Grade level: Pre-service/secondary
Subject: Algebra
Time required: 45 to 60 minutes

TI-Nspire Activity: What's Your Representation?<br>By: Janet B. Andreasen

## Activity Overview

This activity examines contexts for linear and non-linear functions and connections between numerical, graphic, and symbolic representations. Students will predict the type of function from a table, a graph, and an expression. Pre-service teachers will also examine considerations for teaching including what contentspecific and pedagogical knowledge the teacher needs.

## Concepts

- Creating a table from a context
- Best Fit Curve for Linear and Non-linear applications
- Connecting between numerical, graphic, and symbolic representations


## Teacher Preparation

This activity is designed for an Algebra I or Algebra II classroom or pre-service methods course.

- Prior to beginning the activity, students should have some familiarity with linear functions
- Each handheld needs the file Representation.tns
- Student handout for each student/group


## Classroom Management.

This activity is best completed in small groups, although individual completion is also appropriate.

- Student worksheet will guide students through activity and provides a place for students to record their answers.
- TI-Nspire file Representation.tns provides additional directions for each part of the activity.


## TI-Nspire Applications

Graphs \& Geometry, Lists \& Spreadsheets, Data \& Statistics, Notes

## Getting Started

- Open the file Representation.tns
- Follow directions on student worksheet and in the TINspire file.



## Problem 1: Linear Functions

## This starts on screen 1.2

Before moving on to page 1.3, have students complete Question 1:
Make a prediction related to which meal plan is a better choice.
Answer: The meal plan which is the best choice depends on how many meals you plan on eating.

Students will first make a table for each option. Encourage them to use the formulas within the spreadsheet to create their tables. Use caution to not go beyond the number of meals given on the table (14 meals).

## Formulas:

For option 1, enter 12 into cell B1. Then into B2, type =B1+6. Copy cell B2 and paste into each of the other cells in column B down to B15. You can select all the cells and then paste at one time if you like.
For option 2, enter 4 into cell C1. Then into C2, type $=C 1+8$. Copy cell C2 and paste into each of the other cells in column C down to C15.
For option 3, enter 0 into cell D1. Then into D2, type $=D 1+10$. Copy Cell D2 and paste into each of the other cells in column D down to D15. The table should look like the screen shot to the right.

Have students answer the following questions:
Question 2:
Can you determine which meal plan is better? Why? Answer: You can look at the table and see when each option becomes the least expensive. For example, if you look at Option 3 is the cheapest for only having one meal. At two meals, Option 3 and Option 2 are the same cost. At three meals, Option 2 is less expensive.


Question 3:
What type of relationship (e.g. linear, quadratic, exponential, etc.) is each of the meal plans? Why? Answer: These relationships are all linear. There is a constant rate of change. The cost per meal does not change as the number of meals increases.

Move to screen 1.4.
A scatter plot of the data in the table has been created for you.

Have students answer questions 4-6:
Question 4:
Which graph is which?
Answer: The open circles is option 1, the closed circles is option 2, and the plus signs is option 3. Option 1 starts highest and increases the smallest amount each time.
Option 3 starts lowest and increases the fastest.

## Question 5:

According to the graph, which meal plan is best? Why? Answer: Until 2 meals, option 3 is the cheapest. For 2 meals, either option 2 or option 3 is the cheapest. At 3 meals, option 2 is the cheapest. This is true until four meals when option 2 and option 1 are the same cost. For more than 4 meals, option 1 is the cheapest.

## Question 6:

How can you tell from the graphic representation the type of relationship? Why?
Answer: Each graph is a straight line. This makes it linear.
Create a regression line for each meal plan. Follow the steps below:

1. Go to page 1.3. Click on a cell E1 in your table.
2. Click "Menu", "4:Statistics", "1:Stat Calculations". Then choose the type of regression you want based on the relationship observed.
3. For "x list" choose Meals. For "y list" choose the option you want to find the regression for (opt1, opt2, opt3). Label opt1 as F1(x), opt2 as F2(x), and opt3 as F3(x). Begin the results in column F for opt1, column H for opt2, and column J for opt3.
4. Go to page 1.4. Be sure the graph side is active. Click Ctrl-G to show the entry line. Click "Menu", "3:Graph Type", "1:Function".
5. Click in the entry line and press the up arrow three times to show f1(x). Click Enter three times. The equations will show as text on the graph. You can drag them by selecting the text and holding down the "click" button to drag it to a different part of the screen.


Have students answer questions $7-9$.

## Question 7:

What are your three equations:
Answers:
$F 1(x)=6 x+12$
$F 2(x)=8 x+4$
$F 3(x)=10 x+0$
Question 8:
How can you tell from the symbolic equation the type of relationship? Why?
Answer: Each equation has an $x$ and no higher power of $x$. The degree of the equation is one.

## Question 9:

How can you tell from the equations which meal plan is better? Why?
Answer: It is difficult to determine just from the equation.
For a large number of meals, the equation with the smallest slope (smallest rate of increase) would be the better deal.

## Problem Two

This starts on page 2.1. Have students answer question 1 before moving on.

Question 1:
Make a prediction related to which job is a better choice. Answer: Answers may vary. Some may think the video store is better - doubles each day. Some may think the restaurant is better. Some may think the grocery store is better.

