TI-92+ Cabri Geometry for the "Not-Necessarily Advanced" User

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Introduction to *TI-92+ Geometry*

The first hour that you use *TI-92+ Geometry* is the most difficult! Persevere! Once you reach a plateau of skills, the more you use the software, the more you will understand its organization and operation. The purpose of this lesson is to help you reach that first plateau and begin to feel more comfortable with the tools and operations of *TI-92+ Geometry*. More importantly, the purpose of this introduction is to begin building your pedagogical skills as a teacher who can model instructional methods that utilize current dynamic geometry software in the mathematics classroom.
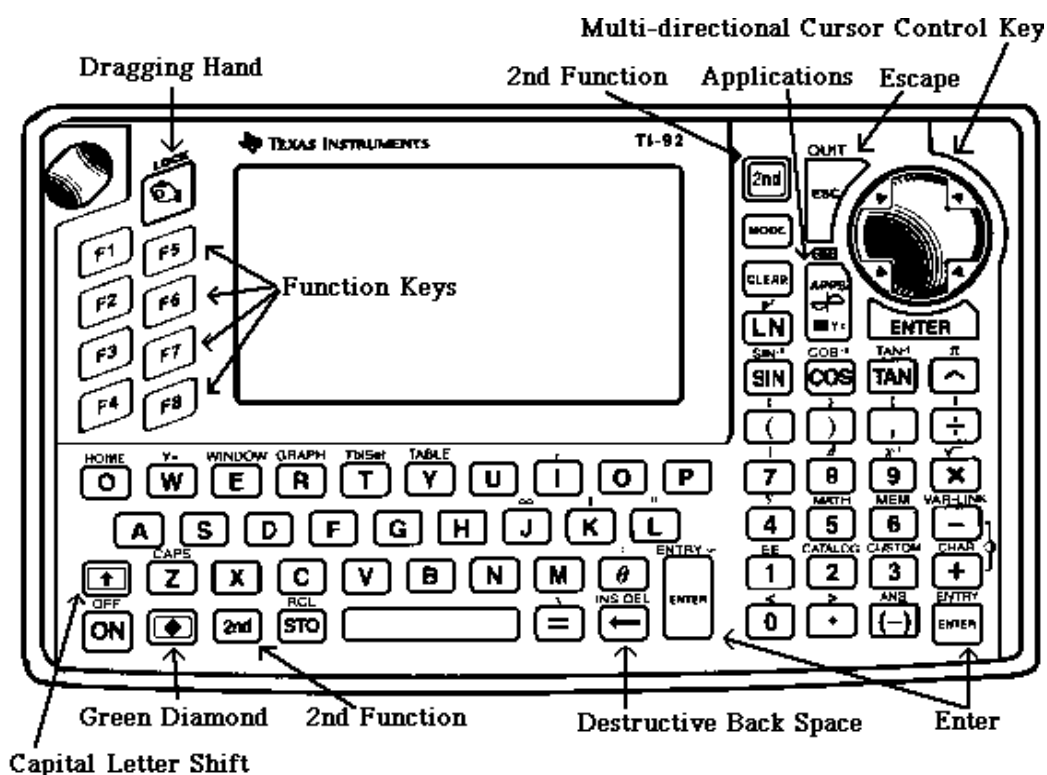
Important Keys for Geometry:

Figure 1: TI-92+ keyboard with the important *TI-92+ Geometry* keys noted

Multi-directional cursor control key: This key is the only tool used to move the cursor and `Pointer` on the drawing screen of *TI-92+ Geometry*. This key will be called the `cursor` key. In *TI-92+ Geometry*, this key moves the cursor in 8 directions as indicated in Figure 2. In all the other TI-92+ Applications, this key moves the cursor in 4 directions only.
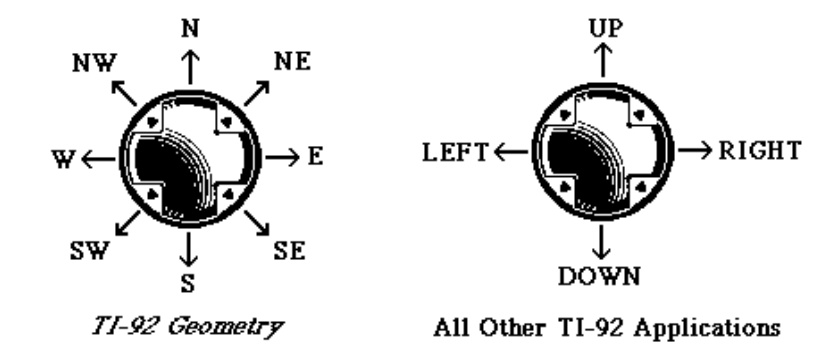

Figure 2: Cursor control operations in *TI-92+ Geometry* and all other TI-92+ Applications

ENTER : This key is used to select items with any of the *TI-92+ Geometry* tools. All three of these keys perform the same operation.

APPS : This key accesses a menu of different TI-92+ applications, including `8:Geometry` on TI-92+s with "older" code or `1:FlashApps` then `ENTER` on TI-92+s with "newer" code.

ESC : This key is used to get you "out" of nearly anything such as a menu or a tool. The most frequent use of this key in *TI-92+ Geometry* is to activate the **Pointer** arrow for dragging and selecting items in the drawing.

2nd : Use this key to access functions listed in yellow above the main keys of the TI-92+.

·· : This is **not** a cursor control key; use this key as a **destructive back-space** when dealing with text and the **delete** key when a geometric object has been **selected**.

⌂ : This is **not** a cursor control key; this is the **alphabetic shift** key used to access capital letters.

◆ : This is called the **green diamond** key; use this key to access keyboard functions written in green. This key also shifts control of the **cursor** key from the **screen cursor** to the `Comment`, `Label`, or `Numerical Edit` cursor.

F1 through **F8** : These keys display the main tool menus or tool boxes for *TI-92+ Geometry*.

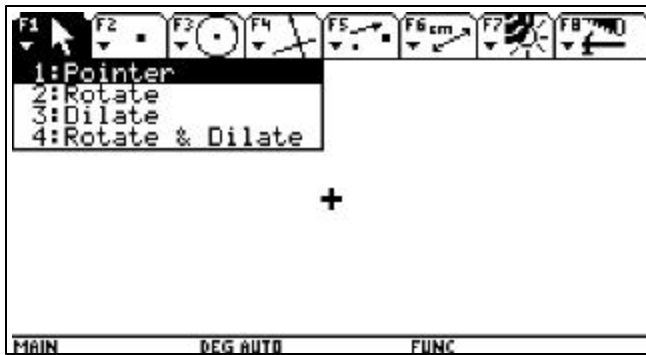Figures 3 through 10 show the eight *TI-92+ Geometry* main menus.

