Interactive dynamic computation using the *Notes* application

T³ Conference: *LEARN ENERGIZE CONNECT*

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Overview

All of these activities include the use of interactive dynamic computation in the *Notes* application to explore mathematical patterns, make and test conjectures about later iterations of the patterns and make connections between the mathematics encountered in the, seemingly unconnected, activities.

Activity 1: Pascal's triangle

In this activity you will create an interactive model of Pascal's triangle in which the number of lines displayed in controlled by a slider.

Instructions:

1. Open a new document and select *Add Notes*.
2. From the *context menu* (i.e. press [ctrl] [menu]), select *Insert Math Box*.
3. In the Math Box, input: **seq( )**. With the cursor inside the brackets, [menu] > Calculations > Probability > Combinations, [enter]. 'seq(**nCr**( ))' will now be displayed.
4. To display, say, the 5th row of Pascal's triangle, edit the input to seq(nCr**(4,*r*),*r*,0,4)**, [enter]. You will note that the 'seq' command substitutes $r = 0$ to $r = 4$, generating $\left\{ {}^4C_0, {}^4C_1, {}^4C_2, {}^4C_3, {}^4C_4 \right\}$.

Generalise this by replacing the '4' with a parameter, *n*. The value of *n* will be controlled by a slider in the *Geometry* application, as follows.

5. To add a *Geometry* application window to the same page, press [doc▾] (or [ctrl] [⌂on] on older grey 'Clickpad' models) > Page Layout > Custom Split [enter]. Press [+] until the screen is split horizontally. Press ▼ (down arrow) until there is a split of about 5:1, then [enter]. Press [ctrl] [tab] to move between top and bottom windows. Select bottom window, [menu] > *Add Geometry*.
6. To add a slider to the *Geometry* application in the bottom window, [menu] > Actions > Insert Slider [enter]. Move the slider to the middle of the window, and [enter]. Input the symbol *n* in place of *v1*, [enter]. Press [ctrl] [menu] > Settings. In the dialog box, set Minimum: 0, Maximum: 12 and Step Size: 1, [enter] [enter].
7. Generalise step 4 above, as follows. In the top window, in a new Math box, store the value of *n* as *a* (i.e. input *n* [ctrl] [var] *a* [enter]). Edit the command 'seq(nCr(4,*r*),*r*,0,4)' to '**seq(**seq(nCr(***n***,*r*),*r*,0,***a***)**,*n*,0,*a*)'.**
8. Change the value of *n* by adjusting the slider. Observe changes to Pascal's triangle as the value of *n* is altered.

- What are some of the number patterns that you observe in Pascals triangle?
- Set $n = 0$. Add another Math box and input 11^n. Observe the value of 11^n and Pascal's triangle as the value of *n* is systematically increased. What connection can you make? How can you predict the value of 11^n from Pascal's triangle for $n > 5$?
- What are some relationships between the numbers in the columns of this version of Pascal's triangle?

Activity 2: Expansion of $(a + b)^n$

In this activity you will create an interactive page where mathematical patterns in the expansion of $(a+b)^n, n \in Z$ may be observed dynamically, as the value of *n* is varied using a slider.

Instructions:

1. Add a new **problem** to your document, with a *Notes* page, as follows. Press [doc▾] > Insert > Problem > Add Notes.
2. To add a *Geometry* window to the same page, press [doc▾] > Page Layout > Custom Split [enter]. Press [+] until the screen is split horizontally. Press ▼ (down arrow) until there is a split of about 5:1, then [enter]. Press [ctrl][tab] to move between top and bottom windows. Select bottom window, [menu] > *Add Geometry*.
3. To add a slider to the *Geometry* application in the bottom window, [menu] > Actions > Insert Slider [enter]. Move the slider to the middle of the window, and [enter]. Input the symbol *n* in place of *v1*, [enter]. Press [ctrl][menu] > Settings. In the dialog box, set Minimum: 0, Maximum: 20 and Step Size: 1, [enter][enter].
4. Select the *Notes* (top) window and input '$\text{expand}\left((a+b)^n\right)$' in a Math Box, as follows. [menu] > Calculations > Algebra > Expand. Inside the brackets, input $(a+b)^n$. The output will show the expansion for the current value of *n*, as shown on the slider.
5. Systematically change the value of *n* and deduce the mathematical patterns as *n* varies.

