Bewildered Babies

ID: 9976

Time required
45 minutes

Activity Overview

Students test the limits of the combinations formula by applying it to a labeling situation. In Problem 1, after making charts and using logic to list possible label arrangements, students compare their results with the output of the combinations formula and ***nCr*** *command. They may formalize the results of this comparison by writing a piecewise function for the number of label arrangements in an optional extension. In Problem 2, they write and simplify factorial expressions to calculate probabilities involving combinations.*

Topic: Data Analysis and Probability

- *Use the Fundamental Counting Principle to calculate the number of outcomes in a sample space.*
- *Use factorial notation to express the number of permutations and combinations of n elements taken r at a time.*
- *Evaluate expressions involving factorials to compute the number of outcomes in a sample space.*
- *Use factorial notation to express the theoretical probability of a simple event in a finite sample space.*

Teacher Preparation and Notes

- *This activity is appropriate for an Algebra 1 classroom. Students should have experience with the Fundamental Counting Principle as well as calculating the probability of complex events.*
- *This activity is intended to be teacher-led with periods of individual student work.*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "9976" in the quick search box.***

Associated Materials

- *Alg1Week32_BewilderedBabies_Worksheet_TINspire.doc*
- *Alg1Week32_BewilderedBabies.tns*
- *Alg1Week32_BewilderedBabies_Soln.tns*

Suggested Related Activities

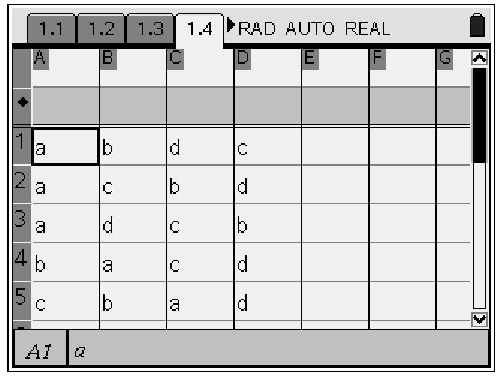
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Too Many Choices (TI-Nspire technology) — 11763*
- *What's Your Combination? (TI-Nspire technology) — 9839*
- *Combinations (TI-Nspire technology) — 8433*
- *Probability (TI-84 Plus and TI-Navigator) — 2146*
- *Permutations and Combinations (TI-84 Plus and TI-Navigator) — 1756*

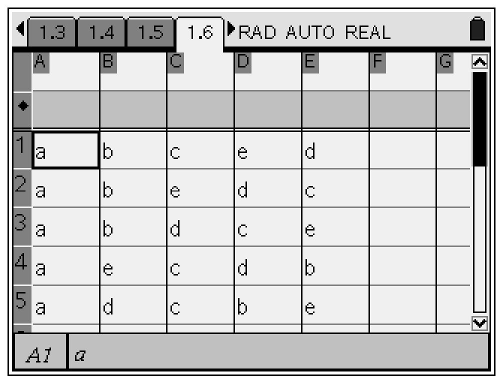
Problem 1 – Finding combinations

The activity begins by presenting the labeling problem. Four babies born during the same night in the same hospital were labeled with four identification bracelets. Somehow, the bracelets were mixed up, and only two are correct. How many different ways can this happen?

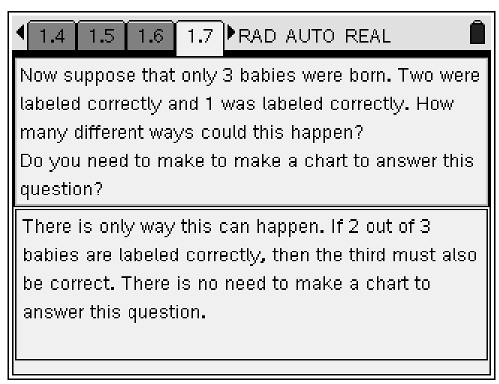
Page 1.3 directs students to make a chart, listing all the different arrangements of the labels.



Demonstrate making the chart on page 1.4. Each column represents a baby, and each row represents a possible arrangement of the labels. Two of the labels must be correct: fill in these cells first. Then fill in the remaining cells so that the labels are incorrect. Continue in an organized fashion to list all 6 possibilities.

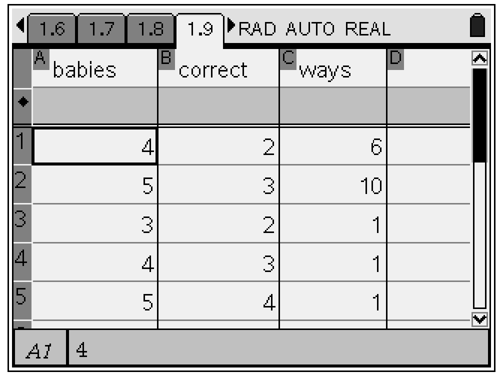


Page 1.5 presents a new situation. What if there were more babies? How many different ways can this happen? Allow students to work independently on page 1.6 to list and count all 10 possible label arrangements for 5 babies born—3 labeled correctly and 2 labeled incorrectly.