Figure 3: **F1: Pointer** menu
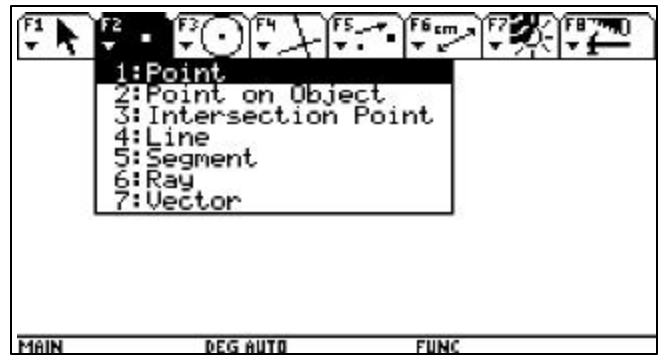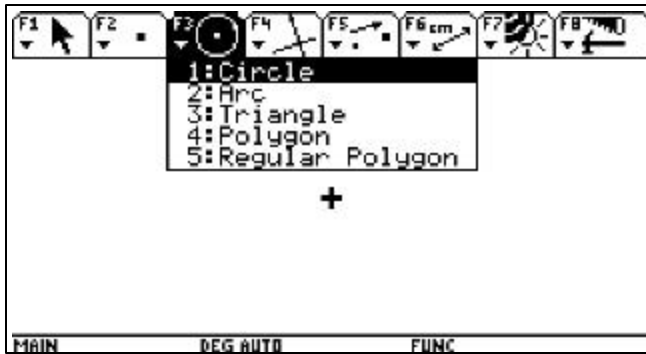

Figure 4: **F2: Points and Straight Objects** menu

Figure 5: **F3: Geometric Figures** menu
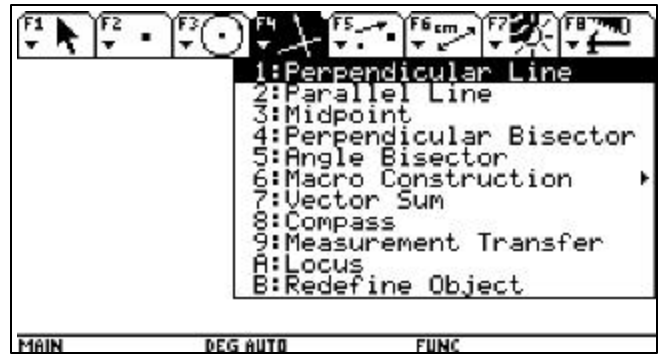

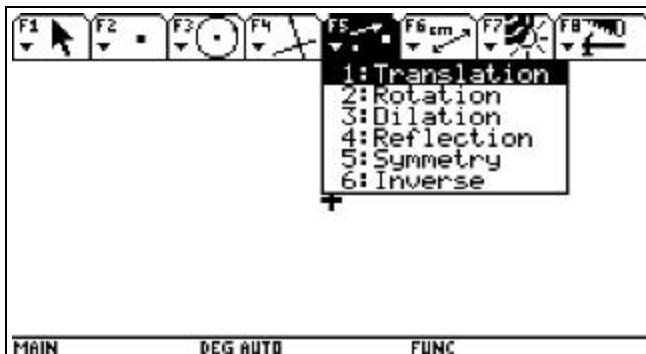Figure 6: **F4: Construction Tools** menu

Figure 7: **F5: Transformations** menu
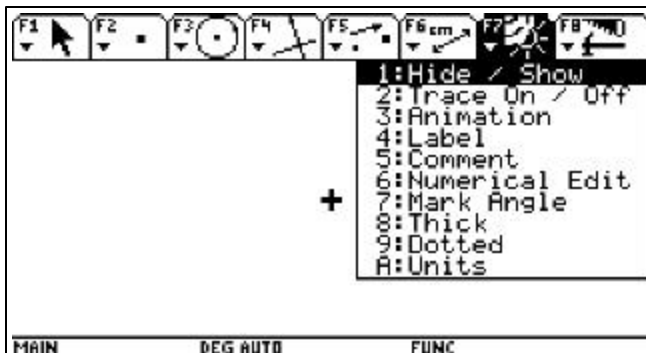

Figure 8: **F6: Measurement** menu

Figure 9: **F7: Display Tools** menu

Figure 10: **F8: File Operations** menu

Opening a New Geometry Session

1. On "older" TI-92+'s, press the $\boxed{\text{APPS}}$ key to access the Applications menu and select 8:Geometry (see Figure 11). A sub-menu appears when you press the $\boxed{8}$ key or when you move the cursor down to the 8:Geometry option and press ENTER. On newer ones, press $\boxed{\blacklozenge}$ $\boxed{\text{APPS}}$ followed by ENTER (see Figure 12) to access the same sub-menu.

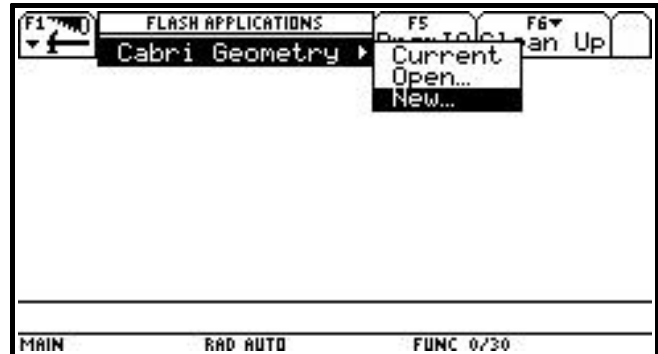Figure 11: **Geometry** sub-menu (older version)   Figure 12: **Geometry** sub-menu (newer version)

   a)   **Current**: This option automatically opens the current **Geometry** file (also referred to as a figure) in the memory of the TI-92+; this is the last file you worked on; there is always a current or last file to work on unless this is the first time you access **Geometry** on a new TI-92+ or the memory of your machine has been reset.

   b)   **Open**: This option allows you to open an existing **Geometry** file saved in the memory of the TI-92+ other than the **Current** file.

   c)   **New**: This option allows you to open a new **Geometry** file (or figure).

