Statistics & Probability

with the

TI-89

Sample Activity: Exploration 1

Brendan Kelly

An Explosive Controversy — The Monty Hall Dilemma

On September 9, 1990, this letter appeared in the *Ask Marilyn* column of *Parade* magazine.[1]

Dear Marilyn:

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

Craig F. Whitacker
Columbia, Maryland

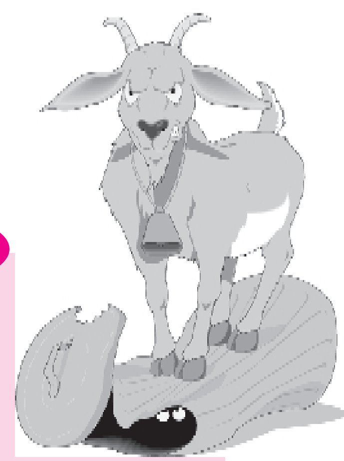
Marilyn vos Savant

Marilyn vos Savant is known for her widely popular column, "Ask Marilyn", which appears in the Sunday magazine *Parade*. In that column she answers questions of a mathematical or scientific nature posed by readers. Some of the counter-intuitive problems she addresses have sparked widespread controversy and stimulated discussion among amateur and professional mathematicians.

The Guinness Book of Records Hall of Fame lists Marilyn's I.Q. of 228 as the highest ever recorded.

Marilyn's Response (without the explanation)

"Dear Craig:
Yes you should switch. The first door has a 1/3 chance of winning but the second door has a 2/3 chance. Here's a good way to visualize..."



DO YOU THINK MARILYN'S ANSWER IS CORRECT?

Think through your answer carefully and do not look at page 9 until you have made a decision. You may assume the following:

- The door behind which the car is placed is chosen randomly.
- The host follows these two rules:
  - — If the car is not behind the chosen door, the host shows the unchosen door behind which the goat is located and then offers the contestant the opportunity to switch.
  - — If the car is behind the chosen door, the host chooses randomly from the unchosen doors hiding the goats and offers the contestant the opportunity to switch.

[1] A detailed account of the entire controversy described on these pages is given in: Marilyn vos Savant. *The Power of Logical Thinking*. New York: St. Martin's Press, 1996.

Sample Responses

The following is a small sample of responses disagreeing with Marilyn's answer. They are reprinted with permission from *Parade* and Marilyn vos Savant, copyright ©1995. The names of the respondents are given in that source, but have not been reproduced here.

❶

Dear Marilyn,

...your answer that you should switch to door number 2 ...is incorrect. Each of doors number 1 and number 2 has 1/2 chance of winning... Your correspondents seem rather rude; I wager your womanhood is a factor!

[Respondent was a professor in the Department of Pure Mathematics at Cambridge University in England.]

❷

Dear Ms. vos Savant

It is apparent from your "Ask Marilyn" column, dealing with probabilities, ... that being smart is no guarantee of being correct. Your analysis of the game-show probabilities, and the analogy involving the pea under a shell, reveals a misunderstanding of the rudiments of probability theory, and an appalling lack of logic. ...

...I urge you to lower your mantle of omniscience and seek the advice of experts when the subject matter is outside your area of expertise. Your ignorant responses are hurting the fight against mathematical illiteracy.

[Respondent was another professor of mathematics.]

❸

Dear Marilyn:

Since you seem to enjoy coming straight to the point, I'll do the same. In the following question and answer, you blew it! Let me explain. If one door is shown to be a loser, that information changes the probability of either remaining choice, neither of which has any reason to be more likely, to 1/2. As a professional mathematician, I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error and in the future being more careful.

[Respondent was a Ph.D. from George Mason University]

❹

You blew it, and you blew it big! Since you seem to have difficulty grasping the basic principle at work here, I'll explain. After the host reveals a goat, you now have a one-in-two chance of being correct. Whether you change your selection or not, the chances are the same. There is enough mathematical illiteracy in this country, and we don't need the world's highest I.Q. propagating more. Shame!

[Respondent was a Ph.D. from the University of Florida.]

