

Exploring Rational Functions

ID: 8968

Time required
45 minutes

Activity Overview

Students investigate the graphs of functions of the form $y = \frac{1}{x - a}$. They will discover that the graph of such a function has a vertical asymptote at $x = a$, and a horizontal asymptote at $y = 0$. They will investigate the graphic and numeric consequences of such asymptotic behavior by observing a trace point on a graph as it moves in response to various inputs for the independent variable.

Topic: Rational & Radical Functions

- Evaluate a rational function for any real value of the variable
- Graph any rational function to verify its domain, range, singularities and asymptotes.

Teacher Preparation & Notes

This investigation could be used as an introduction to graphs of rational functions. The phrases vertical and horizontal asymptotes are introduced in the activity, but the emphasis here is the connections among graphic, numeric, and symbolic representations of rational functions.

- This activity could be used in Algebra 2 or Precalculus as an introduction to rational functions.
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- **To download the student TI-Nspire document (.tns file), go to education.ti.com/exchange and enter “8968” in the keyword search box.**

Associated Materials

- [ExploringRadicalFunctions_Student.doc](#)
- [ExploringRadicalFunctions.tns](#)

Suggested Related Activities

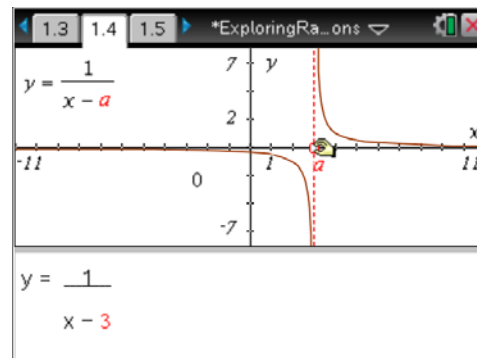
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *The Remainder Theorem Using TI-Nspire CAS (TI-Nspire CAS technology)* — 9254
- *Multiplication & Division of Functions (TI-Nspire technology)* — 10218
- *Polynomial Division (TI-89 Titanium)* — 5505

Problem 1 – Graphing $y = \frac{1}{x - a}$ for various values of a

On page 1.4, students adjust the value of a by dragging a point on the x -axis. They observe how the “jump” in the graph moves along with the point on the x -axis.

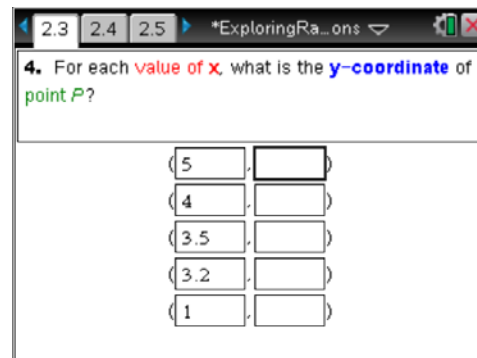
Students should see that $y = \frac{1}{x - a}$ becomes undefined when $x = a$ (since the denominator is 0) and it is at this value of x that the “break,” or vertical asymptote, occurs.



TI-Nspire Navigator Opportunity: Quick Poll
See Note 1 at the end of this lesson.

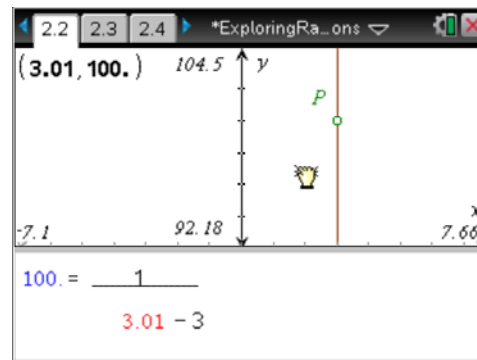
Problem 2 – Behavior Near the Vertical Asymptote

On page 2.2, students investigate the behavior of $y = \frac{1}{x - 3}$ for values of x that get successively closer to 3 from either side. They observe a “trace point” P on the graph. Students will enter the x -coordinates listed on page 2.3 as the x -coordinate of the point on page 2.2. Students must click on the x -coordinate twice, type the new coordinate, and press **enter**. The y -coordinate will update automatically. The idea is to connect the graph, the symbolic form of the function, and the numerical calculations of the outputs. Students may need to be prompted to be attentive to the calculation displayed in the 2nd quadrant on the graph screen.