2. If you choose **New**, a file dialogue box will appear as shown in Figure 13. The word **Variable** seen in this menu stands for the **file name** or **name of the Geometry figure**. Move your cursor down into the window adjacent to the word **Variable** using the **cursor** key and type a name for your new figure. The name may be up to 8 characters; numbers may be used but not as the first character in the name; no punctuation marks like periods, hyphens, or slashes may be used in a valid name. Press ENTER twice to open the new geometry file (figure).
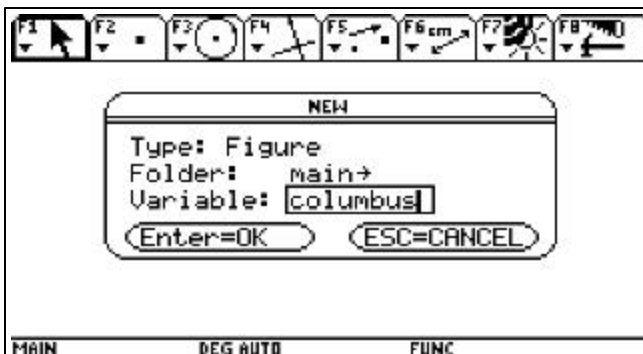

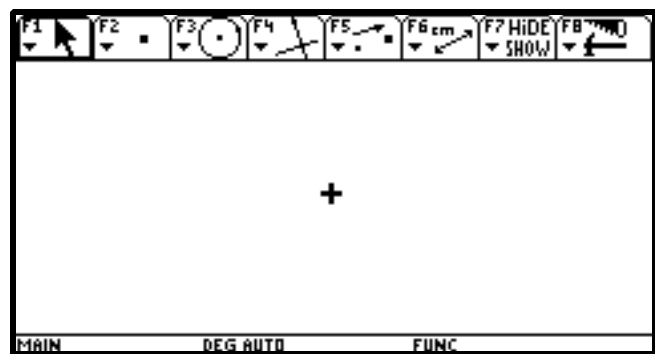Figure 13: Naming a **New Geometry** file   Figure 14: A **New Geometry** figure

3. Figure 14 shows the drawing area of *TI-92 Geometry* with the cursor on the screen. Place your thumb on the **cursor** key and "roll" it around. As the cursor moves around the screen, notice that the speed of the cursor changes after the first few moments. When you first press the **cursor** key, the cursor moves very slowly for the first few pixels. Then, the cursor begins to move quickly across the screen. Practice moving the cursor around the screen.

    To move the cursor in small steps, you will need to learn to "bump" the cursor by tapping the **cursor** key in the direction you wish to go. Actually, when you bump the cursor, you are repeatedly activating the slow moving portion of the cursor's motion algorithm.

Some Things to Know About *TI-92+ Geometry:*

1. Objects can only be moved based on where you have defined them. That means that if you create a point on a line, it can only be dragged up and down that line. If you want to use the point to move the line from side to side, then define the point first, and draw the line through it. (Note: If you hold down the hand key, all the independent/movable points will flash.)

2. The ESC key is a quick way to get the **pointer** tool, used for selecting and dragging objects. When you point at an object with the **pointer** tool and press ENTER, the object you selected will flash.

3. You can LABEL objects without using the 4:LABEL command from the F7 menu by simply type a letter immediately after creating a point, line, or circle.

4. You can add or remove digits of precision in numerical values by selecting the value using the **pointer** tool and then use the + or – keys.

5. You can delete objects as follows:

    a. To undo the last operation only, select D:UNDO from the F8 menu or press ♦Z;

    b. To delete one object, select it with the **pointer** tool, and use the ⌫ key.

    c. To clear the screen without saving, select 8:Clear All from the F8 menu.

    *Note: If you delete an object, any object or measurement that was dependent on the deleted object will also disappear.

6. As the number of objects on your screen increases, the time it takes the TI-92+ to respond also increases. Consider hiding some object using 1:Hide/Show from the F7 Menu

7. If you use ♦ON to turn off the TI-92+ rather than 2nd ON, the TI-92+ will return to the same figure (file) in *TI-92+ Geometry* when turned back on.

Activity: Right Triangles and Circles(!?)

Construct a segment (not too big, not too small) near the center of the screen. Then construct a ray — select 6:Ray from the F2 menu — that does not coincide with the segment but starts at one of its endpoints (See Figure 15). Next, construct a line perpendicular to the ray — select 1:Perpendicular Line from the F4 menu — containing the segment's other endpoint (See Figure 16).
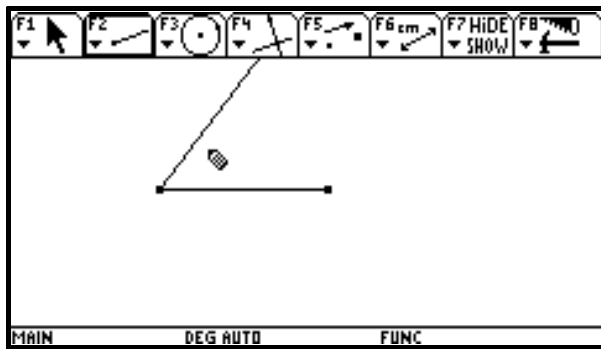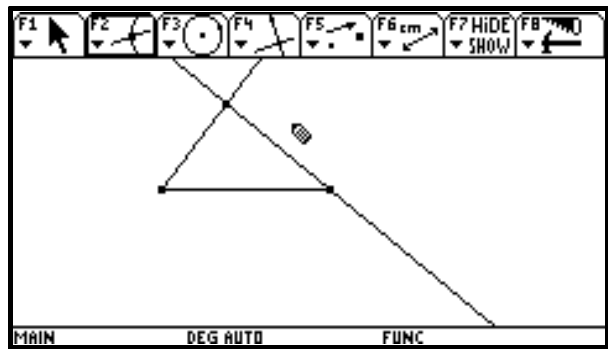
Figure 15

Figure 16

Locate the intersection point of the line and ray, hide the line (but not the ray), and construct the "missing" segment. You now have constructed a right triangle. Select 2:Trace On/Off from the F7 menu and highlight the vertex you just constructed. Now, drag the ray around the screen (See Figure 17).

PROBLEM: What figure is being traced out as you drag the ray across the screen? Why do you think this is happening? Can you explain this phenomenon using properties you know about right triangles? about circles?
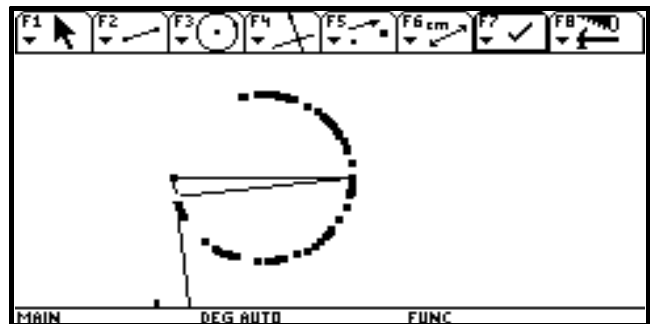

Figure 17

This problem is a twist on the theorem that an angle inscribed in a semicircle is a right angle.