❺

Dear Marilyn:

You are utterly incorrect about the game-show question, and I hope this controversy will call some public attention to the serious national crisis in mathematical education. If you can admit your error, you will have contributed constructively towards the solution of a deplorable situation. How many irate mathematicians are needed to get you to change your mind?

[Respondent was a Ph.D. from Georgetown University.]

- Are these professors and Ph.D.'s correct? If so, explain why Marilyn vos Savant's answer is incorrect.

- If you believe Marilyn vos Savant is correct, explain what error you think the professors have made.

- Why do you think these respondents are so angry?

SAMPLE RESPONSES

Not everyone disagreed with Marilyn's answer. In fact, she reported,

Of the letters from the general public, 92% are against my answer, and of the letters from universities, 65% are against my answer. Overall, nine out of ten readers completely disagree with my reply.

The first response which Marilyn gave to problem poser, Craig Whitaker, was:

Dear Craig:

Yes, you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance. Here's a good way to visualize what happened: Suppose there are a million doors, and you pick door number 1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door number 777,777. You'd switch to that door pretty fast, wouldn't you?

Are you convinced by this argument? The table below presents a different argument based on a consideration of the three possible cases, all of which are equally likely. **We assume in each case that you first choose door 1.**

WHICH OF THESE TWO ARGUMENTS DO YOU CONSIDER MORE CONVINCING? EXPLAIN WHY.

	Case 1 Door 1 car, Door 2 goat, Door 3 goat	**Case 2** Door 1 goat, Door 2 car, Door 3 goat	**Case 3** Door 1 goat, Door 2 goat, Door 3 car
If You Stick	**You win**	**You lose**	**You lose**
If You Switch	You switch from door 1. **You lose**	The host shows you the goat behind door 3, so you switch from door 1 to door 2. **You win**	The host shows you the goat behind door 2, so you switch from door 1 to door 3. **You win**

The table shows that if you stick, you will win on average one-third of the time. If you switch, you will win two-thirds of the time.

In an open letter to her readers, Marilyn invited math classes across America to run simulations of the Monty Hall game for a large number of trials and record the results. Two testimonials to the correctness of Marilyn's answer are presented in the following letters.

Dear Marilyn:

...Your problem was presented to 240 students, who were introduced to it by their science teachers. They then established the experimental design while the mathematics teachers covered the area of probability. Most students and teachers initially disagreed with you, but during practice of the procedure, all began to see that the group that switched won more often. We intend to make this activity a permanent fixture in our curriculum.

Anthony Tamalonis, Arthur S. Sommers
Intermediate School 252
Brooklyn, New York

Dear Marilyn:

Our class, with unbridled enthusiasm, is proud to announce that our data support your position. Thank you so much for your faith in America's educators to solve this.

Jackie Charles,
Henry Grady Elementary
Tampa, Florida

On p. 62 you will run program **montyhal( )** which will enable you to test Marilyn's answer.

• Do you think the results of many simulations present conclusive evidence? Explain why or why not.

Exercises & Investigations

❶. In the Monty Hall Dilemma:

a) What is the probability that the car will be behind door number 1?

b) If you always choose door number 1, on what fraction of the games would you expect to win the car if you do not switch?

c) What is the probability that the car will *not* be behind door number 1?

d) If the car is not behind door number 1 and if you switch from door number 1 to the unopened door, on what percentage of the games would you expect to win the car?

❷. On page 10, it was revealed that of the letters received:

•92% of the letters from the general public and 65% of the letters from universities opposed vos Savant's answer.

•9 out of 10 letters opposed vos Savant's answer.

Use the information above to answer these questions.

a) Were the letters from the universities more likely than those from the general public to agree with vos Savant?

b) What percent of the letters came from the universities?

c) Can we infer that 9 out of 10 readers opposed vos Savant's answer? Explain why or why not.

❸. "Proof by authority" means that we accept the truth of a statement because there is general agreement among the experts that the statement is true.

a) Explain how the Monty Hall controversy reveals potential dangers in proof by authority.

b) Is it correct to infer from the letters sent to Marilyn vos Savant that most of the professional mathematicians who read the article opposed vos Savant's answer. Explain why or why not?

c) Give an example of a statement which we might consider true, based only on the consensus of opinion among experts.

