the Class Discussion questions above using the TI-Nspire.

Problem 2. (a) - (d) (ii)

Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g, and state their domains.
$f(x)=\log x, \quad g(x)=\ln x$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Class Discussion:

What do we notice about the graph of $(f-g)(x)=\log x-\ln x$ when x is between 0 and 1?

Possible Answers: The difference would be positive.

Class Discussion:

Why do we have to exclude $x=1$ from the domain of the function $\left(\frac{f}{g}\right)(x)=\frac{\log x}{\ln x}$? Why does the graph appear to be horizontal?

Possible Answers: When $x=1$, the denominator would be zero. For $\left(\frac{f}{g}\right)(x)=\frac{\log x}{\ln x}$ we use the

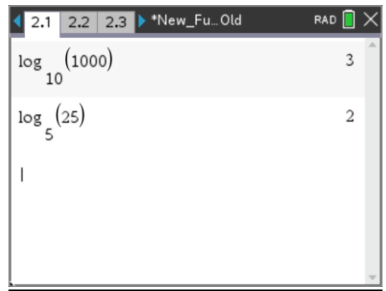
Change of Base rule and $\left(\frac{f}{g}\right)(x) = \log e$ which is a constant.

Teacher Note: Revisit the Class Discussion questions above using the TI-Nspire.

Using the TI-Nspire for Problem 2. (a) - (d) (ii)

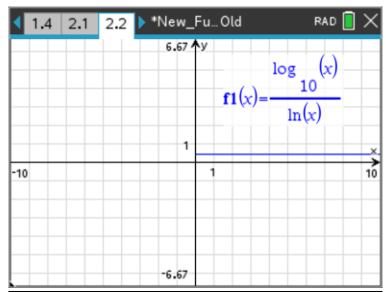
Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g, and state their domains.

$f(x) = \log x, \quad g(x) = \ln x$



Technology Tip: On the calculator application, ctrl 10ˣ allows you to have a logarithmic expression with a base other than 10. If you are working with a common logarithm (base of 10), you have the option of typing 10 in the lower box or moving to the box in parentheses and typing your value or expression. The default for the base is 10.

Also ctrl eˣ allows you to work with the natural logarithmic function.

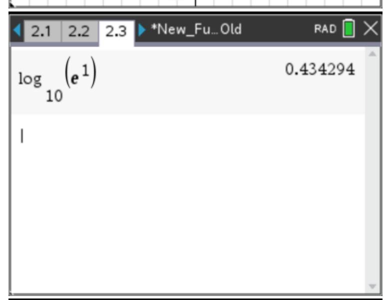


Graph of $\left(\frac{f}{g}\right)(x) = \frac{\log x}{\ln x}$.

In the video, the gridlines are visible.

Technology Tip: To show the gridlines, select menu then View>Grid>Lined Grid

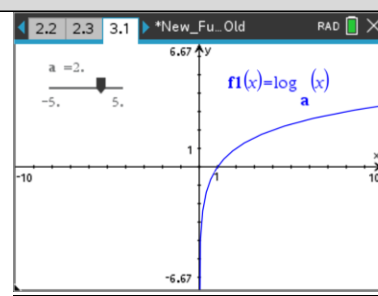
Use Trace to verify the hole at $x=1$. Notice that the y-values are constant. That constant value is $\log(e) = 0.43429$ which is a little less than $\frac{1}{2}$.



Using a Slider on the TI-Nspire

Here is a Youtube Video in the Texas Instruments Education Channel on creating sliders.
https://youtu.be/FM5wpN7t-Ik?si=8JyFjGK3JmpHsRb3

Graph of $f1(x) = \log_a x$ using a slider. By default, the slider starts at $a=1$, and no graph appears until we use the slider to change the base. The settings for the slider may be changed and you may also animate the slider.



Class Discussion:

What point do all of the graphs, log functions with different bases, go through?

Possible Answers:

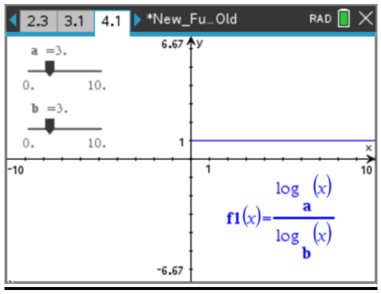
The point (0,1).

Class Discussion:

As you change the value of a, does every graph look like a logarithmic function?

Possible Answers: Every graph of a logarithmic function is a vertical dilation of another logarithmic function. Every logarithmic function is a multiple of another logarithmic function.

Graph of $f1(x)=\dfrac{\log_a x}{\log_b x}$.



Class Discussion:

What do we notice about the graph as the values of A and B change?

Possible Answers: The graph is a constant function for $x>0$ and $x\neq 1$.

Note: *Problems 2. (a) – (d) (iii) and 2. (a) – (d) (iv) are not discussed in the video.*

(iii) Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g, and state their domains.