- What are some relationships between the mathematical patterns observed in Activities 1 and 2?
- What are some advantages of exploring these patterns in a dynamic *Notes* page, rather than in a static *Calculator* page?

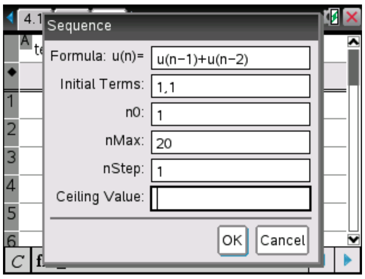
Activity 3: Fibonacci numbers generated in various ways

In this activity you will, firstly, generate the sequence of Fibonacci numbers in the *Lists and Spreadsheet* application. You will then use the connection between the Fibonacci numbers and Pascal's triangle to create an interactive dynamic Fibonacci number generator in a *Notes* page.

3.1 Fibonacci sequence in a spreadsheet using first-principles recursion

1. Add a new **problem** to your document, with a *Lists and Spreadsheets* page, as follows. Press [doc▾] > Insert > Problem > Add Lists and Spreadsheets.
2. Name column A *term* and name column B *fibonacci*. To fill down the *term* number, input the numbers1 and 2, in cells A1 and A2 respectively. From cell A1, press [⇧shift]▼ (down arrow), then [ctrl][☟] then ▼ (down arrow) to fill own to cell A20, then [enter].
3. Press [ctrl][7] to return to the top. To generate the Fibonacci sequence by first-principals recursion, input the number 1 in cells B1 and B2. In cell B3, input the formula =B1+B2 and [enter]. From cell B3, press [ctrl][☟] then ▼ (down arrow) to fill own to cell B20, then [enter]. The formula will automatically update to '=B2+B3' in cell B4, down to '=B18+B19' in cell B20.

3.2 Fibonacci numbers in a spreadsheet using the recurrence relation in *Generate Sequence*



Name column C *fib_2*. From the shaded cell C♦, press [menu] > Data > Generate sequence. In the dialog box, enter the recurrence relation, initial terms and number of terms, as shown.

- Investigate and graph the ratio of consecutive terms.
- Investigate the sum of any 10 consecutive terms, divided by 11.
- How are the Fibonacci numbers related to Pascal's triangle?

3.1 Interactive dynamic Fibonacci numbers in *Notes* application

1. Add a new **problem** to your document, with a *Notes* page. Add a *Geometry* window to the page: press [doc▾] > Page Layout > Custom Split [enter]. Press [+] until the screen is split horizontally. Press ▼ (down arrow) until there is a split of about 5:1, then [enter]. Press [ctrl][tab] to move between top and bottom windows. Select bottom window, [menu] > *Add Geometry*.
2. To add a slider to the *Geometry* application in the bottom window, [menu] > Actions > Insert Slider [enter]. Move the slider to the middle of the window, and [enter]. Input the symbol *n* in place of *v1*, [enter]. Press [ctrl][menu] > Settings. In the dialog box, set Minimum: 0, Maximum: 50 and Step Size: 1, select *Minimise* [enter][enter]. Change document settings to Display Digits: Float 12.
3. From the *Notes* window, press [template key] and select the $\sum_{\square}^{\square}\square$ template. In accordance with the occurrence of Fibonacci numbers in Pascal's triangle, input $\sum_{i=0}^{(n-1)/2}\left(\text{nCr}(n-i-1),i\right)$.

- Verify that the numbers generated using the pattern in Pascal's triangle are identical to the Fibonacci numbers obtained recursively in the spreadsheet.