❹. On page 11 of *The Power of Logical Thinking,* Marilyn states, "Personally, I regarded the probability trial as more of a proof than an experiment, but it worked."

a) What do you think Ms. vos Savant meant by this statement?

b) If two different probability trials yield different probabilities, how would you determine which one is closer to the "true" probability?

c) Is it possible that a large number of trials would yield an incorrect answer? If so, how would you know that the answer is incorrect?

❺. **The Coin Paradox**

A terrorist group determines whether a prisoner will be released by presenting him with a box containing 3 coins. One coin has a "head" on both sides, the other a "tail" on both sides, and the third is a normal coin with a"head" on one side and a "tail" on the other.

Blindfolded, the prisoner randomly selects one coin. If the normal coin is selected, the prisoner is freed.

a) What is the probability that the prisoner will be freed?

b) Suppose the prisoner is shown one side of the coin selected and he sees that it is tails. What is the probability that the other side will show heads and that he will be freed?

c) Is there a conflict between your answers to parts a) and b)? Explain.

❻. **The Problem of Points**

A and **B** play the following game:

A fair coin is tossed twice.
If a head occurs on either toss, then **A** wins.
Otherwise, **B** wins.

What is the probability that **A** wins?

The Brothers & Sisters Paradox

Consider this statement.

Girls have more brothers than boys, because a boy is not a brother to himself.

Assume that both genders are equally likely on the birth of a child. State whether or not you believe this statement is true and give reasons for your answer. Explain how you might determine whether your answer is correct.

Answers to the Exercises in Exploration 1

1. a) 1/3 b) 1/3 c) 1/3 d) 100%

2. a) Assume that the letters which did not oppose vos Savant's answer, were in agreement with it. Then 8% of the letters from the general public and 35% of the letters from the universities agreed with her. Therefore letters from the universities were more than four times more likely to agree with vos Savant's answer.

b) Let the percentage of letters from the universities be x.
Then the percentage of letters from the general public was $100 - x$.
The fraction of letters which opposed vos Savant's answer is:

$$0.65x + 0.92(100 - x)$$

Since 9 out of 10 of the letters opposed vos Savant's answer, then

$$[0.65x + 0.92(100 - x)] = 0.9$$

Solving this linear equation in x yields $x = 2/27$ or 0.0740…
That is, about 7.4% of the letters came from the universities.
Alternatively,:

$$\frac{\text{Number of letters from universities}}{\text{Total number of letters}} = \frac{92-90}{92-65} = \frac{2}{27}$$

That is, about 7.4% of the letters came from the universities.

c) We cannot assume that those who wrote letters constitute a random sample because those who opposed vos Savant's answer may have been more likely to respond. Thus the sample of respondents would contain a disproportionate number of those who opposed the answer. Therefore we cannot assume that 9 out of 10 readers opposed Marilyn vos Savant's answer.

3. a) Since the preponderance of letters voiced opposition to the vos Savant answer, and since the majority of university responses were in opposition, one might be inclined to accept the judgment of these specialists. The fact that Marilyn's solution was correct reminds us that even many of those with the official credentials can be wrong. The history of science teaches us this lesson over and over again.

b) For the reason expressed in the answer to exercise 2c), we cannot assume that the respondents constitute a random sample. That is, many of the mathematicians may have observed that the vos Savant solution was correct and therefore saw no reason to write.

c) Answers will vary. One example may be Fermat's Last Theorem. Few people have the background to check Andrew Wiles' proof of this theorem, but if those who have checked, declare that it is valid, then we might choose to accept its truth — but we're never certain!

4. a) The ultimate test of the probability of an outcome is to run an experiment a large number of times and observe the relative frequency of that outcome. This is not a rigorous (absolute) proof, but a very persuasive verification.

b) Run another large number of trials and repeat until a trend emerges. There is always a small chance that your answer is wrong.

5. a) 1/3
   b) There are six faces on the three coins and all faces are equally likely. Three of the faces show tails and they are equally likely. Only one of these three tails is opposite a head, so the prisoner's probability of survival is only one in three i.e. 1/3.
   c) There is only a conflict if your answers to parts a) and b) differ.

6. For a detailed solution to this classical problem, see page 55.