Additional "Exploration-Type" Problems

1. What are some other ways you can construct a parallelogram? Describe as many different ways as you can think of using the different capabilities of the calculator and indicate (discuss) any properties of parallelograms you used for your construction.

    Here's one possibility. First, select 8:Clear All from the F8 menu to get a fresh screen. Draw two segments with a common endpoint. Now, select 8:Compass from the F4 menu to create two circle (see Figure 18).
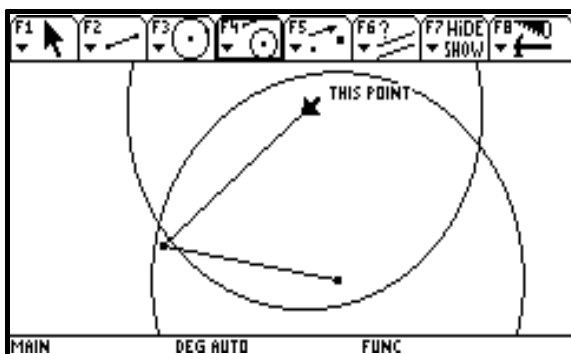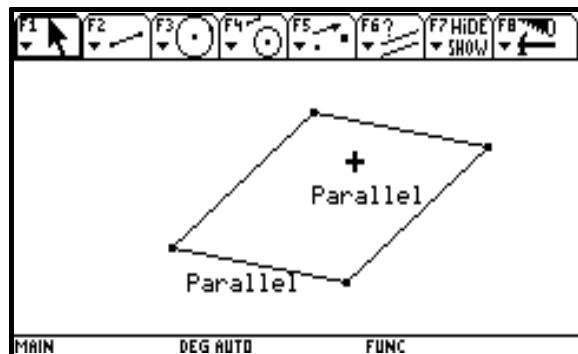

Figure 18

Figure 19

The center of each circle will be the noncommon endpoints and radius of each circle is the length of the segment for which the center is <u>not</u> an endpoint. Next, locate the points of intersection of the circles, hide the circles and the "extra" point of intersection (what quadrilateral is obtained by using this point as the fourth vertex?), and construct the remaining two sides of the parallelogram. Looks good, but are we sure it is a parallelogram? To check, open the `F6` menu, select `8:Check Property` followed by `2:Parallel`, then test the pairs of opposite sides (see Figure 19). What properties of a parallelogram were being used in the construction?)

Note: This activity can be done with other special quadrilaterals such as rhombi, trapezoids, and kites. Work on these types of constructions can lead to discussion of which constructions assure that you will get a particular quadrilateral and which may only obtain a special case of that quadrilateral (e.g., a construction for a rhombus that only produces squares). One other "bonus" with this type of problem is that it can be used to investigate properties of specific quadrilateral OR it can be used in an assessment of students' understanding of the properties of specific quadrilaterals!

2. Construct a quadrilateral of your choice. Locate the midpoints of each side of the quadrilateral then connect the midpoints of adjacent sides to form a new quadrilateral (see Figure 20). Measure the lengths of the sides and the sizes of the angles. What type of quadrilateral was formed? Continue this exploration with several examples of different quadrilaterals and then answer the following.

    a. What type of quadrilateral is formed by connecting the midpoints of adjacent sides of a square? a rectangle? a rhombus? a parallelogram? an isosceles trapezoid? a general trapezoid? a kite? a general quadrilateral?

    b. Under what conditions will the quadrilateral formed by connecting the midpoints of adjacent sides be a square? rectangle? rhombus? trapezoid? kite? parallelogram?
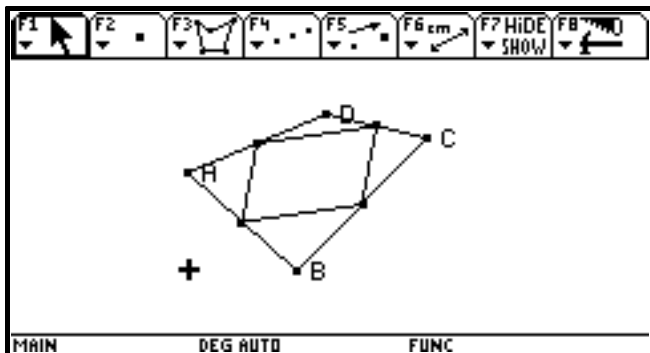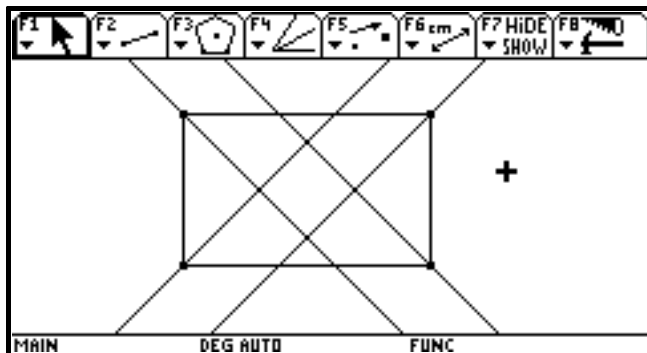

Figure 20   Figure 21

3. Construct each of the following quadrilaterals. Then construct the bisectors of the angles of each quadrilateral. Figure 21 shows this construction for a rectangle. Make a list of as many properties as you can for the points of intersection for each. Try to justify each property on your lists. Are there any properties that are common to each of these quadrilaterals. If so, write your conjecture (in if-then form) and write a justification for it.

    a. square   b. rectangle   c. rhombus   d. parallelogram

    e. kite   f. isosceles trapezoid g. trapezoid   h. general quadrilateral

4. Construct an equilateral triangle and a point on its interior. Determine the distance from this point to each side of the triangle. Make a conjecture about the relationship between the sum of these measures and the length of the altitude of the triangle. Can you justify your conjecture?

5. Construct a triangle. Locate the midpoints of two sides of the triangle. Choose an arbitrary point on the remaining side and draw the segments from each midpoint to that point. Make a conjecture about the relationship between the areas of the quadrilateral and two triangles created by this construction. Can you justify your conjecture?

6. Construct a square. Construct the midpoints of two consecutive sides. Construct the point on one of the remaining sides such that the distance from this point to the vertex of the square opposite the previously chosen consecutive sides is one-fourth the length of the side of the square. Construct the triangle whose vertices are the three points just constructed. Make a conjecture about the relationship between the areas of the square and triangle. Can you justify your conjecture?

