Let's Count the Ways

| **Overview – Activity ID: 8938**	**Math Concepts**	**Materials**
Students will be introduced to the different ways to calculate numbers of outcomes, including using the counting principle. They will also evaluate expressions using permutations of data without repeat elements, both manually and on the TI-34 MultiView Calculator.	• counting principle • fractions • factorials • permutations	• TI-34 MultiView™ • pencil • paper

Activity

Give the students an example of when they might need the multiplication counting principal.

You are trying to decide what you want for lunch in the cafeteria today, and you feel overwhelmed by the options. You decide upon a sandwich, but still, there are many choices. You can select wheat or white bread, then ham, turkey, or roast beef. How many choices do you really have if you're allowed one type of meat?

Most students will create sketches of some kind to come up with their answers. Here is the beginning of one example of what students might attempt. This is called a "tree diagram."
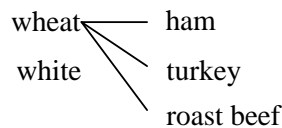
You can count the possibilities using a diagram. On wheat, you can choose ham, turkey, or roast beef. So there are three possible sandwiches on wheat bread. There are also three possible sandwiches on white bread since white can be paired with ham, turkey, or roast beef, also. We can see there are six total possibilities.

Now, what if we had the option of four different types of bread and five types of meat? If offered white, wheat, rye, or multigrain bread, then ham, turkey, roast beef, chicken, or corned beef, how many sandwich possibilities would there be (still allowing one type of meat per sandwich)? A diagram similar to the one we just used is still possible, but the more options we have, the more complicated the diagram becomes.

Introduce the counting principle.

When determining the number of possible outcomes, multiply the number of ways each separate possibility can occur.

For instance, in the initial example, there were two types of bread and three types of meat. Multiply 2 • 3 to find that there are six possible sandwich combinations. In the next example, there were four types of bread and five types of meat. Multiply 4 • 5 to see that there are 20 possible combinations.

In both previous examples, we were simply counting numbers of possibilities. Next, introduce the concept of permutations before giving a formal definition.

In both sandwich problems, we were simply counting possibilities. There was no need to consider order. However, sometimes order is important. For instance, consider this example: A track coach is trying to decide the order of his sprinters for the 4 × 100-meter relay. How many possibilities are there?

Discuss the relevance of order. Even though each runner runs one leg, the order "Josh, Jake, Lucas, Cody" is different from "Josh, Jake, Cody, Lucas," etc.

There are four choices for the runners of the first leg; then for the second leg, there are three options remaining since one runner is already chosen to run the first leg. Continuing, there are two options for the third leg, then one runner left for the last leg.

(4 runners) • (3 runners) • (2 runners) • (1 runner) = 24

This is a "factorial." The factorial n! is defined as the product of all positive integers in descending order from n to 1. For example, 4! = 4 • 3 • 2 • 1.

Show how to use the calculator to calculate a factorial. Now, move on to examples in which not all objects are used.

The above example is also called a "permutation." Any particular arrangement of the elements of a set is a permutation. But not all permutations use all the objects or elements. What if the track coach in the previous example were trying to decide which sprinters should run the 4 × 100-meter relay, but instead of having just four runners, he had nine possible runners to fill those four spots?

Start again with the counting principle.

If only three of the boys were good starters, the first leg could be run by any of those three runners. Then the second leg could be filled by any of the eight remaining runners. If we continue this pattern, we will find that
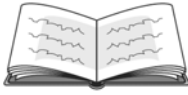*(3 runners) • (8 runners) • (7 runners) • (6 runners)*
= 1,008.

Follow these steps:

1. Press 4.

2. Press K ' ' to access the factorial command.

3. The calculator screen should display this:

   ```
                        DEG
   PRB  RAND
   1:nPr
   2:nCr
   3:!
   ```
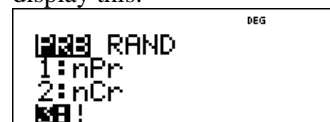
4. Press ?   ?   .

Discuss how this is completely different from 9!, as 9! would be $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, and the above is $3 \cdot 8 \cdot 7 \cdot 6$.

We define the number of permutations of r objects taken from a set of n objects as follows:

$$_nP_r = \frac{n!}{(n-r)!}$$

In the example of nine runners and four possible slots to fill, n = 9 and r = 4. Therefore, we could say this:

$$_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

Show the students on a chalkboard or overhead projector how to simplify this fraction first, then multiply what remains. The answer will be 3,024, just as before.

Now that you've seen how large the numbers can get and how many factors there are, you can see why using a calculator can simplify the process. Let's use the TI-34 MultiView.

Do one more example with the students, again stressing that order is important and reminding them we are strictly dealing with problems that have no repeat elements.

Let's try one more example. How many four-letter arrangements can be made from the letters in the word "movies"? The four-letter arrangements do not need to be words, so "movs" qualifies as a legitimate arrangement.

