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Time required: 30 minutes

Activity Overview

This lesson is intended to allow students to investigate the angle & arc relationships when 2 chords in a circle intersect. Pages include a statement of the theorem, a dynamic geometry demonstration, several problems that apply the theorem, and a 2-column geometric proof of the theorem.

Teacher Preparation

This lesson is created for use in a middle school or high school geometry class.

- Inscribed angles have a vertex on the circle and a measure equal to one-half the measure of the intercepted arc.
- Angles formed by intersecting chords have a measure equal to one-half the sum of the measures of the intercepted arcs. This is the premise of this lesson.
- Since the geometry application does not have an “arc measure” tool, the measure of an arc is equated to the measure of the central angle that intercepts the arc.

Classroom Management

- This lesson is intended to allow students to investigate the angle-arc relationships using the TI-Nspire Geometry Application.
- The student worksheet contains additional diagrams that allow the student to work on each of the problems, as well as the geometric proof.

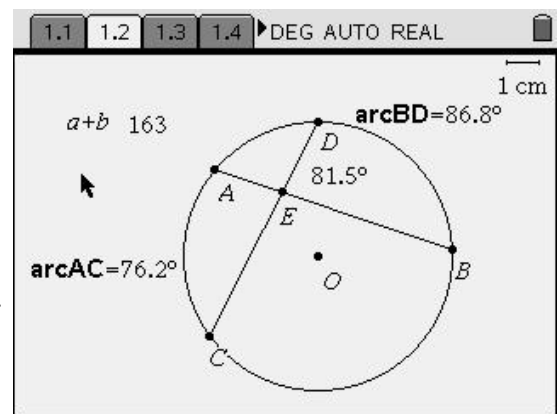
On page 1.1, the theorem is stated for the students. Most students will experience greater success working with the variety of angles formed in circles if the location of the vertex (inside, outside, or on the circle) is emphasized. In this case, the intersection of the chords causes the vertex to lie *inside* the circle.

1.1 1.2 1.3 1.4 ▶ DEG AUTO REAL

Angles Formed by Chords in a Circle

THEOREM: The measure of an angle formed by two chords intersecting within a circle is equal to one-half the sum of the measures of the intercepted arcs.

On page 1.2, the Geometry Application allows the student to manipulate the measures of the arcs by dragging the endpoints of the chords to different locations. As the measures of the arcs change, so does the sum, and the student is expected to be able to confirm that the angle formed by the chords is in fact, equal to one-half of this sum.



The questions on pages 1.3-1.5 each require use of the theorem and correspond to the diagrams provided on the student worksheet.

Answers:

1.3 – $m \angle BED = 81^\circ$

1.4 – $m \angle DEB = 50^\circ$

1.5 – $m \text{ arc } AC = 76^\circ$

1.1 1.2 1.3 1.4 ▶ DEG AUTO REAL

Question

If $m \text{ arc } AC = 42^\circ$ and $m \text{ arc } BD = 120^\circ$, find $m \angle BED$.

Answer ▼

1.1 1.2 1.3 1.4 ▶ DEG AUTO REAL

Question

If segment AB is a diameter and $m \text{ arc } AC = 30^\circ$, find $m \angle DEB$ if $m \text{ arc } AD = 110^\circ$.

Answer ▼

1.2 1.3 1.4 1.5 ▶ DEG AUTO REAL

Question

If $m \angle DEB = 53^\circ$, $m \text{ arc } DB = 3x + 3$, & $m \text{ arc } AC = 10x - 14$, find the measure of the larger of these 2 arcs.

Answer ▼

Page 1.6 instructs the student to complete the geometric proof that follows.

Page 1.7 illustrates a 5 step geometric proof of the theorem. It is necessary to draw an additional chord (AD) in order to clearly identify the inscribed angles and triangle ADE referred to in the proof.

The missing items are: Reason #2 – An exterior angle of a triangle is equal to the sum of the 2 remote interior angles.

Statements	Reasons
1. Circle O with chords AB and CD, that intersect at E.	1. Given
2. $m\angle AEC = m\angle BAD + m\angle CDA$	2.
3. $m\angle BAD = \frac{1}{2}m$	3.

Reason #3 – The measure of an inscribed angle is equal to one-half the measure of the intercepted arc.

Statement #4 - $m\angle AEC = \frac{1}{2}m \text{ arc } BD + \frac{1}{2}m \text{ arc } AC$.

Statements	Reasons
3. $m\angle BAD = \frac{1}{2}m \text{ arc } BD$ and $m\angle CDA = \frac{1}{2}m \text{ arc } AC$	3.
4. $m\angle AEC =$	4. Substitution
5. $m\angle AEC = \frac{1}{2}(m \text{ arc } BD + m \text{ arc } AC)$	5. Greatest common factor

The inclusion of the 5th step is only necessitated in order to state the theorem's conclusion in its factored form.

Statements	Reasons
$m\angle CDA = \frac{1}{2}m \text{ arc } AC$	
4. $m\angle AEC =$	4. Substitution
5. $m\angle AEC = \frac{1}{2}(m \text{ arc } BD + m \text{ arc } AC)$	5. Greatest common factor