7. An equilic quadrilateral is a quadrilateral with one pair of opposite sides congruent such that if these two sides were extended, they would meet to form a 60° angle. The two noncongruent sides are called the bases. A sample equilic quadrilateral is shown in Figure 22. Construct your own equilic quadrilateral. Write the sequence of steps you used for this construction. Then refer to the figure to answer each of the following.
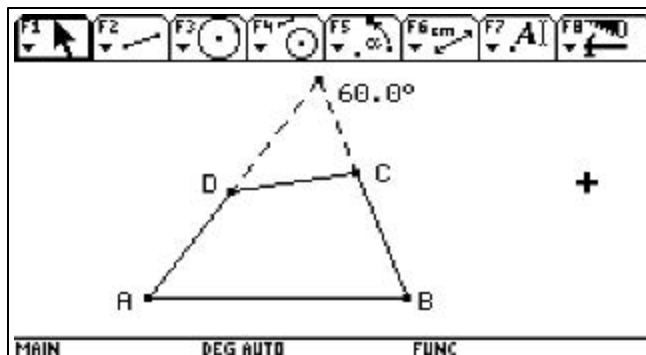

Figure 22

a. Can $\overline{AB}$ be parallel to $\overline{CD}$? If so, draw a model; if not, explain.
b. Can $\overline{AB}$ be congruent to $\overline{CD}$? If so, draw a model; if not, explain.
c. Can $\overline{AB}$ or $\overline{CD}$ be congruent to $\overline{BC}$ and $\overline{AD}$? If so, draw a model; if not, explain.
d. Can diagonals $\overline{AC}$ and $\overline{BD}$ be congruent? If so, draw a model; if not, explain.
e. What are the sum of the measures of ∠A and ∠B? of ∠C and ∠D?
f. Can ∠A or ∠B be right angles? Can both? If so, draw a model; if not, explain.
g. Construct equilic quadrilateral ABCD with diagonals $\overline{AC}$ and $\overline{BD}$. Construct the midpoints of $\overline{AB}$, $\overline{AC}$, $\overline{CD}$ and $\overline{BD}$. What can you say about the relationship between these four midpoints? between any three of these midpoints? (Hint: You may wish to draw segments connecting these midpoints.)
h. Construct equilic quadrilateral ABCD. Draw an equilateral ?ABE such that point E is on the side of $\overleftrightarrow{AB}$ opposite that in which C and D lie. How are points C, D, and E related?

Activity: Macros and Centers of a Triangle

Every triangle has many, MANY different "centers." Four of the centers most commonly explored in geometry are the circumcenter, the incenter, the orthocenter, and the centroid. Let's look at the construction of these centers and see how *TI-92+ Cabri Geometry* **Macros** can be used to make it easy to do such constructions over and over in different figures.

The circumcenter is the center of the circumcircle of a triangle. It is the intersection point of the perpendicular bisectors of the sides of the triangle. Construct a triangle — select 3:Circle from the F3 menu, and then press ENTER to "place" the first vertex of the triangle. Now move the pointer to another spot on the screen, press ENTER to place the second vertex, move the pointer to yet another spot on the screen, and then press ENTER to place the third vertex and complete the construction.
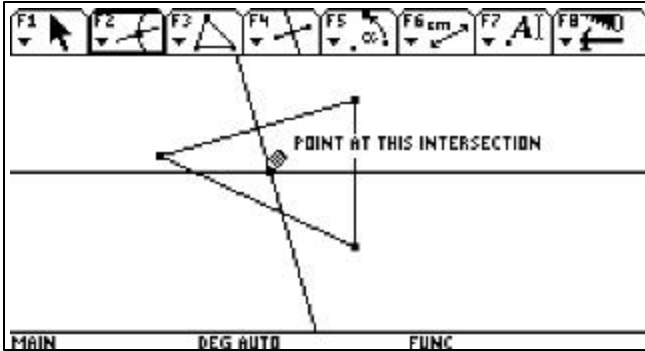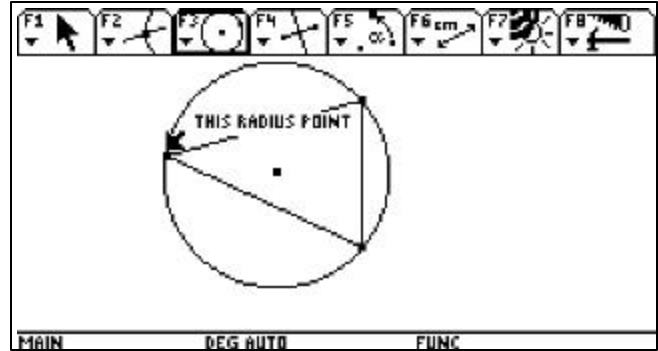

Figure 23

Figure 24

Next construct the perpendicular bisectors of any two of the sides of the triangle— select 4:Perpendicular Bisector from the F4 menu, highlight one side, then highlight a second side. Select 3:Intersection Point from the F2 menu, to find the circumcenter (see Figure 23). Hide the two lines — select 1:Hide/Show from the F7 Menu. To construct the circumcircle, select 1:Circle from the F3 menu, move the pointer to the circumcenter (wait for the message THIS CENTER POINT), then press ENTER to "place" the center of the circle there. Next, move the pointer to one the vertices of the triangle (wait for the message THIS RADIUS POINT), and then press ENTER to complete the construction (see Figure 24).
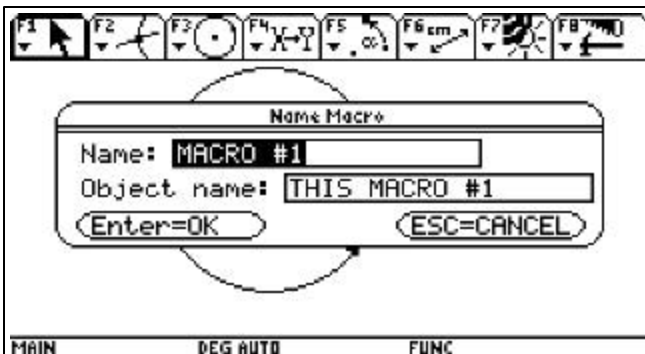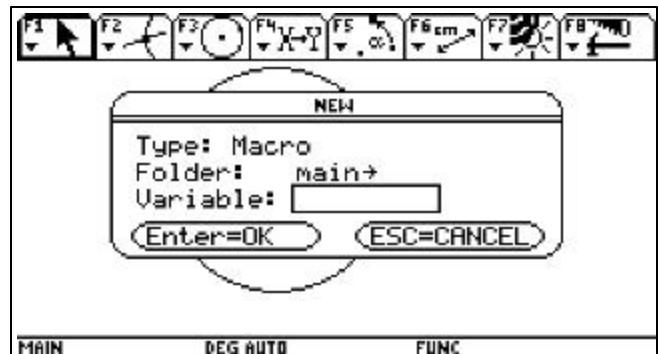

Figure 25

Figure 26

Now we are ready to create a macro for constructing the circumcenter for any triangle. Select 6:Macro Construction from the F4 menu, then select 2:Initial Objects and highlight the triangle. Again select 6:Macro Construction from the F4 menu, but now select 2:Final Object and highlight the circumcenter. Finally, select 6:Macro Construction from the F4 menu, followed by 4:Define Macro. A screen like the one in Figure 25 will appear. Type in a Name (a descriptive name like circumcenter is helpful) and an Object name (circumcenter can be used here too) and press ENTER twice. A new screen like the one in Figure 26 will appear.

As with the case of a TI-92+ Geometry figure, the name may be up to 8 characters. You should again choose a descriptive name, like circumct, to help you remember what the macro does. After typing in a name, press ENTER twice to complete the creation of your circumcenter macro. To test the macro, open a new TI-92+ geometry figure and construct a triangle. Select 6:Macro Construction from the F4 menu, then select 1:Execute Macro. A highlighted list should appear (see Figure 27) that shows all the different macros available. Choose the macro you wish to use (in this case circumcenter) and highlight the necessary initial objects (in this case, the triangle). Voila, you now have the circumcenter of your triangle (see Figure 28)
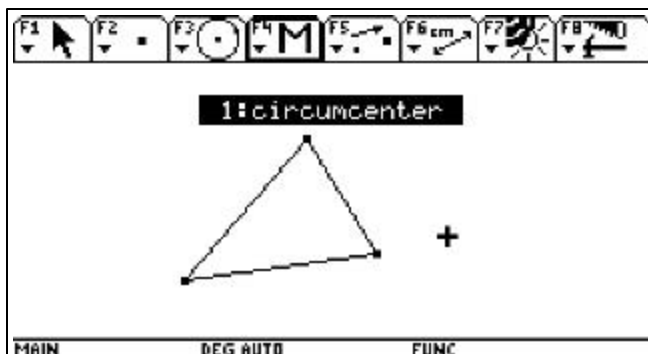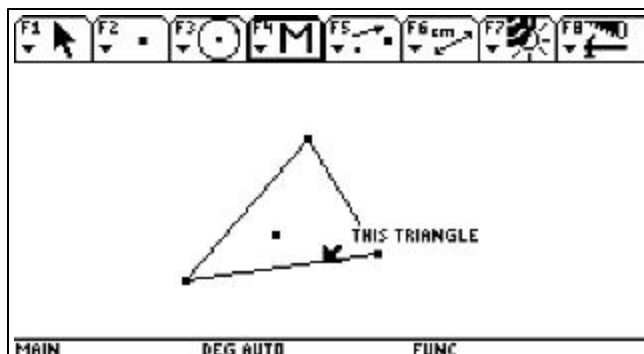

Figure 27

Figure 28

When using the Execute Macro command, the list will only contain those macros created during that session unless you open the specific macros you want. To do this, select 1:Open from the F8 menu. When the dialog box appears, select Macro as the Type and then open any macros you want from those that are available. Repeat this process to open multiple macros. Also, a macro will not work unless the exact initial objects specified during its definition are used. In the case of our circumcenter macro, it will not work if you just have three points or three segments that form a triangle. A triangle has to be constructed on the screen to use this macro. Given this, you should choose your initial objects wisely so that you can be flexible in your use of macros with different figure. For example, we might have been better off creating our circumcenter macro from three points rather than a triangle.

Additional "Macros and Centers of a Triangle" Problems

1. Construct the other three centers mentioned. Use your constructions to create macros for constructing each of these centers.

2. Construct the circumcenter and incenter of a triangle. Create macros for each of these constructions. While you are at it, use what you learn from these constructions to create a macro for constructing a circle through 3 non-collinear points (include the center of the circle in your macro).

3. Construct a triangle along with the midpoints of its three sides. Use your macro to create a circle through the three midpoints. This is the Nine-Point Circle. Now try to figure out the other six special points of the Nine-Point Circle. One hint I'll give you to start with is that they come in two sets of three. More hints will be given as necessary.

4. Construct a triangle. Use your macros to construct the circumcircle and Nine-Point Circle for this triangle. What relationships can you find between these two circles?

5. Construct a triangle. Use your macros to construct the circumcenter, incenter, orthocenter, and centroid of the triangle. What relationships can you find between these four points. Next, use your macro to construct the Nine-Point Circle for this triangle. Hide the circle, leaving just its center. Now what do you observe?

6. Construct a triangle. Next, construct a point S on the interior of your triangle and reflect this point over each side of the triangle. Construct the triangle with vertices at these three reflected points. Use your Macros to construct the circumcenter of your original triangle and the new "reflected" triangle. Now drag point S around the interior of your original triangle until the circumcircles coincide. What do you observe?

7. This activity is a variation of Activity 5. Again, construct a triangle. Use your macros to construct the circumcenter of this triangle. Next, construct a point on the circumcircle and reflect this point over each side of the triangle. What do you notice about these points? Does your conjecture hold as you move the point on the circumcircle? Select 2:Trace On/Off from the F7 menu and highlight the three points. What do you notice about where the three traces intersect?

8. Construct a triangle. Construct lines containing the sides of the triangle. Use your macros to construct the circumcenter and orthocenter of this triangle. Construct segments connecting the orthocenter to a vertex of the triangle. Construct a segment from the circumcenter that is perpendicular to the line containing the side opposite the vertex you just connected to the orthocenter. What do you observe? Can you prove your conjectures?!?

9. Construct a triangle. Use your macros to construct the circumcircle and orthocenter of this triangle. Next, construct a point on the circumcircle, the segment connecting this point to the orthocenter, and the midpoint of this segment. Select 2:Trace On/Off from the F7 menu and highlight the midpoint. What do you notice about the figure traced out by this midpoint?

10. Construct a triangle. Construct the three lines that contain the sides of the triangle. Use your macros to construct the circumcircle of the triangle. Place a point P on the circumcircle. Construct the lines through P perpendicular to the lines that contain the sides of the triangle. Construct the points where these perpendicular lines intersect the corresponding lines that contain the sides of the triangle. What do you observe about these points? Draw a line through them, then select 2:Trace On/Off from the F7 menu and highlight the line. What do you notice about the trace of this line as point P is moved around the circumcircle?

Activity: Exploring Conics

Construct a circle — select 1:Circle from the F3 menu, press ENTER to "place" the center of the circle, move the pointer in any direction to make the circle as large as you wish, and then press ENTER to complete the construction. Label this point *C*.

Next construct a point in the interior of the circle, but not at the center (label it *F*), and a point on the circle (label it *A*) — select 1:Point from the F2 menu. Finally, construct a segment connecting points *A* and *F* and the perpendicular bisector of segment *AF* — select 4:Perpendicular Bisector from the F4 menu and then highlight the segment (see Figure 29).
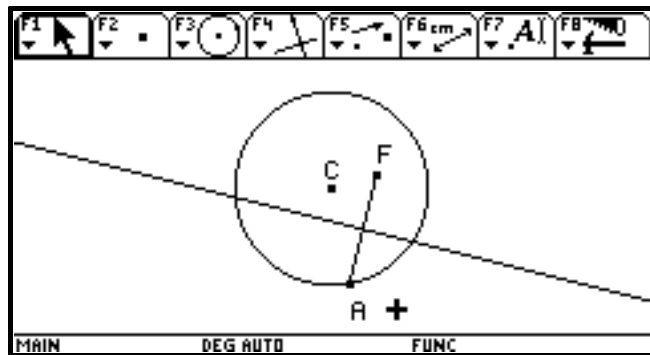
Figure 29

Select 2:Trace On/Off from the F7 menu and highlight the line you just constructed. Next select 3:Animate from the F7 menu, select point *A* on the circle, hold down the "hand" key in the upper left corner while moving the pointer in a direction approximately tangent to your circle at point *A*. A "spring" will appear as you move the pointer (see Figure 30).
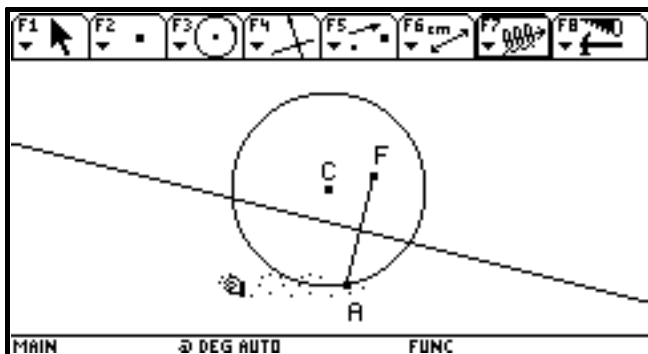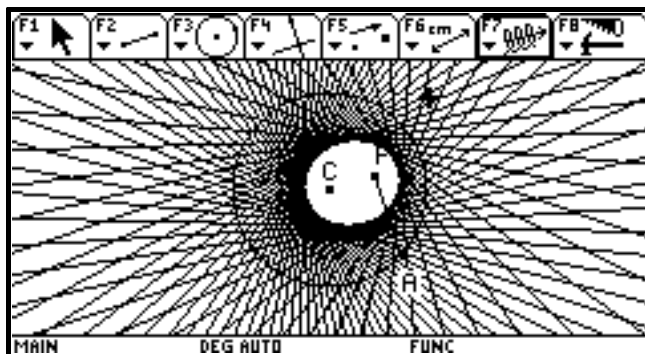

Figure 30

Figure 31

Let go of the "hand" key. Depending on the size of the circle and the position of your points, you should get an image like the one shown in Figure 31. What figure is being traced out by all the different images of the perpendicular bisector (called an *envelope* of lines) as the point moves around the circle?

Next question: Can you think of a way to prove your conjecture? To do this, we must first decide what point on the perpendicular bisector of segment *AF* is tracing out the ellipse (oops, gave away the answer) as point *A* moves around the circle. One likely candidate is point of intersection of the perpendicular bisector and segment *CA*, which is a radius of the circle (see Figure 32).
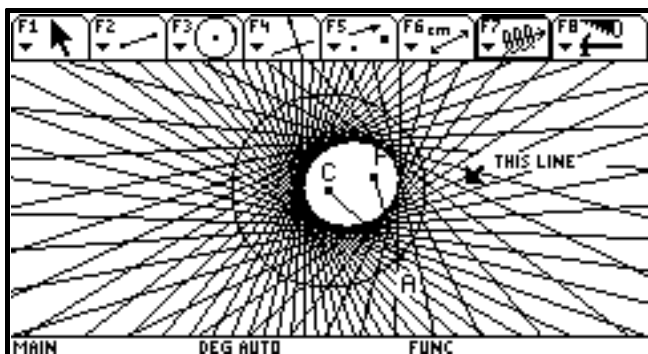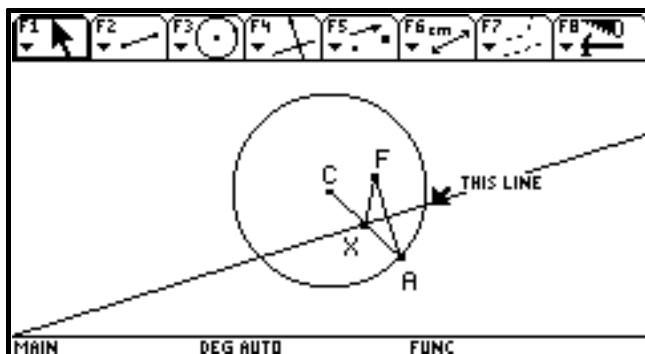

Figure 32

Figure 33

Press the Clear key to clear the locus of lines and construct segment *CA*. Then construct the point of intersection of segment *CA* and the perpendicular bisector (label it *X*) — select 3:Intersection Point from the F2 menu then move the point to the point of intersection (you will this the message "point at this intersection") — and the segment between this point and point *F* (see Figure 33).

Since point *X* is on the perpendicular bisector of segment *AF*, *XF* = *XA*. Thus, *CX* + *XA* = *CX* + *XF*. But *CX* + *XA* = *CA*, the radius of the circle, so *CX* + *XF* always has the same length as the radius no matter where point *X* is as point *A* moves around the circle! By definition, the figure traced out by point *X* must be an ellipse (since *CX* + *XA* is constant).