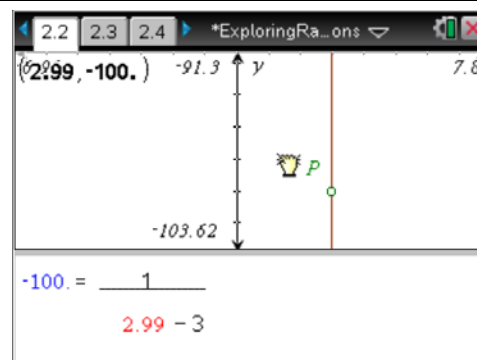
TI-Nspire Navigator Opportunity: Live Presenter
See Note 2 at the end of this lesson.

In Question 5, students enter a value for x that sends the trace point off the screen. They are to drag the coordinate plane down to find where the point went. To do this, move the cursor to an empty space near the top of the screen and press and hold the **CLICK** key (⌨) until a hand appears to “grab” the plane. Then press down on the NavPad to drag the screen down. Press **[esc]** or the click key, and repeat the process to drag the plane down again. It may take 10 or more repetitions to bring point P into view. If students need to know where they are on the graph, press **MENU > Window > Window Settings**.



TI-Nspire Navigator Opportunity: Live Presenter
See Note 3 at the end of this lesson.

After entering 2.99 for the x -coordinate, students should return to the standard window (⌨ > **Window > Zoom – Standard**), then drag the plane in the opposite direction as before. Again, it may take several repetitions to bring point P into view.



While this may seem tedious at first glance, there is some merit in students seeing that the graph is nearly vertical and watching as point P eventually comes into view. They should gain a feel for the behavior of the graph of a rational function near its vertical asymptote.

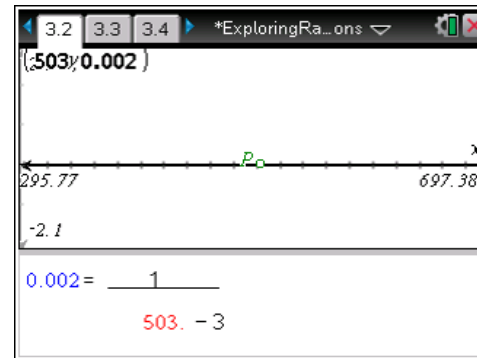
In Questions 7 and 8, students should notice that they can make the magnitude of the y -coordinate of point P arbitrarily large.

Problem 3 – Horizontal Asymptote

In Problem 3, students investigate how a trace point changes when values of x get progressively larger and progressively smaller. Again, a connection should be made among the three representations: the graph flattening out, the symbolic form, and the numeric calculation of the output. They should notice the absolute value of the denominator getting very large, and so the outputs from $y = \frac{1}{x - 3}$ approach 0.

5	0.5
13	0.1
23	0.05
103	0.01
1	-0.5

After entering several values for x and watching the point and the calculated output adjust as before, students record their results and then enter a large value for x that moves the point off screen again—this time far to the right. Again, they can grab the plane and pull it to the left until the point comes back into view. Then they repeat the maneuver with a negative input that is large in magnitude.



Finally, students observe that they could make point P get as close as they want to the x -axis (that is, make the y -coordinate of P as close to zero as desired) by making x sufficiently large in magnitude. This is the essence of a horizontal asymptote.

Students should see that the horizontal asymptote occurs at $y = 0$.

TI-Nspire Navigator Opportunities

Note 1

Problem 1: Quick Poll

Use Quick Poll to send Questions 1 to 3 to students so student responses can be collected and checked for understanding.

Note 2

Problem 2: Live Presenter

Use *Live Presenter* to demonstrate how to enter x -values on page 2.2 as well as aide in the discussion with students as they work through questions 4-9.

Note 2

Problem 2: Live Presenter

Use *Live Presenter* with Question 5 to belabor the fact that point P is a LONG way from the x -axis.