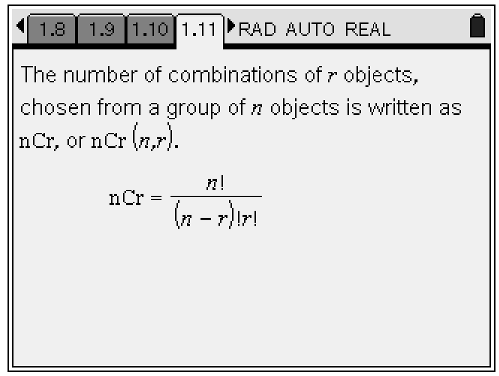


Page 1.7 presents still another situation. What if there were 3 babies and 2 were labeled correctly? How many different ways can this happen? This situation represents a trick question: if 2 out of 3 babies are labeled correctly, the third must also be correct. However, most students will not realize this immediately. Allow them to work independently to make a chart. They will soon see that there is only a single possible arrangement: that all 3 babies are labeled correctly. Afterwards, discuss the nature of this situation. What made it different from the others? Was it necessary to make a chart of outcomes?

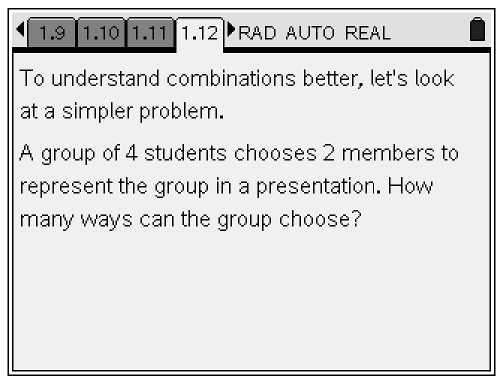
Now that students understand the scenario and the tools at their disposal, they should work independently to complete the table on page 1.9 by finding the number of different ways each situation could occur.



Explain that the problem of choosing which babies to label correctly is a combinations problem, as is any situation where we must choose members from a group and the order in which we choose does not matter. If students have prior exposure to the concepts of permutations and combinations, discuss why this situation is modeled by combinations, not permutations. Page 1.11 presents the formula for calculating the number of combinations.

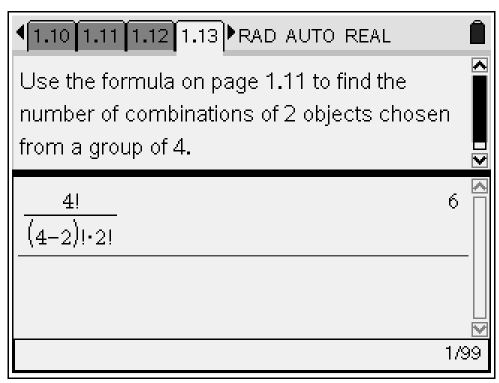


Page 1.12 presents a simpler example to solidify understanding and allow students to practice finding combinations.

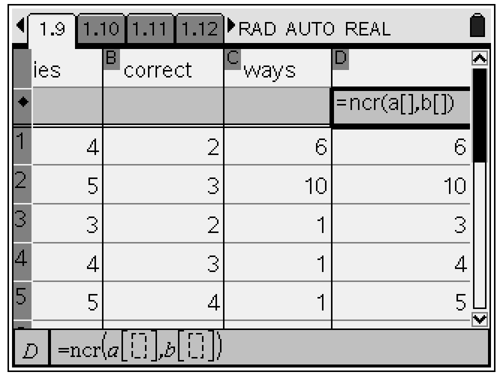


Demonstrate applying the combinations formula on page 1.13 and the **nCr** command on page 1.14. The screenshot to the right shows both.

Note: Press ctrl + tab to move between panes on a single screen.



Reconnect the idea of combinations with the baby labeling problem. Students are to type **=nCr(a[],b[])** in the formula bar of Column D. Compare the results with the numbers on Column C.



When (for what numbers of correctly labeled babies) can we use the combinations formula to find the number of ways? (when at least 2 babies are labeled incorrectly)

When can we ***not*** *use the combinations formula?* (when 1 or 0 babies are labeled incorrectly) Why? (Because there is only one correct label for each baby, it is not possible to label all but one of the babies correctly. So we do not really choose 3 out of 4 babies to label correctly.)

Students should test additional values in Columns A and B to see if this is always true.