There are two ways to "see" that point *X* actually does trace out an ellipse as point *A* moves around the circle. One way is to repeat the steps we used earlier for tracing the locus generated by the perpendicular bisector as point *A* moved around the circle. Select 2:Trace On/Off from the F7 menu and select point *X* (also be sure the perpendicular bisector is no longer highlighted). Then select 3:Animate from the F7 menu, select point *A* on the circle, hold down the "hand" key in the upper left corner, move the pointer "away" from point *A*, and let go of the "hand" key (see Figure 34).
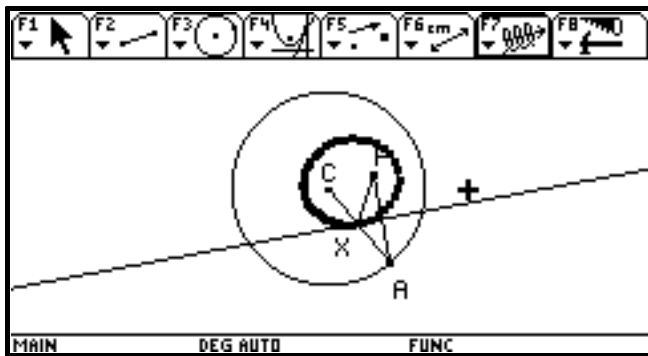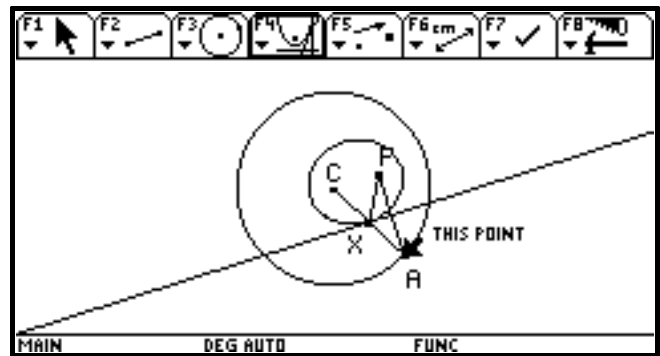

Figure 34

Figure 35

The second method involves the Locus command. Once again press the Clear key to clear the locus that was just generated. Next, select 2:Trace On/Off from the F7 menu and select point *X* so the image of this point will no longer be traced. Now select 4:Locus from the F4 menu, select point X, and finally select point A. The Locus command draws the locus of points generated by the point selected first (point *X*) as the point selected second (point *A*) moves along its path (see Figure 35).

The results of these two activities lead to other interesting questions. For instance, how might one modify the original drawing (Figure 29) to have the envelope of lines form a different but related conic? How might one create an entirely different drawing to construct an envelope of lines that depicts a third conic? For other figures, are there ways to use the Trace On/Off and Animate or Locus commands could be used to produce the actual conics (as in the second part of the last activity) rather than envelopes of lines that "trace out" the conics?

Activity: Ratios in Right Triangles

1. Use the `Ray` tool to construct a horizontal ray with endpoint A. Use the `Point` tool to place point D on the screen above and to the right end of the ray and construct a second ray $\overrightarrow{AD}$. Use the `Point on Object` tool to draw point C on the first ray. Use the `Perpendicular Line` tool to construct a line perpendicular to $\overrightarrow{AC}$ at C. Use the `Intersection` Point tool to mark the intersection of $\overrightarrow{AD}$ and the perpendicular line through C as point B.

2. Use the `Segment` tool to draw $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. These segments represent the hypotenuse of the right triangle and the adjacent side and opposite side of ∠A, respectively (see Figure 36). Use the `Distance & Length` tool to measure the sides of ∆ABC. Use the `Mark Angle` and `Angle` tools to mark and measure ∠BAC. Use the `Hide/Show` tool to hide the rays and perpendicular line, but not point D (see Figure 37).
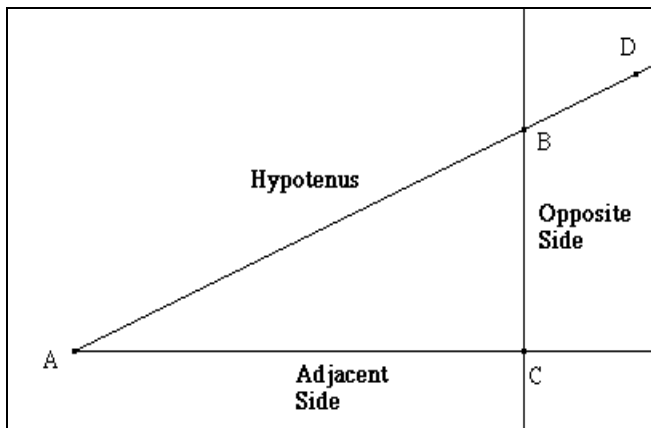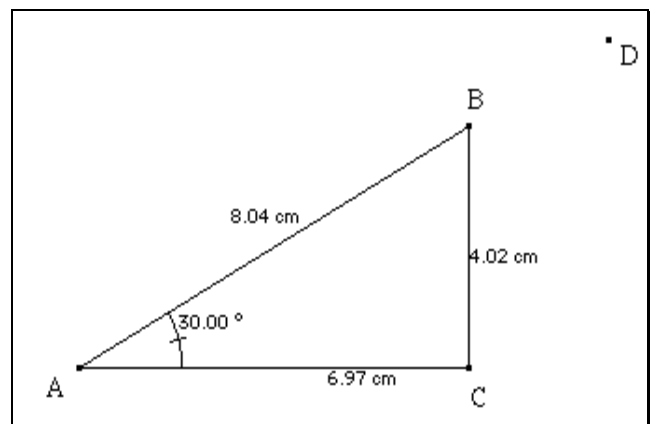

Figure 36: Right triangle nomenclature   Figure 37: Construct and measure ∆ABC

3. Drag point C toward point A. What changes and what does not? Use the `Calculate` tool to compute the ratio of the length of the opposite side $\overline{BC}$ and the hypotenuse $\overline{AB}$ of the right triangle. Drag point C a few more times. What seems to happen?

4. Change the measure of ∠BAC by moving point D. What happened to the ratio of the length of the sides? Drag point C again and explain what happens. Summarize what you have discovered about the ratio of the length of the side opposite ∠A and the length of the hypotenuse of the right triangle.

5. Choose other combinations representing any two sides of the triangle in any order and compute their ratios. What happens to the side lengths and ratios as point C is moved? What happens when ∠BAC changes? Summarize your findings.

6. Create a table of measurements using the `Collect Data` tool. Enter the lengths of the sides and the ratios in the table. Use the `Data View` tool to see the table and the triangle on a split screen. Drag point C and make several more entries in your table. How are the many different sized triangles related as point C is moved ? Explain.

7. What happens to the ratios in the triangle when ∠BAC approaches 0° or 90°?

8. Make a new type of construction to investigate what happens when the measure of ∠BAC is between 90° and 180°? Compare this to values between 0° and 90°. Explain your findings.