Somewhere in the Middle

ID: 11523

Time Required
15 minutes

Activity Overview

In this activity, students will explore the Mean Value Theorem (MVT). Students will find out when the tangent line is parallel to the secant line passing through the endpoints of an interval to help them find the values of c *guaranteed to exist by the MVT. Students will also test functions where the hypotheses of the MVT are not met.*

Topic: Mean Value Theorem

- *Continuity and Differentiability*
- *Slope of Secant Line as Average Rate of Change of a Function Over an Interval*
- *Slope of Tangent Line as Instantaneous Rate of Change of a Function at a Point*

Teacher Preparation and Notes

- *On page 1.5, students may need assistance with the syntax for solving equations in the calculator application. Alternatively, students may do these calculations by hand on separate sheets of paper.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11523" in the keyword search box.***

Associated Materials

- *SomewhereInTheMiddle_Student.doc*
- *SomewhereInTheMiddle.tns*
- *SomewhereInTheMiddle_Soln.tns*

Suggested Related Activities

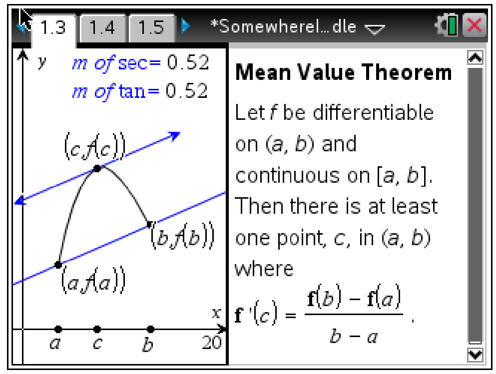
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *The Mean Value Theorem (TI-Nspire CAS technology) — 10044*

The Mean Value Theorem

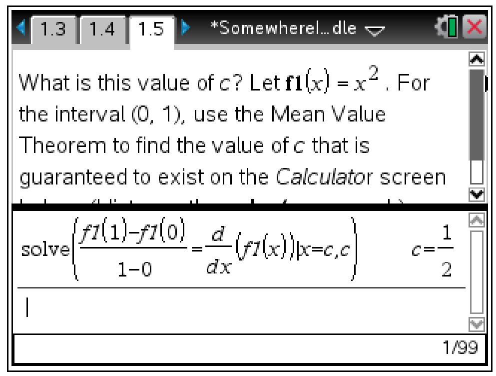
On page 1.3, students are introduced both to the Mean Value Theorem and a graph demonstrating the theorem. If students have trouble understanding the theorem, try explaining it using the terms *secant* and *tangent:*

> Let *f* be differentiable on (*a*, *b*) and continuous on [*a*, *b*]. Then, there exists a value *c* such that the tangent line at *f*(*c*) is parallel to the secant line passing through *f*(*a*) and *f*(*b*).

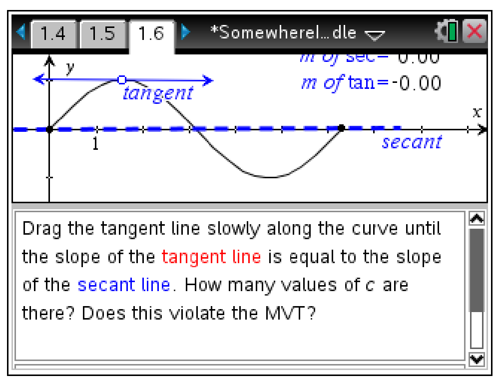


On page 1.4, students are to grab and drag the point of tangency until the slope of the tangent line equals the slope of the secant line.

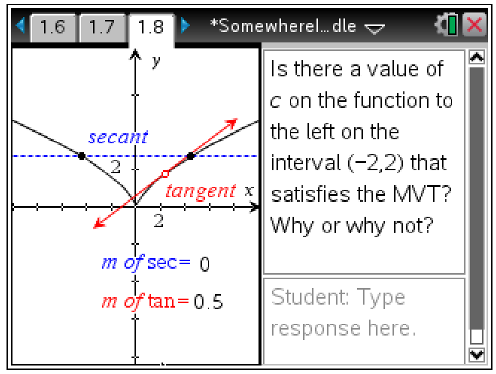
Students are also asked to find this value of *c* using page 1.5. Students should use the **Solve** command (**MENU > Algebra > Solve**). The input for this command should appear as shown to the right. Make sure that students use the derivative command and not division for $\frac{d}{dx}$ (**MENU > Calculus > Derivative**). Alternatively, students can do this by hand on a separate sheet of paper.



Students are to repeat this same process on pages 1.6 and 1.7. However, there will be more than one value for *c*. Students should find that this does not violate the MVT because the theorem states that there is *at least* one value, not *only* one value.

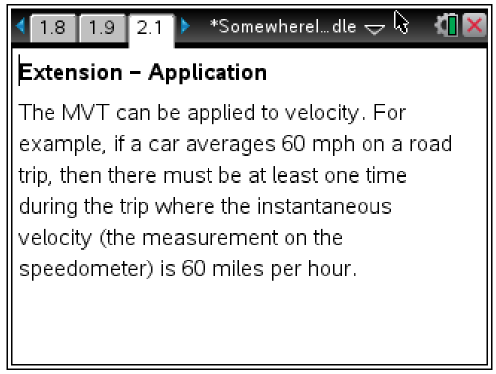


Students should try to apply the MVT to the functions on pages 1.8 and 1.9. For both of these functions, there is no value *c*. The function on page 1.8 is not differentiable at $x = 0$, so it does not meet all of the hypotheses of the MVT. The function on page 1.9 is not continuous on the given interval, so it, too, does not meet all of the hypotheses of the MVT.



Extension – Application

On page 2.1, students are introduced to an application of the Mean Value Theorem involving average and instantaneous velocity. Students are to apply this to a horse race where two horses finish in a tie.



Student Answers

1. The hypotheses are met. $c = \frac{1}{2}$
2. There are 2 values of *c*. The MVT states that there is at least one value, but there may be more than one.
3. $c = \frac{\pi}{2}, \frac{3\pi}{2}$.
4. There is not a value of *c* because the function is not differentiable on the interval.
5. There is not a value of *c* because the function is not continuous on the interval.
6. Let $f(t)$ represent the position of the first horse and $g(t)$ represent the position of the second horse at time t. Let $h(t) = f(t) - g(t)$. At the beginning of the race and at the end of the race $h(t) = 0$. Therefore, by the MVT, there must be at least one time, $t = c$, where $h'(c) = 0$, i.e. $f'(c) = g'(c)$.