Polar Necessities

ID: 12558

Time Required
15 minutes

Activity Overview

Students will explore what is necessary to understand the calculus of polar equations. Students will graphically and algebraically find the slope of the tangent line at a point on a polar graph. Finding the area of a region of a polar curve will be determined using the area formula.

Topic: Polar Equations

- *Find the slope of a polar equation at a particular point.*
- *Find the area of polar equation.*

Teacher Preparation and Notes

- *To graph a polar* equation, press [MODE] and set the **Graph** option to **Polar**.
- *Polar equations are a BC topic. AB teachers may enjoy using this activity after the AP* exam or using with students in your AB class who want to prepare for the BC exam. After completing the activity, students should be more successful with AP questions like multiple-choice 98BC19, 73BC40, and free response 05BC2, 03BC3, 93BC4, 90BC4, 84BC5.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "12558" in the keyword search box.***

Associated Materials

- *PolaNecessities_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Polar Functions (TI-89 Titanium) — 3195*
- *Cardioid Patterns – Discover Using Graphs (TI-Nspire technology) — 10149*

Part 1 – Plotting Coordinates & Exploring Polar Graphs

Students begin the activity by plotting points on a polar graph. This should be a refresher of polar coordinates for most students. Students practice using the calculator to graph a polar equation.

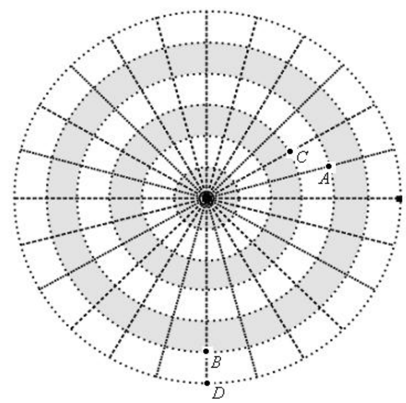
<u>*Discussion Questions*</u>

- *What do you think it means to have a negative angle, like* $\left(-\frac{\pi}{3}, 3\right)$*?*
- *What about if r was negative? For example, move to* $\left(\frac{\pi}{2}, -6\right)$*.*

Student Solutions

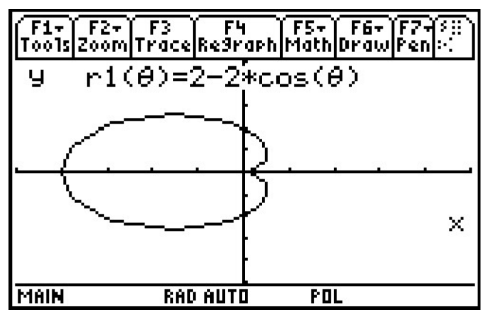
1. See image at right

2. If $r(\theta) = \cos(\theta)$, $r\left(\frac{\pi}{3}\right) = 0.5$.



3. a heart or cardiod

4. A circle is in the form $r = a$, where a is a constant.
A polar rose with even petals is in the form $r(\theta) = a \cdot \sin(n\theta)$, where n is even.
A polar rose with odd petals is in the form $r(\theta) = a \cdot \sin(n\theta)$, where n is odd.
A limaçon with an inner loop comes form $r(\theta) = b + a \cdot \cos(\theta)$, where $b < a$.

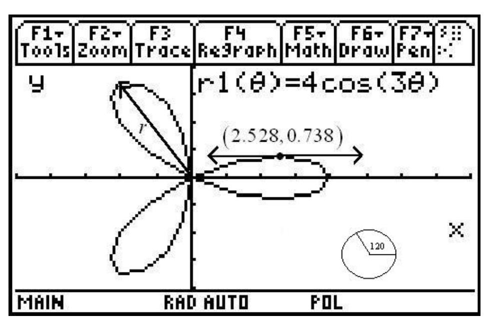


Part 2 – Slopes of Polar Graphs

Students should recall what they know about the slope of a line and find the locations of the vertical and horizontal tangents. Problem 5 introduces the topic of slope. It will be of value to have students consider the meaning of $\frac{dr}{d\theta}$.

<u>*Discussion Questions*</u>

- *What does is mean if* $\frac{dr}{d\theta}$ *is negative?*
- *Using the graph of* **r1** *to the right, where on the polar rose is* $\frac{dr}{d\theta}$ *negative?*



Problems 6 and 7 lead students to see that the slope of the tangent is

$$\frac{dy}{dx} = \frac{\dfrac{d\left(r \cdot \sin(\theta)\right)}{d\theta}}{\dfrac{d\left(r \cdot \cos(\theta)\right)}{d\theta}}.$$

Student Solutions

5. $\dfrac{dy}{dx}$

6. $x = r \cdot \cos(\theta)$ and $y = r \cdot \sin(\theta)$

7. a. $\dfrac{dy}{d\theta} = 0,\ \dfrac{dx}{d\theta} \neq 0$

b. There are four horizontal tangents.

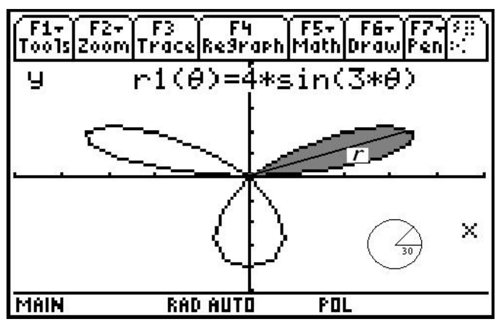
c. $\tan^{-1}\left(\dfrac{0.738}{2.528}\right) \approx 0.284$

d. **solve(*d*(r1(θ)*sin(θ),θ)=0,θ)|0<θ<π** gives $\theta = 0.284$ or $\theta = 1.103$ or $\theta = 2.039$ or $\theta = 2.858$
solve(*d*(r1(θ)*cos(θ),θ)=0,θ)|0<θ<π<π gives $\theta = 0.912$ or $\theta = 1.571$ or $\theta = 2.229$ or $\theta = 3.14159$

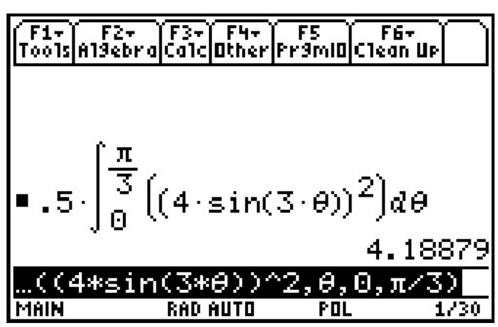
8. $\left.\dfrac{d(\mathbf{r1}(\theta)\cdot\sin(\theta))}{d\theta}\right|_{\theta=\frac{2\pi}{3}} = -2$, $\left.\dfrac{d(\mathbf{r1}(\theta)\cdot\cos(\theta))}{d\theta}\right|_{\theta=\frac{2\pi}{3}} = -2\sqrt{3}$, $\dfrac{dy}{dx} = \dfrac{\sqrt{3}}{3} \approx 0.557$

Part 3 – Area of Polar Graphs

In this part of the activity, the formula for the area of a polar curve is given and students will use this to find the area of the shaded region. One of the most difficult parts of this topic is finding the limits of integration.



Students should know the polar area formula for exams. If students are asked to evaluate the integral, it is likely that it will be calculator active. Non-calculator questions will have them get the set up correct.



If time allows, explain how this formula is derived from triangles instead of rectangles, like Riemann sums.

Student Solutions

9. 0 to $\dfrac{\pi}{3}$

10. 4.189