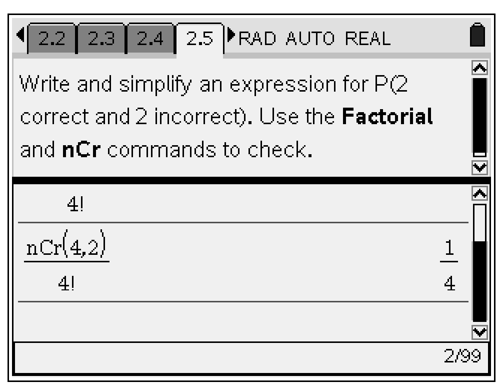
Student Solutions

1. There are 6 ways to label the babies so that 2 bracelets are correct and 2 are incorrect.
2. There are 10 ways to label 5 babies so that 3 are labeled correctly and 2 are labeled incorrectly.
3. No. If 2 of the 3 babies are labeled correctly, the third must be also.
4. There is one way to label 3 babies so that 2 of them are labeled correctly.
5. List **WAYS** should read {6, 10, 1, 1, 1, 3, 4, 5, 1, 1}.
6. The students can choose 2 students from the group of 4 in 6 different ways.
7. When fewer than 2 babies are labeled incorrectly, the number of ways is not equal to the number of combinations. Because there is only one correct label for each baby, it is not possible to label all but one of the babies correctly.

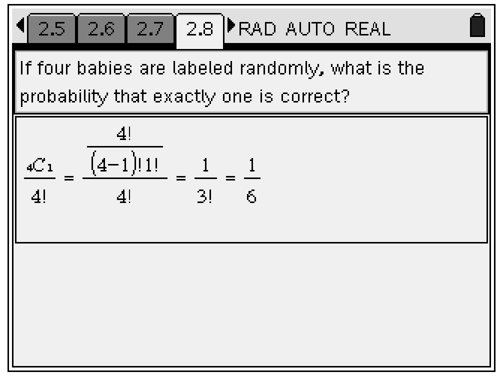
Problem 2 – Finding probabilities

Pages 2.1 through 2.4 step students through the process of using combinations to find probabilities. The probability situation is presented on page 2.1. The bracelets have been assigned to the babies randomly. What is the probability that 2 will be correct and 2 will be incorrect? Page 2.2 reminds students of the definition of probability as the ratio of the number of favorable outcomes to the number of possible outcomes. Page 2.3 guides students to write a factorial expression (4!) for the number of possible outcomes.

Page 2.4 directs them to use the combinations formula to write and expression for the number of favorable outcomes. Remind students that the number of favorable outcomes in this situation means the number of different ways that the favorable outcome (2 correctly labeled and 2 incorrectly labeled) can occur.



Students calculate the probability on page 2.5.



Pages 2.6 and 2.7 list additional probability problems.

Student Solutions

8. The number of possible outcomes is 4! = 24.

9. The number of favorable outcomes is ${}_4C_2 = 6$.

10. P(2 correct and 2 incorrect) = $\frac{1}{4}$

11. a. P(all correct) = $\frac{1}{4!} = \frac{1}{24}$

b. P(3 correct) = $\frac{1}{4!} = \frac{1}{24}$

c. P(exactly 1 correct) = $\frac{{}_4C_1}{4!} = \frac{\frac{4!}{(4-1)!1!}}{4!} = \frac{1}{3!} = \frac{1}{6}$

d. P(all incorrect) = 1 – P(exactly 1 correct) – P(exactly 2 correct) – P(all correct)

$= 1 - \frac{1}{6} - \frac{1}{4} - \frac{1}{24} = \frac{24}{24} - \frac{4}{24} - \frac{6}{24} - \frac{1}{24} = \frac{13}{24}$

e. P(at least 1 incorrect) = 1 – P(all correct) = $1 - \frac{1}{24} = \frac{23}{24}$

f. P(at least 1 correct) = 1 – P(all incorrect) = $1 - \frac{13}{24} = \frac{11}{24}$

Extension – Writing a piecewise function

Remind students of the definition of piecewise function: a function where different rules apply to different values of the independent variable. When the number of babies is 4, there were 3 different ways to find the number of arrangements. Which way we use depends on the number of babies that are correctly labeled. With this guidance, student should write the following functions.

$$f(x) = \begin{cases} 13 & \text{if } x = 0 \\ \dfrac{4!}{(4-x)!\,x!} & \text{if } x = 1, 2 \\ 1 & \text{if } x = 3, 4 \end{cases} \qquad g(x) = \begin{cases} 94 & \text{if } x = 0 \\ \dfrac{5!}{(5-x)!\,x!} & \text{if } x = 1, 2, 3 \\ 1 & \text{if } x = 4,\ 5 \end{cases}$$

The most difficult part of writing this function is calculating the number of ways when $x = 0$. Have students refer back to the table on page 1.9 as well as the probabilities they calculated on page 2.7. One way is as follows: if there are 24 possible arrangements, and the probability of none of the bracelets being incorrect is $\frac{13}{24}$, there must be 13 different ways to label the babies so that none of the bracelets are correct.