Go to page 2.2. Using the scenario given on page 2.2, students will make the table. They can again use the formula feature if they wish. Allow students to figure out the formulas for themselves.
Video: Make cell B1 = 20. In B2, type =B1*2. Copy cell B2 and paste into cells B3-B25.
Grocery: Make cell C1 $=80$. In C2, type $=C 1+10$. Copy cell C2 and paste into cells C3-C25.
Restaurant: Make cell D1 = 40 and D2 = 50. In cell D3, type $=D 2+(D 2-D 1)+10$. Copy cell D3 and paste into cells D4 - D25.

Have students answer questions 2 and 3.

## Question 2:

Can you determine which job is better? Why? Answer: For days 1 - 3, the grocery store pays better. On day 4, the video store pays better. This is true from that point onward.


## 

You are looking for a summer job. You have three options.
Video Store: $\$ 20$ the first day, double the pay each day.

Grocery Store: $\$ 80$ the first day, $\$ 10$ more each day.
Restaurant: $\$ 40$ the first day, $\$ 50$ the second day, $\$ 70$ the third day, $\$ 100$ the fourth day,

| 1.3 | 1.4 | 2.2 Rad auto real el |  |
| :---: | :---: | :---: | :---: |
| s | ${ }^{8}$ video | ${ }^{\text {c grocery }}$ | $\square_{\text {restaurant }}$ |
| - 7 (u) |  |  |  |
| 11. | 20 | 80 | 40 |
| 22. | 40 | 90 | 50 |
| 33. | 80 | 100 | 70 |
| 44. | 160 | 110 | 100 |
| 55. | 320 | 120 | 140 |
| B1 | 20 |  |  |

## Question 3:

What type of relationship (e.g. linear, quadratic, exponential, etc.) is each of the jobs? Why?
Answer: Video store is exponential (powers of 2 times the original amount), Grocery store is linear (constant rate of change), and restaurant is quadratic (constantly changing rate of change). The students may not be able to figure this out right away. They can use the graphic representation to help them if needed.

Move to screen 2.3. A scatter plot of the table has been created here. Have students answer questions 4-6.

## Question 4:

Which graph is which?
Answer: The $x$ 's is the grocery store. The plus signs are the restaurant. The squares are the video store.

## Question 5:

According to the graph, which job is best? Why?
Answer: Until day4, the grocery store is better. From day 4 onward, the video store is better.


## Question 6:

How can you tell from the graphic representation the type of relationship? Why?
Answer: The grocery store is linear - it is a straight line. The restaurant looks like half of a parabola, so it is quadratic. The video store is increasing very quickly, so it may be exponential.

Create a regression line for each job. Follow the same steps as in problem 1, being sure to choose the type of regression for the relationship you observed. Use F4(x) for video, F5(x) for grocery, and F6(x) for restaurant. Graph F4, F5, and F6. Note: If students are not sure of the type of relationship, they can experiment with these regression lines. If they guess wrong, it will not match the data exactly. They can go back and guess a different relationship. Just be sure that they replace the same function each time, so F4 is always the video, etc.

Have students answer questions 7 - 10 .

## Question 7:

How well did you predict the relationship? What helped you to determine which relationship to use?
Answers will vary
Question 8:
What are your three equations:

## Answers:

$F 4(x)=10(2)^{x}$
$F 5(x)=10 x+70$
$F 6(x)=5 x^{2}+50+40$

## Question 9:

How can you tell from the symbolic equation the type of relationship? Why?
Answer: F4, the variable is in the exponent, so it is exponential. F5, the highest degree is 1, so it is linear. F6, the highest degree is 2, so it is quadratic.

Question 10:
How can you tell from the equations which job is better? Why?
This is also difficult to tell, although exponential functions grow very quickly, so it is a logical choice to say it is better.

## Pedagogical Considerations - This is particularly relevant for preservice teachers to consider.

Question 1: What were the mathematical goals of this task?
Question 2: Why are those goals important?
Question 3: What prerequisite knowledge must students have in order to do this activity?
Question 4: What prerequisite mathematics content knowledge, technological knowledge and pedagogical knowledge must the teacher have in order to facilitate this activity effectively?

Question 5: How might you accomplish the same goal in a non-dynamic environment?
Question 6: What are advantages and disadvantages of using the dynamic environment?
Question 7: NCTM's Technology Principle indicates that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." How does this relate to the mathematics activity?

Question 8: How does the Technology Principle and this activity relate to with NCTM's Principle on Learning?

Question 9: NCTM's Standard for Representation indicates that "Instructional programs from prekindergarten through grade 12 should enable all students to: create and use representations to organize, record, and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; use representations to model and interpret physical, social, and mathematical phenomena. How does this mathematics activity allow students to interact with Representation?

## Student TI-Nspire File (Representation.tns)



| 1.1 | 1.2 | 1.3 |
| :--- | :--- | :--- |
| The university has a meal plan for the |  |  |
| cafeteria. There are three options: |  |  |
| Option 1: Cost $\$ 12$, plus $\$ 6$ per meal |  |  |
| Option 2: Cost $\$ 4$, plus $\$ 8$ per meal |  |  |
| Option 3: Cost $\$ 0$, plus $\$ 10$ per meal |  |  |
| Create a table for each meal plan. |  |  |
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