



Science Objectives

- Students will explore a simulation of static forces.
- Student will find vector components.
- Students will find resultant and equilibrant force vectors.

Vocabulary

- compression
- equilibrant
- resultant
- statics
- tension
- vector component

About the Lesson

- This lesson simulates a 3-way tug o' war where students supply a third equilibrant force to balance two other forces. This will take place on a compass direction system.
- As a result, students will:
 - Compare Cartesian (x,y) and compass (N,E) direction systems.
 - Determine vector components in a compass system.
 - Find resultant and equilibrant vectors.
 - Solve a set of 3-way tug o' war problems by determining and setting an equilibrant to two other forces, and then checking their answer.

TI-Nspire™ Navigator™


- Send out the *3-Way_Tug_o_War.tns* file.
- Monitor student progress using Screen Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- *3-Way_Tug_o_War.tns* documents
- TI-Nspire™ Technology







TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use minimized sliders
- Drag points
- Use ScratchPad  for calculations.

Tech Tips:

Slider use:

- Go to a page with sliders
-  until slider is selected,
-  to activate slider
- Use arrow keys to change values or just type value
-   to release slider

Lesson Materials:

Student Activity

- 3-Way_Tug_o_War_Student.doc
- 3-Way_Tug_o_War_Student.pdf

TI-Nspire document

- 3-Way_Tug_o_War.tns



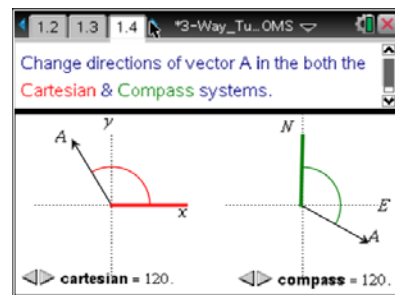
Discussion Points and Possible Answers

Teaching Tip: Spend a few moments to set the context by tying three cords to a small ring and have three students pull in different directions to keep the ring over a defined mark on the table. Discuss the concept of magnitude of force and directions and how they all can change if one changes.

Tech Tip: The default angle mode is radians. Set devices to work in degrees by pressing **[on]** > **Settings** > **Settings** > **Graphs and Geometry** and set both Graphing Angle and Geometry Angle to Degree.

Move to page 1.4.

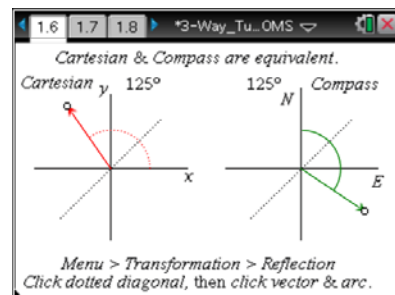
1. In order to see angle measurements in Degrees, press **[on]** > **Settings** > **Settings** > **Graphs and Geometry** and set both Graphing Angle and Geometry Angle to Degree.
2. Students can change direction of the vectors in both systems using the appropriate slider. Have students explore different directions in one system and predict the direction in the other. They should then set the direction with the other slider.



Tech Tip: If students tab too often, the Entry Line may appear at the bottom of the screen. Press **[ctrl]** **[G]** to hide it. To hide the chevron, move the cursor over the chevron and press **[ctrl]** **[menu]** > **Hide Chevron**.

Move to page 1.6.

3. Reflect both the vector and the angle arc across the diagonal using **Menu** > **Transformation** > **Reflection**. This requires separate reflections for vector and arc. Drag vector tips to change initial direction. Compare Cartesian and compass to show equivalence.



Teaching Tip: Show the equivalence of Cartesian and compass systems by taking a blank piece of paper and drawing axes with a dark marker. Label the axes (N,E) or (x,y). Hold the paper up by the 1st and 3rd quadrant corners for the students to see and flip the paper across the corners. Point out that North goes to x (or x goes to North), and East and y also interchange.



Move to page 1.7.

To find a component of a vector we can use simple trigonometric functions. For vector A with magnitude A units and direction Θ , the components can be called A_x , A_y in the Cartesian system or A_N , A_E in the compass system.

A_x or $A_N = A \cos \Theta$ and A_y or $A_E = A \sin \Theta$. Once you understand this, you need to remember to use the correct angle for the direction system you are using.

N.B.: Do not use reference angles! Use the angle (up to 360°) in the direction system as given. Practice as needed.

Move to page 1.8.

Study the example question, diagram, and solution on page 1.8 before trying examples on pages 1.9. Ensure that students understand the direction convention and the component notation, and can calculate the component values correctly.

Move to page 1.9.

Q2. A plane is flying with a speed of 220 km/h at a compass heading of 140° . What are the velocity vector components (A_N and A_E)?

Answer: $A_N = -169$ km/h (the minus “-“ means “South”); $A_E = 141$ km/h

Move to page 1.11

Students can drag vectors A and B to change them and have as many practice problems as they wish. Have them calculate and add the components until they are competent and confident.

Move to page 1.12. (Direct students who are having difficulties to a diagram on page 4.1 of the .tns file to help answer the following question.)

Q3. Two ropes are pulling on the same tree stump. A represents a force of 500 units pulling in a direction of 30° . B represents a force of 200 units pulling in a direction of 100° . What are the components of the resultant vector along the N and E directions? (Use ScratchPad for your calculations.)