$f(x)=x^3+2x^2,\quad g(x)=-2x^2-1$

(iv)) Find (a) $f+g$, (b) $f-g$, (c) fg, and (d) f/g, and state their domains.

$f(x)=\sqrt{4-x},\quad g(x)=|x+3|$

Sample Solution:

Refer to the Teacher Solutions Document for the full solutions to these problems.

Problem 3. (a) - (d) (i)

Find (a) $f\circ g$, (b) $g\circ f$, (c) $f\circ f$, and (d) $g\circ g$, and state their domains.

$f(x)=2^x, \quad g(x)=3x$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Class Discussion:

What is happening with the graph of $(f \circ f)(x)$ as x decreases without bound?

Possible Answers: *As x decreases without bound, $(f \circ f)(x)$ approaches 1.*

$\lim_{x \to -\infty} (f \circ f)(x) = 1$

Teacher Note: Revisit the Class Discussion question above using the TI-Nspire.

Problem 3. (a) - (d) (ii)

Find (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$, and state their domains.

$f(x)=2^x, \quad g(x)=\log x$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Graph $f(x)=2^x, g(x)=\log x$, and $(f \circ g)(x)$

Class Discussion:

What is happening with the graph of $(f \circ g)(x)=2^{\log x}$ as x approaches 0 from the right?

Does the graph of $(f \circ g)(x)$ have a hole at $x=0$?

Possible Answers: As x approaches 0 from the right, $(f \circ g)(x)=2^{\log x}$ approaches 0.

$\lim_{x \to 0^+} (f \circ g)(x) = 0$

The graph of $(f \circ g)(x)$ does have a hole at $x=0$.

(d) $f(x)=2^x, g(x)=\log x$, and $(g \circ g)(x)=\log(\log x)$

Class Discussion:

What is happening with the graph of $(g \circ g)(x)$ as x approaches 1 from the right?

Does the graph of $(g \circ g)(x)$ *have a vertical asymptote at* $x=1$?

Possible Answers: The graph of $(g \circ g)(x)=\log(\log x)$ approaches negative infinity as x approaches 1 from the right. The graph has a vertical asymptote $x=1$.

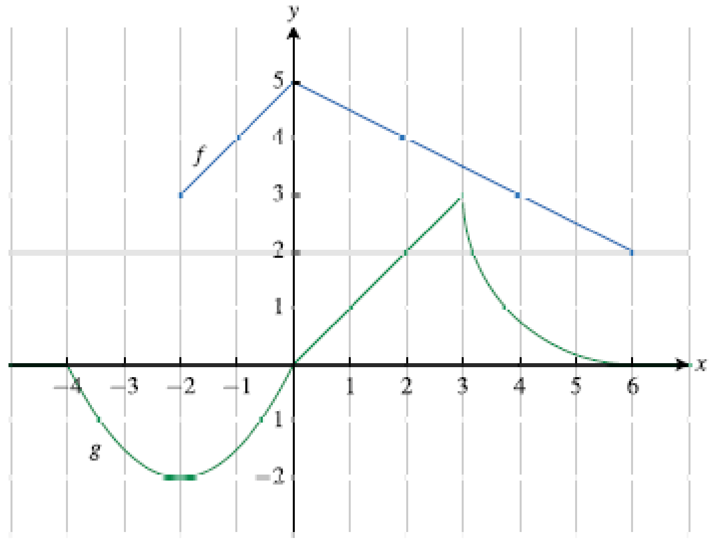
$\lim_{x \to 1^+}(g \circ g)(x) = -\infty$

Teacher Note: Revisit the Class Discussion questions above using the TI-Nspire.

Note: *The following problems, 4 and 5, are not discussed in the video.*

Problem 4. (a) – (i)

Use the graphs of f and g to evaluate each expression or explain why it is undefined.



(a) $f(g(2))$ (b) $g(f(4))$ (c) $(f \circ g)(-2)$

(d) $(g \circ f)(6)$ (e) $(g \circ g)(-2)$ (f) $(f \circ f)(0)$

(g) $(g \circ f)(5)$ (h) $(f \circ g \circ f)(4)$ (i) $(g \circ f \circ g)(-2)$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 5. (a) – (d)

Problem 5. (a)

Find a formula for the inverse of the function.

$f(x)=1+\sqrt{3+7x}$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 5. (b)

Find a formula for the inverse of the function.

$$f(x)=\frac{4x-1}{2x+3}$$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 5. (c)

Find a formula for the inverse of the function.

$$f(x)=\sqrt{1-x^2},\ 0\le x\le 1$$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 5. (d)

Find a formula for the inverse of the function.

$$f(x)=3+\log_2 x, \qquad x>0$$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The graphing application can be used to verify transformations.
- The calculator application can be used to evaluate compositions.
- The graphing application can be used to explore the graphs of compositions.

For more videos from the AP Precalculus Live series, visit our playlist
https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI