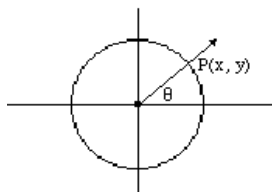Name _____

Date _____

EXPLORATIONS

ACTIVITY 1

Exploring the Unit Circle

The *unit circle* is a circle with a radius of one and its center at the origin; it is defined by the relation $x^2 + y^2 = 1$. Any point on the circle, *P*(*x*, *y*), represents an ordered pair that satisfies the given equation.

If a ray is drawn from the center of the circle through a point on the circle, then that ray and the ray that is the positive *x*-axis create a central angle, 0. As you move the point *P*(*x*, *y*) around the circle in a counterclockwise direction, the central angle and the coordinates of the point create a unique mapping called the *wrapping* function.

The angle and the *x*-coordinate of the point create another mapping function, as do the angle and the *y*-coordinate. These latter two mapping functions are examined in this activity using the Cabri Geometry Application.

Instructions

1. Press ON to turn on the Voyage™ 200 Personal Learning Tool (PLT). To reset to the default settings, press 2nd MEM F1, select 1:RAM, select 2:Default, and then press ENTER ENTER.

2. To begin a new geometry session, press APPS, and select Cabri Geometry, and then select 3:New.

    In the NEW dialog box, press ⊙ to move the cursor to the Variable box. Type a variable name such as *unit*. Press ENTER ENTER to open a new geometry window.

3. To change the format of the window, press F8 and select 9:Format, which opens the Geometry Format dialog box.

    To turn on the coordinate axes, press ⊙ and select 2:RECTANGULAR. Press ⊙ to highlight FIX 2. Press ⊙ and select 4:FIX 4. Press ENTER to return to the geometry window.

Exploring the Unit Circle *(Continued)*

4. To set up a split screen, press F8 and select B:Data View. The split screen displays with the geometry screen on the left and the data cells on the right (see figure at right). Notice that the selected screen (geometry view) has a thick line border.

   Press 2nd [⊞] to select the data view. Clear any data by pressing F1, selecting 8:Clear Editor, and then pressing ENTER. Press 2nd [⊞] to return to the geometry screen.
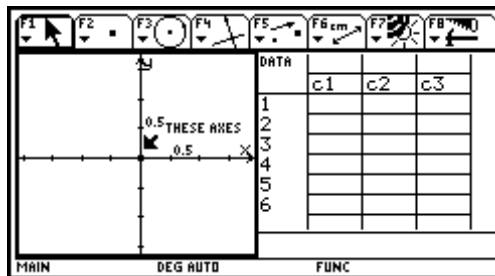
5. To construct the circle, press F3 and select 1:Circle. Move the cursor to the origin until "THIS CENTER POINT" is displayed, and press ENTER. Press ⊙ to expand the circle until the point (1, 0), which is the second tick mark on the positive x-axis, is selected and press ENTER. The ray through the points (0, 0) and (1, 0) is the initial side of the angle.
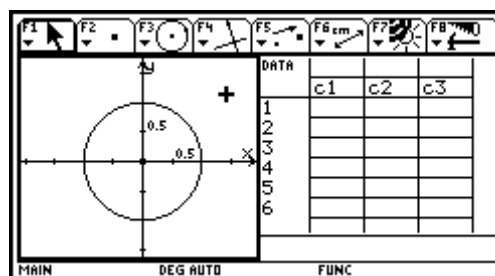
6. To display the equation of the circle, press F6 and select 5:Equation & Coordinates. Select the circle by moving the cursor until "EQUATION OF THIS CIRCLE" is displayed and then pressing ENTER.

   You can reposition the equation by pressing F1 and selecting 1:Pointer. Select the equation with the pointer, and "grab" it by pressing and holding ✋. Use the arrow keys to move the equation. Release ✋ when the desired position is reached.
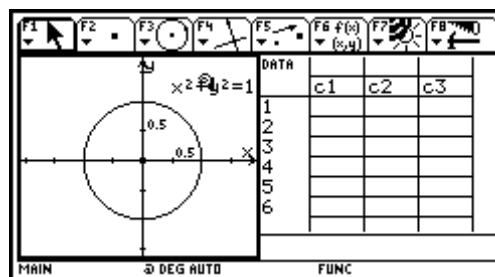
7. To create the terminal side of the central angle, press F2 and select 5:Segment. Move the cursor to select the origin as the first point and press ENTER. Move the cursor to a point on the circle in the first quadrant, and press ENTER. The point $P(x, y)$ is created.
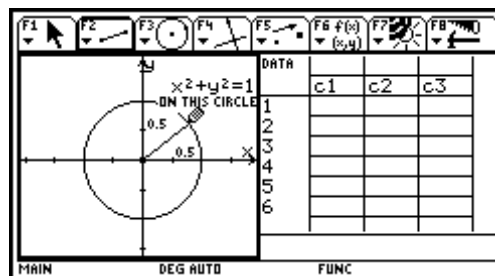
Exploring the Unit Circle *(Continued)*

8. To define the angle to be measured, press F7 and select 7:Mark Angle. Using the arrow keys, move the cursor to select the point (1, 0), and press ENTER. Repeat for the origin and the point $P(x, y)$. You *must* select points in this order when defining the angle.

9. To display the value of the angle, press F6 and select 3:Angle. Move the cursor until "THIS ANGLE" is displayed and press ENTER.

   Reposition the value by pressing F1 and selecting 1:Pointer. Move the pointer to select the number, and use ⌖ to grab the value and drag it to a new location.

10. To display the coordinates of the point, press F6 and select 5:Equation & Coordinates. Select the point $P(x, y)$ on the circle and press ENTER. Reposition the value as in step 9.

11. To define the values for the data table, press F6, select 7:Collect Data, and then select 2:Define Entry. Move the cursor to select the angle measure, and press ENTER. A dashed box encloses the angle measure.

    Move the cursor to select the x-coordinate of the point, and press ENTER. A dashed box encloses the x-coordinate. Repeat for the y-coordinate.

12. To store data in the table, press F6, select 7:Collect Data, and then select 1:Store Data. Next, press F7 and select 3:Animation. Move the cursor to select the point $P(x, y)$, and grab the point by pressing and holding ⌖. Drag the animation spring clockwise and release ⌖. After one complete rotation, press and hold ESC until the animation ends.
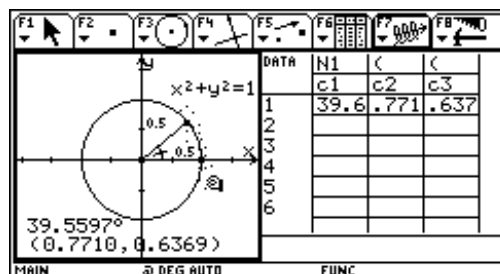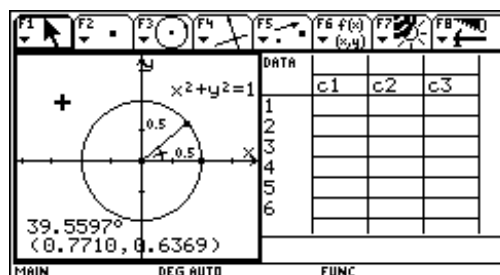
Exploring the Unit Circle *(Continued)*

13. To plot the angle versus the x-coordinate, press [2nd] [⊞] to move to the data table. Press [♦] [Y=] [F1] and select 8:Clear Functions. Then press [ENTER]. Press ⊖ to highlight Plot 1 and press [ENTER].

    Change the mark by pressing ⊖ ⊙ and selecting 3:Plus. Type **C1** in the x box and **C2** in the y box, and press [ENTER] [ENTER].

    To display the graph of the angle versus the x-coordinate, press [♦] [WINDOW] [F2] and select 9:ZoomData. Sketch the scatter plot in the area to the right.
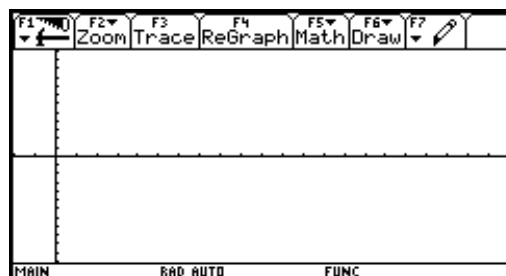
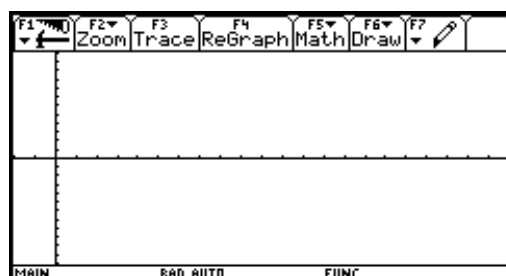The graph of the angle vs. the *x*-coordinate

14. To plot the angle versus the y-coordinate, press [♦] [Y=], highlight Plot 1, and press [F4] to deselect the plot.

    Press ⊖ to highlight Plot 2 and press [ENTER]. Set up this plot the same as Plot 1, except type **C3** in the y box, and then press [ENTER] [ENTER].

    Press [♦] [GRAPH] [F2] and select 9:ZoomData to display the graph of the angle versus the y-coordinate of the point $P(x, y)$. Sketch the scatter plot in the area to the right.

The graph of the angle vs. the *y*-coordinate

15. To graph a function with the scatter plot, press [♦] [Y=] and move the cursor to y1 = . Define y1 = sin(x) by pressing [SIN] **X** [)] [ENTER]. The cursor moves to y2 =. Define y2 = cos(x) by pressing [COS] **X** [)] [ENTER]. Press [MODE], and then press ⊖ to highlight RADIAN. Press ⊙, select 2:DEGREE, and press [ENTER]. Finally, press [♦] [GRAPH].

Exploring the Unit Circle *(Continued)*

Questions

1. With Plot 2 selected and Plot 1 deselected, graph $\sin(x)$ and $\cos(x)$ with the scatter plot. How do the two functions compare with the scatter plot of the angle versus the y-coordinate?

2. With Plot 1 selected and Plot 2 deselected, graph $\sin(x)$ and $\cos(x)$ with the scatter plot. How do the two functions compare with the scatter plot of the angle versus the x-coordinate?

3. Beginning at 0, what is the next value of x for which each graph repeats itself? This is the period of $\sin(x)$ and $\cos(x)$. To what value does the number relate on the unit circle?

4. The amplitude of $\sin(x)$ and $\cos(x)$ is defined as the maximum value minus the minimum value divided by two. What is the amplitude of the two graphs? To what value on the unit circle does the amplitude relate?

5. If the activity were repeated with a radius of 1.5, how would the two graphs change? What would be their new equations?

6. For a circle with its center at the origin and radius of r, what would be the equation relating the angle and the x-coordinate? The angle and the y-coordinate?

Exploring the Unit Circle *(Continued)*

Extensions

Repeat the activity with a different center for the circle, such as (1, .5). Name the new geometry session *Unit 2*.

1. Press F8 and select 9:Format. Turn on the grid and press ENTER. Move the axis to the left by grabbing the origin with ✍ and dragging it to the left about 1 unit.

2. Use the point on the grid at (1, .5) as the center of the circle, and expand the circle to the point (2, .5).

   The initial side of the angle must be drawn parallel to the x-axis from the center of the circle to the grid point at (2, .5). This creates a vertical shift in the two graphs.

Repeat the activity with the initial side not along the positive x-axis but at some other angle, such as ß. Name the new geometry session *Unit 3*.

1. Draw a segment from the center to a point on the circle. To find the value of ß, press F6 and select 3:Angle.

2. Move the cursor to the three points that define the angle and press ENTER at each point. This creates a phase shift, or horizontal translation, in the two graphs.

   The general equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center, and r is the radius. The general equations of the two functions are $f_1(x) = A\sin(x - C) + D_1$ and $f_2(x) = A\cos(x - C) + D_2$.

   How do the values h, k, r and ß relate to A, C, D_1, and D_2?