Answer: $R_N = 398$ units; $R_E = 447$ units

Problem 2: Force Vectors in Balance: The Equilibrant

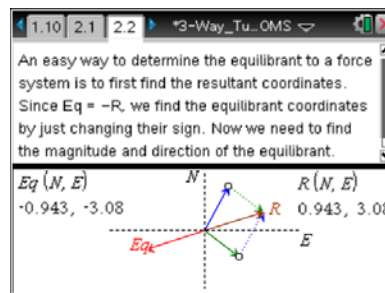
Move to page 2.1.

Forces are an important application of vectors. In the study of statics, many forces can be acting simultaneously, but they all balance each other. To maintain that balance, we sometimes need to apply a balancing force called the equilibrant (E_q). The equilibrant is the force that is equal in magnitude but opposite in direction to the resultant of all the other forces: $E_q = -R$.



Move to page 2.2.

An easy way to determine the equilibrant to a force system is to first find the coordinates for the resultant. In other words, express the equilibrant in terms of vector components along the (x,y) or along the (N,E) axes. Since $E_q = -R$, you can then find the equilibrant coordinates by just changing their signs.



4. Drag vector tips to see how resultant and equilibrant coordinates relate. Coordinates of the equilibrant are the negative of the coordinates of the resultant. This is because the equilibrant is equal in magnitude but opposite in direction to the resultant.

Now you need to find the magnitude and direction of the equilibrant. Use the Pythagorean theorem to find the magnitude of the equilibrant and trigonometry (\tan^{-1}) to find its direction. Provide extra practice as needed, based on student experience.

Finding an Equilibrant Force summary:

- Find component coordinates for each force.
- Add all component coordinates to find resultant coordinates in each direction.
- Change the sign of resultant coordinates to get equilibrant coordinates in each direction.
- Use the Pythagorean theorem to find the magnitude of the equilibrant (and resultant).
- Use the *E* and *N* equilibrant coordinates to find the angle of the equilibrant: $\Theta = \tan^{-1} (E/N)$.
- Reference the angle in a diagram to find correct quadrant and adjust for direction.

A table as shown below may be helpful:

$A_{\text{north}} =$	$A \cos(\Theta_A) =$	
$A_{\text{east}} =$	$A \sin(\Theta_A) =$	
$B_{\text{north}} \square$	$B \cos(\Theta_B) =$	
$B_{\text{east}} =$	$B \sin(\Theta_B) =$	
$E_{q_{\text{north}}} =$	$A_{\text{north}} + B_{\text{north}} =$	
$E_{q_{\text{east}}} =$	$B_{\text{north}} + B_{\text{east}} =$	
$E_{q \text{ mag}} =$	$\text{Sq rt} (E_{q_{\text{north}}}^2 + E_{q_{\text{east}}}^2) =$	*
$E_{q \text{ dir}} =$	$\text{Tan}^{-1}(E_{q_{\text{east}}} / E_{q_{\text{north}}}) + 360 =$	*

* $E_{q \text{ mag}}$ and $E_{q \text{ dir}}$ together specify the equilibrant.

Teaching Tip: A review of the information above with further practice might be needed by less experienced students. A separate print copy of the above table will also be helpful for many students.

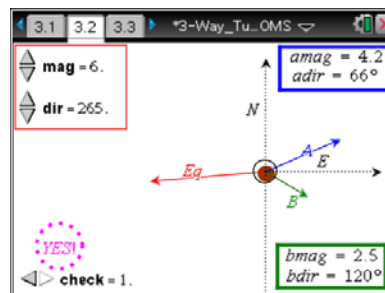
Teaching Tip: A working spreadsheet to solve the problem is provided as an aid for some students, or to help the teacher more quickly find the solution. See page 4.2 of the .tns file.



Move to page 3.2.

- When you have your materials ready, move to page 3.2. Click the START button to see the forces applied by *A* and *B*. Do your calculations, then provide your equilibrant force using the sliders to set your magnitude (**mag**) and direction (**dir**). When you are ready, set the CHECK slider to 1 to check your work.

Click START to try again.



Tech Tip: Once a slider is activated (with) students can use the arrow keys to change slider values, or type a value directly.

Tech Tip: When students click START, the system generates random values for vectors *A* and *B* up to a maximum of 5 units of magnitude and directions within the respective quadrants. When **check** slider is 0, no result is visible. Students set their equilibrant answer with the **mag** and **dir** sliders. When they set check slider to 1, the result (brown circle) appears. If it is within a small tolerance, the YES stamp appears indicating success. Students can manipulate the equilibrant with the **check** set to 1 to find the correct answer.

Extension

Move to page 3.3.

Now imagine solving a 4, 5 or 6-way tug o' war!

If you look at bridges or towers you will find struts, beams, and cables all in balance. Some members may be under tension (pulling); others may be under compression (pushing). An engineer calculated the range of forces in each of these so that the components would have the appropriate strength to be safe. Investigate a simple structure by sketching it, researching some data, and making some reasonable assumptions. Then calculate the forces needed to balance at a joint. Don't forget that weight of materials is a force.

For a complete analysis, it is likely that torques would also need to be considered. That is beyond the scope of this activity.



TI-Nspire Navigator Opportunities

Use TI-Nspire Navigator Screen Capture to monitor student. During the lesson, Screen Capture can be used to illustrate example student work as a focus for discussing concepts or difficulties.

Wrap Up

Have students explain and model the concept of force vector components, resultant, and equilibrant using cords tied to a ring.

Assessment

- Formative assessment and practice occurs with the questions embedded in the .tns file. Further practice is encouraged as needed and assigned by the teacher.
- Summative assessment may consist of completing all work for a 3-Way Tug o' War as generated on page 3.2 of the .tns file, or as assigned by the teacher.