We have seven letters and want arrangements of four letters. So, for the first letter there are seven choices, for the second letter there are six choices, etc. Using our formula for permutations, we find this:
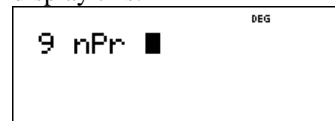
$$_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$$

We could use the counting principal: $7 \cdot 6 \cdot 5 \cdot 4$,

or on the calculator, we can more easily find $_7P_4 = 840$.

Follow these steps:

1. Press 9.
2. Press K ? to access the permutations command.
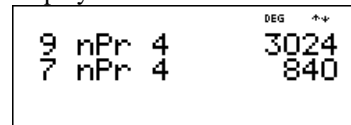3. The calculator screen should display this:

   ```
                          DEG
    9 nPr ■
   ```

4. Press 4 ? .

Follow these steps:

1. Press & & ? to access and copy the previous entry.
2. Press $ $ $ 7 ? .
3. The calculator screen should display this:

   ```
                      DEG  ↑↓
    9 nPr 4        3024
    7 nPr 4         840
   ```

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Directions: Use the TI-34 MultiView™ to evaluate expressions using permutations and factorials.

For problems 1-6, evaluate or simplify. To demonstrate understanding, you must first write out the factors. Then you may use your calculator. Problem 1 has been started for you to demonstrate.

1. $8!$

   $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$

2. $4! + 5!$

3. $7!$

4. $3(6!)$

5. $\dfrac{9!}{3!}$

6. $\dfrac{10!}{5!}$

For problems 7 and 8, determine the number of permutations in each given situation. Notice that in both situations, repeats are not allowed.

7. The student council is in the process of electing officers. How many ways can a president, vice-president, secretary, and treasurer be selected if there are 19 members in the group?

8. You would like a cone with two scoops of sherbet and are wondering how many different ways there are to use two *different* flavors to create your own distinct taste. The possibilities are cherry, grape, lime, watermelon, orange, blueberry, and pineapple. A scoop of blueberry on the bottom, then a scoop of orange on top tastes different than if the scoop of orange is on the bottom. Find the number of two-flavor possibilities.

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In problems 9-11, use any method you prefer to find the number of different arrangements in each given situation.

9. It's time for you to set your class schedule for next fall. If you need to take English, French, U.S. history, Art II, geometry, and weight lifting and you have a six-period day, in how many different orders could you take your classes?

10. Each locker in your school has a 10-digit lock with the numbers 1 through 10 each appearing once. How many different 3-digit arrangements are possible if no single number is to be used twice on the same locker?

11. You are about to move into your first home! You have narrowed down your color scheme, but you're having trouble deciding which colors to use and want to know how many options you're considering. Your carpet options are tan or white; your paint colors are warm muffin, Aztec brick, almond brown, and stone brown; and you also need to decide between chrome or brass hardware. How many total possibilities are you considering?

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ANSWER KEY

1. 8!

   $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$

2. 4! + 5!

   $4 \cdot 3 \cdot 2 \cdot 1 + 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 24 + 120 = 144$

3. 7!

   $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5{,}040$

4. 3(6!)

   $3(6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 2{,}160$

5. $\dfrac{9!}{3!} = \dfrac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 60{,}480$

6. $\dfrac{10!}{5!} = \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 30{,}240$

7. The student council is in the process of electing officers. How many ways can a president, vice-president, secretary, and treasurer be selected if there are 19 members in the group?

   $_{19}P_4 = \dfrac{19!}{(19-4)!} = 93{,}024$

8. You would like a cone with two scoops of sherbet and are wondering how many different ways there are to use two *different* flavors to create your own distinct taste. The possibilities are cherry, grape, lime, watermelon, orange, blueberry, and pineapple. A scoop of blueberry on the bottom, then a scoop of orange on top tastes different than if the scoop of orange is on the bottom. Find the number of two-flavor possibilities.

   $_7P_2 = \dfrac{7!}{(7-2)!} = 42$

9. It's time for you to set your class schedule for next fall. If you need to take English, French, U.S. history, Art II, geometry, and weight lifting and you have a six-period day, in how many different orders could you take your classes?

   $6! = 720$

10. Each locker in your school has a 10-digit lock with the numbers 1 through 10 each appearing once. How many different 3-digit arrangements are possible if no single number is to be used twice on the same locker?

    $_{10}P_3 = 720$

11. You are about to move into your first home! You have narrowed down your color scheme, but you're having trouble deciding which colors to use and want to know how many options you're considering. Your carpet options are tan or white; your paint colors are warm muffin, Aztec brick, almond brown, and stone brown; and you also need to decide between chrome or brass hardware. How many total possibilities are you considering?

    $2 \cdot 4 \cdot 2 = 16$