Activity 4: Continued square roots

In this activity you will generate continued square roots of the form $S_n=\sqrt{a_1+\sqrt{a_2+\sqrt{a_3+...\sqrt{a_n}}}}$, where $a_1\in N$ and $a_1=a_2=a_3=a_n$. You will investigate the values of a_1 for which $\lim_{n\to\infty}\left(S_n\right)$ is an integer. You will make connections to other areas of mathematics, including previous activities.

Instructions:

As a preliminary to the investigation, you will explore the convergence of $\sqrt{2+\sqrt{2+\sqrt{2+...\sqrt{2}}}}$ in the *Calculator* application.

1. Add a new **problem** to your document, with a *Calculator* page. Change document settings to Display digits: Float 6.
2. Input $\sqrt{2.}$ (include the decimal point after the 2 to get a rational approximation), then [enter].
3. Copy the previous input to the entry line and edit it to $\sqrt{2.+\sqrt{2}}$, [enter]. Repeat this procedure for 4 more iterations.

- Does $\sqrt{2+\sqrt{2+\sqrt{2+...\sqrt{2}}}}$ appear to be converging to a value? Is this value an integer?

Next, you will use the *Notes* application to investigate other values of *a* for which the infinitely recursive expression $\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+...}}}}$ converges to an integer value.

4. Add a *Notes* page to the problem: press [ctrl][doc▾] (or [ctrl][I]) > Add Notes.
5. Add a *Geometry* window to the same page: press [doc▾] > Page Layout > Custom Split [enter]. Press [+] until the screen is split horizontally. Press ▼ (down arrow) until there is a split of about 3:1, then [enter]. Press [ctrl][tab] to move between top and bottom windows. Select bottom window, [menu] > *Add Geometry*.
6. Add a slider to the *Geometry* application in the bottom window: [menu] > Actions > Insert Slider [enter]. Move the slider to the middle of the window, and [enter]. Input the symbol *a* in place of *v1*. Press [ctrl][menu] > Settings. In the dialog box, set Minimum: 2, Maximum: 110 and Step Size: 1.

7. Copy the last input in the *Calculator* page and paste it in the *Notes* window. Edit the expression to $1.\times\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a}}}}}}\ldots$ (include 1.× to get a rational approximation).
8. Use the slider to systematically change the value of *a*.

- For what values of *a* does the recursive expression converge to an integer?
- What are some patterns in these values of *a*?
- How can you predict the values of *a* for which convergence to an integer value will occur?
- Explain why these particular values of *a* result in convergence to an integer.
- What is special about the case where *a* = 1?

Activity 5: Factors of $x^n - 1$

In this activity you will look for patterns in the factorisation of $x^n - 1,\ n \in N$.

Instructions:

1. Add a new **problem** to your document, with a *Notes* page. Add a *Geometry* window to the page: press [doc▾] > Page Layout > Custom Split [enter]. Press [+] until the screen is split horizontally. Press ▼ (down arrow) until there is a split of about 5:1, then [enter]. Press [ctrl][tab] to move between top and bottom windows. Select bottom window, [menu] > *Add Geometry*.
2. Add a slider to the *Geometry* application in the bottom window: [menu] > Actions > Insert Slider [enter]. Move the slider to the middle of the window, and [enter]. Input the symbol *n* in place of *v1*, [enter]. Press [ctrl][menu] > Settings. In the dialog box, set Minimum: 0, Maximum: 64 and Step Size: 1, select *Minimise* [enter][enter]. Set the value of *n* to *n* = 1.
3. From the *Notes* window: [menu] > Calculation > Algebra > Factor. Then edit to '$\text{factor}\left(x^n - 1\right)$'.

- Systematically vary the value of *n*. What patterns can you observe?
- Predict the factors when *n* = 71, *n* = 81, *n* = 128. Check your predictions by editing the value of *n* on the slider.
- What are some advantages of carrying out this exploration in *Notes*, rather than *Calculator*?